IIT-JAM 2018 (Solutions)

Q1. – Q10. carry one mark each.

Q1. Let $f(x,y)=x^3-2y^3$. The curve along which $\nabla^2 f=0$ is

(a) $x=\sqrt{2}y$ (b) $x=2y$

(c) $x=\sqrt{6}y$ (d) $x=\frac{-y}{2}$

Ans.: (b)

Solution: $\nabla^2 f=\frac{\partial^2}{\partial x^2}(x^3-2y^3)+\frac{\partial^2}{\partial y^2}(x^3-2y^3)+0$

$\nabla^2 f=6x-12y$

$\because \nabla^2 f=0 \Rightarrow 6x-12y=0 \Rightarrow x=2y$

Q2. A curve is given by $\vec{r}(t)=t\hat{i}+t^2\hat{j}+t^3\hat{k}$. The unit vector of the tangent to the curve at $t=1$ is

(a) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (b) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i}+2\hat{j}+2\hat{k}}{3}$ (d) $\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$

Ans.: (d)

Solution: Let $\hat{n}$ be a unit vector tangent to the curve at t.

$$\hat{n}=\frac{d\vec{r}/dt}{|d\vec{r}/dt|}=\frac{\hat{i}+2t\hat{j}+3t\hat{k}}{\sqrt{1+4t^2+5t^2}} \Rightarrow \text{at } t=1, \hat{n}=\frac{\hat{i}+2j+3\hat{k}}{\sqrt{14}}$$

Q3. There are three planets in circular orbits around a star at distances $a, 4a$ and $9a$, respectively. At time $t=t_0$, the star and the three planets are in a straight line. The period of revolution of the closest planet is T. How long after t_0 will they again be in the same straight line?

(a) $8T$ (b) $27T$ (c) $216T$ (d) $512T$

Ans. : (c)

Solution: $T_1=ka^{3/2}=T$, $T_2=k(4a)^{3/2}=8T$, $T_3=k(9a)^{3/2}=27T$

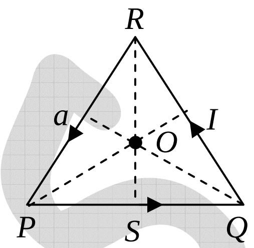
Common time that all three star will meet again is $t_0=T_1\times T_2\times T_3=216T$, which is LCM of all time period.

Q4. A current I is flowing through the sides of an equilateral triangle of side a. The magnitude of the magnetic field at the centroid of the triangle is

(a) $\frac{9\mu_0 I}{2\pi a}$ (b) $\frac{\mu_0 I}{\pi a}$ (c) $\frac{3\mu_0 I}{2\pi a}$ (d) $\frac{3\mu_0 I}{\pi a}$

Ans.: (a)

Solution: $RS = \sqrt{a^2 - a^2/4} = \frac{\sqrt{3}}{2}a$ and $OS = \frac{RS}{3} = \frac{\sqrt{3}}{6}a$

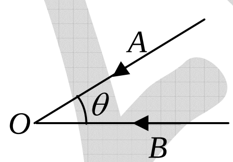


For segment PQ

$$B_{PQ} = \frac{\mu_0 I}{4\pi\left(\frac{\sqrt{3}}{6}a\right)} \times 2\sin 60^0 = \frac{3\mu_0 I}{2\pi a} = B_{QR} = B_{RP}$$

$$B = 3B_{PQ} = \frac{9\mu_0 I}{2\pi a}$$

Q5. Two vehicles A and B are approaching an observer O at rest with equal speed as shown in the figure. Both vehicles have identical sirens blowing at a frequency f_s. The observer hears these sirens at frequency f_A and f_B, respectively from the two vehicles. Which one of the following is correct?



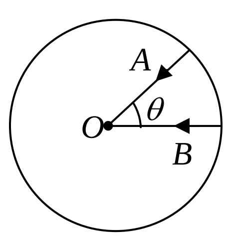
(a) $f_A = f_B < f_s$ (b) $f_A = f_B > f_s$

(c) $f_A > f_B > f_s$ (d) $f_A < f_B < f_s$

Ans.: (b)

Solution: Doppler shift

$$f_A = f_s\left(\frac{v_s}{v_s - v_A}\right), \qquad f_B = f_s\left(\frac{v_s}{v_s - v_B}\right)$$



$$V_A = V_B < V_s$$

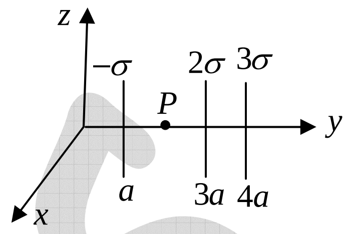
$$\therefore f_A = f_B > f_s$$

Q6. Three infinite plane sheets carrying uniform charge densities $-\sigma, 2\sigma, 3\sigma$ are parallel to the $x-z$ plane at $y = a, 3a, 4a$, respectively. The electric field at the point $(0, 2a, 0)$ is

(a) $\frac{4\sigma}{\varepsilon_0}\hat{j}$ (b) $-\frac{3\sigma}{\varepsilon_0}\hat{j}$ (c) $-\frac{2\sigma}{\varepsilon_0}\hat{j}$ (d) $\frac{\sigma}{\varepsilon_0}\hat{j}$

Ans.: (b)

Solution: The electric field at the point $P(0, 2a, 0)$ is



$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0}\right)\left(-\hat{j}\right) = -\frac{3\sigma}{\varepsilon_0}\hat{j}$$

Q7. Two boxes A and B contain an equal number of molecules of the same gas. If the volumes are V_A and V_B and λ_A and λ_B denote respective mean free paths, then

(a) $\lambda_A = \lambda_B$ (b) $\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$ (c) $\frac{\lambda_A}{V_A^{1/2}} = \frac{\lambda_B}{V_B^{1/2}}$ (d) $\lambda_A V_A = \lambda_B V_B$

Ans. : (b)

Solution: $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P} = \frac{1}{\sqrt{2}\pi d^2 n} = \frac{V}{\sqrt{2}\pi d^2 N} \Rightarrow \lambda \propto V$, where $n = \frac{N}{V}$

$$\lambda_A = KV_A,\ \lambda_B = KV_B$$

$$\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$$

Q8. Let T_g and T_e be the kinetic energies of the electron in the ground and the third excited states of a hydrogen atom, respectively. According to the Bohr model, the ratio $\frac{T_g}{T_e}$ is

(a) 3 (b) 4 (c) 9 (d) 16

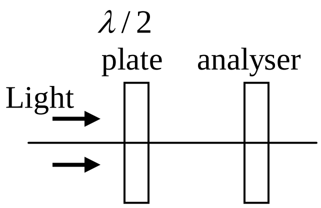
Ans. : (d)

Solution: From Bohr model the kinetic energy and Total energy $\langle E\rangle$ and kinetic energy $\langle T\rangle$

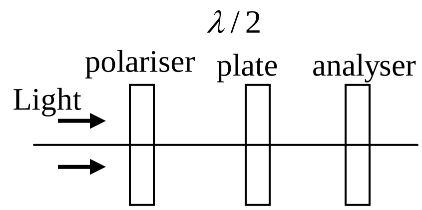
$$\langle T\rangle = -\frac{\langle E\rangle}{2} \text{ where } E_g = \frac{E_0}{1},\ E_e = \frac{E_0}{16} \Rightarrow \frac{T_g}{T_e} = \frac{E_g}{F_e} = \frac{16}{1} = 16:1$$

Q9. Which of the following arrangements of optical components can be used to distinguish between an unpolarised light and a circularly polarised light?

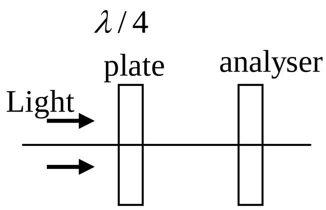
(a)



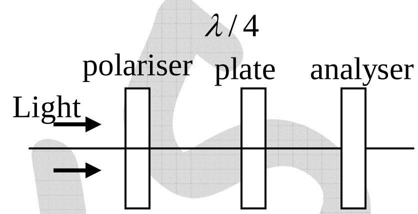
(b)



(c)



(d)



Ans.: (c)

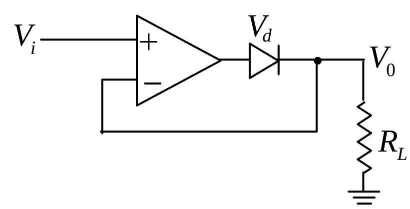
Solution: (i) In configuration (A), output will be linearly polarized for both

(ii) In configuration (B), output will be linearly polarized for both

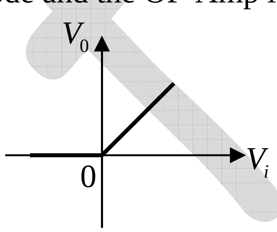
(iii) In configuration (C), output will be linearly polarized of constant intensity if input is unpolarised whereas it is linearly polarized with intensity varying from zero to maximum if input is circularly polarized.

(iv) In configuration (D) output will be linearly polarized for both.

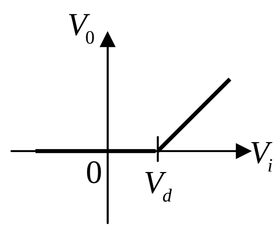
Q10. Which one of the following graphs shows the correct variation of V_0 with V_i? Here, V_d is the voltage drop across the diode and the OP-Amp is assumed to be ideal.



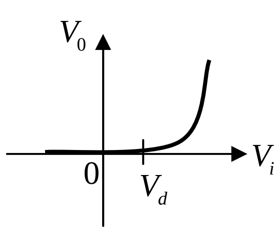
(a)



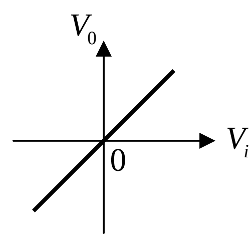
(b)



(c)



(d)

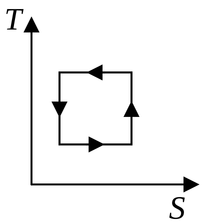


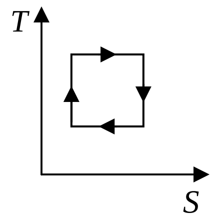
Ans. : (a)

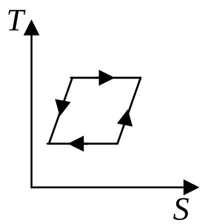
Solution: During positive half cycle it behaves as voltage follower i.e. $v_0 = v_i$, during negative half cycle $v_0 = 0$.

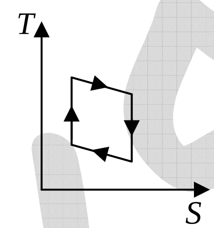
Q11. – Q30. carry two marks each.

Q11. Which one of the figures correctly represents the $T-S$ diagram of a Carnot engine?

(a)


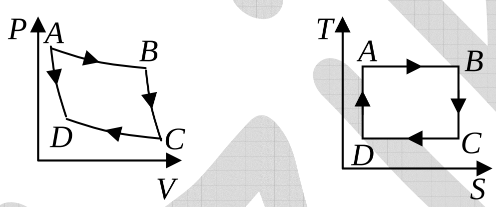
(b)


(c)


(d)


Ans. : (b)

Solution:



$A \to B$ Isothermal expansion

$B \to C$ Adiabatic expansion

$C \to D$ Isothermal compression

$D \to A$ Adiabatic compression

Q12. The plane of polarisation of a plane polarized light rotates by 60^0 after passing through a wave plate. The pass-axis of the wave plate is at an angle α with respect to the plane of polarization of the incident light. The wave plate and α are

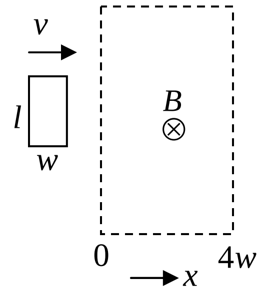
(a) $\frac{\lambda}{4}, 60^0$ (b) $\frac{\lambda}{2}, 30^0$ (c) $\frac{\lambda}{2}, 120^0$ (d) $\frac{\lambda}{4}, 30^0$

Ans.: (b)

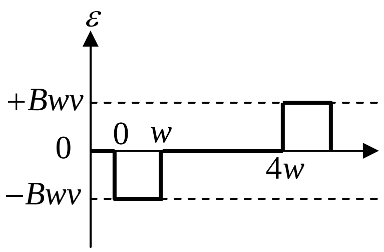
Solution: When plane polarized light is incident on the $\pi/4$ plate, it converts it into circularly polarized light, whereas $\pi/2$ plate rotates is by angle 2α, where α is angle between fast axis and polarization direction.

Given, $2\alpha = 60^0 \Rightarrow \alpha = 30^0$.

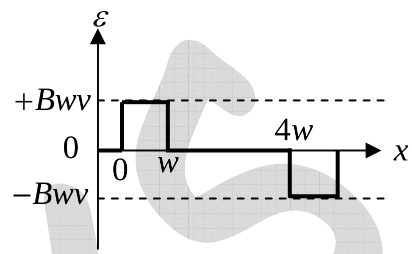
Q13. A rectangular loop of dimensions l and w moves with a constant speed of v through a region containing a uniform magnetic field B directed into the paper and extending a distance of $4w$. Which of the following figures correctly represents the variation of emf (ε) with the position (x) of the front end of the loop?



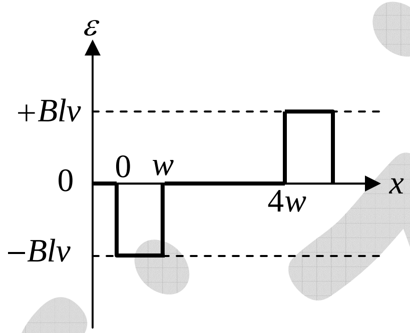
(a)



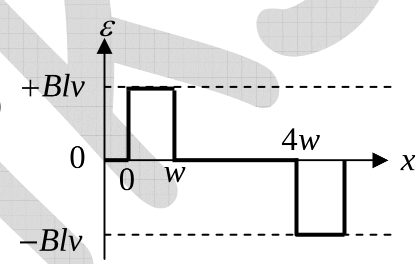
(b)



(c)

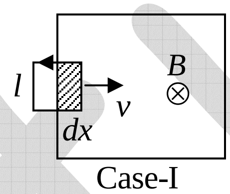


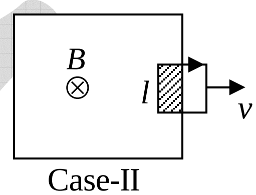
(d)



Ans.: (c)

Solution:





Case-I: at $x = 0, \phi_1 = Blw$ and at, $x = dx, \phi_2 = Bl(w - dx)$

$$\Rightarrow \Delta\phi = Bldx \Rightarrow \varepsilon = -\frac{d\phi}{dt} = Blv$$

Case-II: $|\varepsilon| = Blv$ and direction will be opposite.

When loop is inside there is no flux change so, $\varepsilon = 0$.

Q14. The equation of state for one mole of a non-ideal gas is given by $PV = A\left(1+\frac{B}{V}\right)$, where the coefficient A and B are temperature dependent. If the volume changes from V_1 to V_2 in an isothermal process, the work done by the gas is

(a) $AB\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$

(b) $AB\ln\left(\frac{V_2}{V_1}\right)$

(c) $A\ln\left(\frac{V_2}{V_1}\right)+AB\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$

(d) $A\ln\left(\frac{V_2-V_1}{V_1}\right)+B$

Ans. : (c)

Solution: $PV = A\left(1+\frac{B}{V}\right)$

$$P=\left(\frac{A}{V}+\frac{AB}{V^2}\right)$$

$$W = PdV = \int_{V_1}^{V_2}\frac{A}{V}dV+\int_{V_1}^{V_2}\frac{AB}{V^2}dV$$

$$= A\ln\frac{V_2}{V_1}+AB\cdot\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$$

Q15. An ideal gas consists of three dimensional polyatomic molecules. The temperature is such that only one vibrational mode is excited. If R denotes the gas constant, then the specific heat at constant volume of one mole of the gas at this temperature is

(a) $3R$ (b) $\frac{7}{2}R$ (c) $4R$ (d) $\frac{9}{2}R$

Ans.: (c)

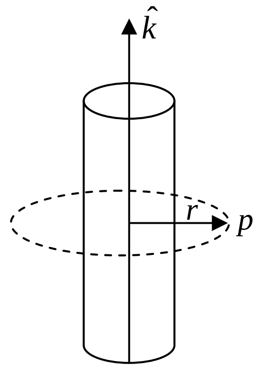
Solution: For a polyatomic gas

$$C_P = (4+f)R$$

$$C_V = (3+f)R$$

As, $f = 1, C_V = 4R$

Q16. A long solenoid is carrying a time dependent current such that the magnetic field inside has the form $\vec{B}(t) = B_0 t^2 \hat{k}$, where $\hat{k}$ is along the axis of the solenoid. The displacement current at the point P on a circle of radius r in a plane perpendicular to the axis



(a) is inversely proportional to r and radially outward

(b) is inversely proportional to r and tangential

(c) increases linearly with time and is tangential.

(d) is inversely proportional to r^2 and tangential

Ans.: (b)

Solution: $\because \oint \vec{E}\cdot d\vec{l} = -\int \frac{d\vec{B}}{dt}\cdot d\vec{l}$

$$\Rightarrow E\times 2\pi r = -2B_0 t\times \pi R^2 \Rightarrow E = \frac{-B_0 t R^2}{r}$$

$$\because J_d = \varepsilon_0 \frac{\partial E}{\partial t} \Rightarrow J_d = \frac{-\varepsilon_0 B_0 R^2}{r} \Rightarrow J_d \propto \frac{1}{r}$$

Q17. Consider an ensemble of thermodynamic systems each of which is characterized by the same number of particles, pressure and temperature. The thermodynamic function describing the ensemble is

(a) Enthalpy (b) Helmholtz free energy

(c) Gibbs free energy (d) Entropy

Ans. : (c)

Solution: The variable

$G = H - TS$

$dG = dH - TdS - SdT$

$= TdS + VdP - TdS - SdT$

$dG = VdP - SdT$

Q18. Given a spherically symmetric charge density $\rho(r) = \begin{cases} kr^2, & r < R \\ 0, & r > R \end{cases}$ (k being a constant), the electric field for $r < R$ is (take the total charge as Q)

(a) $\frac{Qr^3}{4\pi\varepsilon_0 R^5}\hat{r}$ (b) $\frac{3Qr^2}{4\pi\varepsilon_0 R^4}\hat{r}$ (c) $\frac{5Qr^3}{8\pi\varepsilon_0 R^5}\hat{r}$ (d) $\frac{Q}{4\pi\varepsilon_0 R^5}\hat{r}$

Ans.: (a)

Solution: $\because \oint_S \vec{E}.d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow |\vec{E}| \times 4\pi r^2 = \frac{1}{\varepsilon_0}\left(\int_0^r kr^2 \times 4\pi r^2 dr\right)$

$$\Rightarrow |\vec{E}| \times 4\pi r^2 = \frac{1}{E_0} \times 4\pi k \frac{r^5}{5} \Rightarrow |\vec{E}| = \frac{kr^3}{5\varepsilon_0}$$

$$\because Q = \int_0^R kr^2 \times 4\pi r^2 dr = 4\pi k \frac{R^5}{5} \Rightarrow k = \frac{5Q}{4\pi R^5}$$

$$\Rightarrow |\vec{E}| = \frac{5Q}{4\pi R^5} \times \frac{r^3}{5\varepsilon_0} = \frac{Qr^3}{4\pi\varepsilon_0 R^5}$$

Q19. An infinitely long solenoid, with its axis along $\hat{k}$, carries a current I. In addition there is a uniform line charge density λ along thee axis. If $\vec{S}$ is the energy flux, in cylindrical coordinates $(\hat{\rho}, \hat{\phi}, \hat{k})$, then

(a) $\vec{S}$ is along $\hat{\rho}$

(b) $\vec{S}$ is along $\hat{k}$

(c) $\vec{S}$ has non zero components along $\hat{\rho}$ and $\hat{k}$

(d) $\vec{S}$ is along $\hat{\rho} \times \hat{k}$

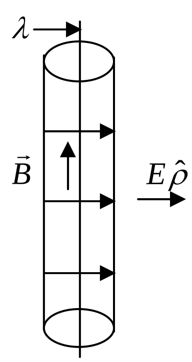
Ans. : (d)

Solution: $\vec{E} = E\hat{\rho}$

$\vec{B} = B\hat{k}$

$\vec{S} \propto \vec{E} \times \vec{B}$

$\vec{S} \propto \hat{\rho} \times \hat{k}$



Q20. Consider two waves $y_1 = a\cos(\omega t - kz)$ and $y_2 = a\cos[(\omega+\Delta\omega)t - (k+\Delta k)z]$. The group velocity of the superposed wave will be ($\Delta\omega \ll \omega$ and $\Delta k \ll k$)

(a) $\frac{(\omega-\Delta\omega)}{(k-\Delta k)}$ (b) $\frac{(2\omega-\Delta\omega)}{(2k+\Delta k)}$ (c) $\frac{\Delta\omega}{\Delta k}$ (d) $\frac{(\omega+\Delta\omega)}{(k+\Delta k)}$

Ans. : (c)

Solution: $y_1 = a\cos(\omega t - kz)$, $y_2 = a\cos[(\omega+\Delta\omega)t - (k+\Delta k)z]$

$$\Rightarrow y = y_1 - y_2 = 2a\cos\left[\frac{\Delta\omega t - \Delta kz}{2}\right] \times \cos\left[\frac{2\omega+\Delta\omega}{2}t - \frac{2k+\Delta k}{2}z\right]$$

$$v_g = \frac{\Delta\omega/2}{\Delta k/2} = \frac{\Delta\omega}{\Delta k}$$

Q21. Consider a convex lens of focal length f. A point object moves towards the lens along its axis between $2f$ and f. If the speed of the object is V_0, then its image would move with speed V_I. Which of the following is correct?

(a) $V_I = V_o$; the image moves away from the lens.

(b) $V_I = -V_o$; the image moves away from the lens.

(c) $V_I > V_o$; the image moves away from the lens.

(d) $V_I < V_o$; the image moves away from the lens.

Ans. : (c)

Solution: $V_I = \left(\frac{f}{f-u}\right)^2 V_0$ and $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For, $u > f$, $v_i > V_0$ and u decreases than v increases.

$\therefore V_I > V_0$ and image moves away from the lens.

Q22. A disc of radius R_1 having uniform surface density has a concentric hole of radius $R_2 < R_1$. If its mass is M, the principal moments of inertia are

(a) $\frac{M\left(R_1^2 - R_2^2\right)}{2}, \frac{M\left(R_1^2 - R_2^2\right)}{4}, \frac{M\left(R_1^2 - R_2^2\right)}{4}$

(b) $\frac{M\left(R_1^2 + R_2^2\right)}{2}, \frac{M\left(R_1^2 + R_2^2\right)}{4}, \frac{M\left(R_1^2 + R_2^2\right)}{4}$

(c) $\frac{M\left(R_1^2 + R_2^2\right)}{2}, \frac{M\left(R_1^2 + R_2^2\right)}{4}, \frac{M\left(R_1^2 + R_2^2\right)}{8}$

(d) $\frac{M\left(R_1^2 - R_2^2\right)}{2}, \frac{M\left(R_1^2 - R_2^2\right)}{4}, \frac{M\left(R_1^2 - R_2^2\right)}{8}$

Ans. : (b)

Solution: $I_{zz} = \int_{R_1}^{R_2} dm\, r^2 = \frac{M}{\pi\left(R_2^2 - R_1^2\right)} \int_{R_1}^{R_2} 2\pi r . r^2 dr = I_{ZZ} = \frac{M\left(R_2^2 + R_1^2\right)}{2}$

$I_{xx} + I_{yy} = I_{zz}$

By symmetry $I_{xx} = I_{yy}$. Therefore, $I_{xx} = I_{yy} = \frac{M\left(R_2^2 + R_1^2\right)}{4}$

Q23. The function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ is expanded as a Fourier series of the form $a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$. Which of the following is true?

(a) $a_0 \neq 0, b_n = 0$

(b) $a_0 \neq 0, b_n \neq 0$

(c) $a_0 = 0, b_n = 0$

(d) $a_0 = 0, b_n \neq 0$

Ans.: (b)

Solution:- $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$

$$a_0 = \frac{1}{2\pi}\left\{\int_{-\pi}^{0} x dx + \int_{0}^{\pi} -x dx\right\} = \frac{-\pi}{2} \Rightarrow a_0 \neq 0$$

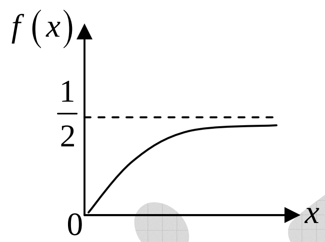
$$b_n = \frac{1}{\pi}\left\{\int_{-\pi}^{0} x\sin nx dx + \int_{0}^{\pi} -x\sin nx dx\right\}$$

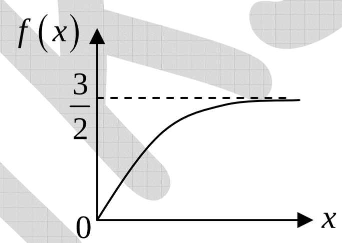
$$= \frac{1}{\pi}\left\{\left(\frac{-x\cos nx}{n} + \frac{\sin nx}{n^2}\right)\Bigg|_{-\pi}^{0} \left(\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right)\Bigg|_{0}^{\pi}\right\}$$

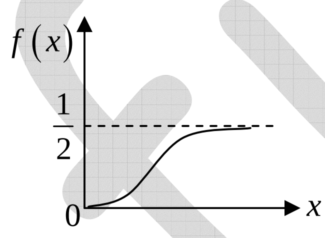
$$= \frac{1}{\pi}\left\{\frac{2\pi\cos n\pi}{n}\right\} \Rightarrow b_n = \begin{cases} \frac{2}{n}; & n = \text{even} \\ \frac{-2}{n}; & n = \text{odd} \end{cases}$$

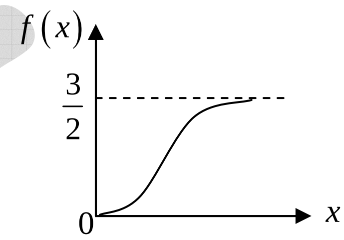
Thus, $b_n \neq 0$

Q24. Which one of the following curves correctly represents (schematically) the solution for the equation $\frac{df}{dx} + 2f = 3 : f(0) = 0$?

(a)


(b)


(c)


(d)


Ans.: (b)

Solution:- $\frac{df}{dx} + 2f = 3;\ f(0) = 0 \Rightarrow \frac{df}{3-2f} = dx \Rightarrow \frac{-1}{2}\ln|3-2f| = x + A$

Since, $f(0) = 0 \Rightarrow A = \frac{-1}{2}\ln|3|$

$$\Rightarrow x = \frac{1}{2}\ln\left|\frac{3}{3-2f}\right| \Rightarrow f = \frac{3}{2}\left(1 - e^{-2x}\right)$$

Now, we can see, at $x = 0,\ f = 0,$ at $x = \infty,\ f = \frac{3}{2}$

Thus option (b) is correct one.

Q25. The mean momentum $\vec{p}$ of a nucleon in a nucleus of mass number A and atomic number Z depends on A, Z as

(a) $\vec{p} \propto A^{\frac{1}{3}}$ (b) $\vec{p} \propto Z^{\frac{1}{3}}$ (c) $\vec{p} \propto A^{-\frac{1}{3}}$ (d) $\vec{p} \propto (AZ)^{-\frac{2}{3}}$

Ans. : (c)

Solution: The radius of a nucleus can be combined as $\frac{\lambda}{2\pi}$ (greater than the wavelength of electron)

The moment $p = \frac{h}{\lambda}$

$\lambda - R = R_0 A^{1/3}$ which implies $p \propto \frac{h}{R_0} \cdot A^{-1/3}$.

As, $p \propto A^{-1/3}$

Q26. The Boolean expression $\left(\overline{AB}\right)\left(\bar{A}+B\right)\left(A+\bar{B}\right)$ can be simplified to

(a) $A+B$ (b) $\bar{A}B$ (c) $\overline{A+B}$ (d) AB

Ans.: (c)

Solution: $Y = \left(\overline{AB}\right)\left(\bar{A}B\right)\left(A+\bar{B}\right) = \left(\overline{AB}\right)\left(\bar{A}\bar{B}+AB\right)$

$= \left(\bar{A}+\bar{B}\right)\left(\bar{A}\bar{B}+AB\right) = \bar{A}\bar{B}+\bar{A}\bar{B} = \bar{A}\bar{B} = \overline{A+B}$

Q27. Consider the transformation to a new set of coordinates (ξ, η) from rectangular Cartesian coordinates (x, y), where $\xi = 2x+3y$ and $\eta = 3x-2y$. In the (ξ, η) coordinate system, the area element $dxdy$ is

(a) $\frac{1}{13} d\xi d\eta$ (b) $\frac{2}{13} d\xi d\eta$ (c) $5 d\xi d\eta$ (d) $\frac{3}{5} d\xi d\eta$

Ans.: (a)

Solution:- $\frac{J(\xi,\eta)}{J(x,y)} = \begin{vmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} \\ \frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -13$

$\frac{J(x,y)}{J(\xi,\eta)} = \frac{-1}{13} = J$

Since, area element in $\xi - \eta$ system is, $dA = |J| d\xi d\eta = \frac{1}{13} d\xi\, d\eta$

Q28. A particle of mass m is in a one dimensional potential $V(x)=\begin{cases}0, & 0<x<L\\ \infty, & \text{otherwise}\end{cases}$.

At some instant its wave function is given by $\psi(x)=\frac{1}{\sqrt{3}}\psi_1(x)+i\sqrt{\frac{2}{3}}\psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ are the ground and the first excited states, respectively. Identify the correct statement.

(a) $\langle x\rangle=\frac{L}{2};\langle E\rangle=\frac{\hbar^2}{2m}\frac{3\pi^2}{L^2}$

(b) $\langle x\rangle=\frac{2L}{3};\langle E\rangle=\frac{\hbar^2}{2m}\frac{\pi^2}{L^2}$

(c) $\langle x\rangle=\frac{L}{2};\langle E\rangle=\frac{\hbar^2}{2m}\frac{8\pi^2}{L^2}$

(d) $\langle x\rangle=\frac{2L}{3};\langle E\rangle=\frac{\hbar^2}{2m}\frac{4\pi^2}{3L^2}$

Ans.: (a)

Solution: $\langle E\rangle=\dfrac{\frac{1}{3}\times(E_0)+\frac{2}{3}4E_0}{\frac{1}{3}+\frac{2}{3}}=\dfrac{\frac{9E_0}{3}}{\frac{3}{3}}=3E_0$ Where, $E_0=\dfrac{\pi^2\hbar^2}{2mL^2}$

$$\langle E\rangle=\frac{3\cdot\pi^2\hbar^2}{2mL^2}=\frac{3\pi^2\hbar^2}{2mL^2}$$

$$\langle X\rangle=\frac{1}{3}\langle\psi_1|X|\psi_1\rangle+\frac{2}{3}\langle\psi_2|X|\psi_1\rangle+\frac{i}{\sqrt{3}}\sqrt{\frac{2}{3}}\langle\psi_1|X|\psi_2\rangle-\frac{i}{\sqrt{3}}\sqrt{\frac{2}{3}}\langle\psi_2|X|\psi_1\rangle$$

$$\Rightarrow\frac{1}{3}\frac{L}{2}+\frac{2}{3}\frac{L}{2}=\frac{L}{2}$$

Q29. A raindrop falls under gravity and captures water molecules from atmosphere. Its mass charges at the rate $\lambda m(t)$, where λ is a positive constant and $m(t)$ is the instantaneous mass. Assume that acceleration due to gravity is constant and water molecules are at rest with respect to earth before capture. Which of the following statements is correct?

(a) The speed of the raindrop increases linearly with time

(b) The speed of the raindrop increases exponentially with time

(c) The speed of the raindrop approaches a constant value when $\lambda t\gg 1$

(d) The speed of the raindrop approaches a constant value when $\lambda t\ll 1$

Ans.: (c)

Solution: Applying impulse momentum m equation.

$$mg\times dt=(m+dm)(v+dv)-mv\Rightarrow mg=\frac{mdv}{dt}+v\frac{dm}{dt}$$

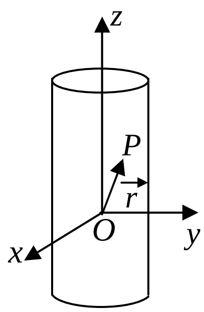
or, $g=\frac{dv}{dt}+v\frac{(dm)}{mdt}\Rightarrow\frac{dv}{dt}-g+\lambda v=0,\lambda=\frac{dm}{mdt}$

$$\Rightarrow\int_{v_0}^{v}\frac{dv}{g-\lambda v}=\int_0^t dt \text{ at } t=0, V=V_0\Rightarrow\ln(g-\lambda v)-\ln|g-\lambda v_0|=-\lambda t$$

$$\Rightarrow\frac{g-\lambda v}{g-\lambda v_0}=e^{-\lambda t}\Rightarrow V=\frac{g}{\lambda}+\left(V_0-\frac{g}{\lambda}\right)e^{-\lambda t}$$

As, $\lambda t>>1, e^{-\lambda t}\to 0$, so $V\to\frac{g}{\lambda}$ (which is constant)

Q30. A particle P of mass m is constrained to move on the surface of cylinder under a force $-k\vec{r}$ as shown in figure (k is the positive constant). Which of the following statements is correct? (Neglect friction.)



(a)Total energy of the particle is not conserved.

(b) The motion along z direction is simple harmonic.

(c) Angular momentum of the particle about O increases with time.

(d) Linear momentum of the particle is conserved.

Ans. : (b)

Solution: $\vec{F}=-k\vec{r}=-k(r\hat{r}+z\hat{z})$

$F_r=-kr$

$F_\theta=0$

$F_z=-kz\Rightarrow m\ddot{z}=-kz$ the motion along z is simple harmonic motion.

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q31. Let matrix $M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$. If $\det(M) = 0$, then

(a) M is symmetric

(b) M is invertible

(c) one eigenvalue is 13

(d) Its eigenvectors are orthogonal

Ans.: (a), (c), (d)

Solution:- Since, $M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$,

If $|M| = 0 \Rightarrow 36 - 6x = 0 \Rightarrow x = 6$

Hence, $M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$

(a) Here, $M = M^+$, so it is symmetric matrix

(b) Determinant $(M) = 0$, so noninvertible matrix

(c) For eigenvalue-

$$M - \lambda I = 0 \Rightarrow \begin{vmatrix} 4-\lambda & 6 \\ 6 & 9-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (4-\lambda)(9-\lambda) - 36 = 0 \Rightarrow \lambda = 0, \lambda = 13$$

(d) Eigen vectors for distinct eigen values for a symmetric matrix are orthogonal.

Q32. Let $f(x) = 3x^6 - 2x^2 - 8$. Which of the following statements is (are) true?

(a) The sum of all its roots is zero

(b) The product of its roots is $-\frac{8}{3}$

(c) The sum of all its roots is $\frac{2}{3}$

(d) Complex roots are conjugates of each other.

Ans.: (a), (b), (d)

Solution:- $f(x) = 3x^6 - 2x^2 - 8$

Now, $3x^6 - 2x^2 - 8 = 0$

$\Rightarrow x^6 - \frac{2}{3}x^2 - \frac{8}{3} = 0$

$\Rightarrow Ax^6 + Bx^5 + Cx^4 + Dx^3 + Ex^2 + Fx + G = 0$

$x^6 + 0.x^5 + 0.x^4 + 0.x^3 - \frac{2}{3}x^2 + 0.x - \frac{8}{3} = 0$

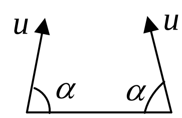
Here, sum of roots $= \left(-\frac{B}{A}\right) = 0$

And product of roots $= \frac{G}{A} = \left(\frac{-8}{3}\right)$

Since all coefficient are real, then complex roots are conjugate to each other.

Hence, options (a), (b) and (d) are correct.

Q33. Two projectiles of identical mass are projected from the ground with same initial angle (α) with respect to earth surface and same initial velocity (u) in the same plane. They collide at the highest point of their trajectories and stick to each other. Which of the following statements is (are) correct?



(a) The momentum of the combined object immediately after collision is zero.

(b) Kinetic energy is conserved in the collision

(c) The combined object moves vertically downward.

(d) The combined object moves in a parabolic path.

Ans. : (a), (c)

Solution: At the highest point there is only Horizontal velocity. In horizontal direction there is not any External force. So momentum in the horizontal direction is conserved.

(c) After collision whole system will full under gravitation.

Q34. Two beams of light in the visible range $(400\ nm - 700\ nm)$ interfere with each other at a point. The optical path difference between them is $5000\ nm$. Which of the following wavelengths will interfere constructively at the given point?

(a) $416.67 nm$ (b) $555.55\ nm$ (c) $625\ nm$ (d) $666.66\ nm$

Ans.: (a),(b) and (c)

Solution: $\delta = \frac{2\pi}{\lambda}(p.d)$

For constructive interference $\delta = 2n\pi$ where n is integer

$\therefore 2n\pi = \frac{2\pi}{\lambda}(p.d)$

$\Rightarrow \lambda = \frac{p.d}{n} = \frac{5000nm}{n}$

for, $n = 8, \lambda = 625\,nm$

$n = 9, \lambda = 555\cdot 55\,nm$

$n = 10, \lambda = 500\,nm$

$n = 11, \lambda = 454\cdot 5\,nm$

$n = 12, \lambda = 416\cdot 67\,nm$

Thus, correct options are (a),(b) and (c)

Q35. Which of the following relations is (are) true for thermodynamic variables?

(a) $TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$ (b) $TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$

(c) $dF = -SdT + PdV$ (d) $dG = -SdT + VdP$

Ans. : (b), (d)

Solution: $S = S(T,V) \Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$

$TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$ (a) is correct.

$S = S(T,P) \Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial P}\right)_T dP$

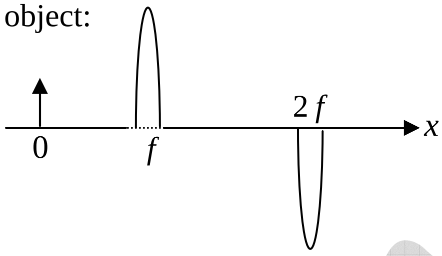
$TdS = T\left(\frac{\partial S}{\partial T}\right)_P dT + T\left(\frac{\partial S}{\partial P}\right)_T dP$

$= C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$, (b) is correct.

$dF = -SdT - PdV$ so (c) is incorrect

$dG = -SdT + PdV$ so (d) is correct

Q36. Consider a convex lens of focal length f. The lens is cut along a diameter in two parts. The two lens parts and an object are kept as shown in the figure. The images are formed at following distances from the object:



(a) $2f$ (b) $3f$ (c) $4f$ (d) ∞

Ans.: (b), (c) and (d)

Solution: For first lens $\frac{1}{v'}-\frac{1}{u}=\frac{1}{f}$

For second lens $\frac{1}{v}-\frac{1}{u'}=\frac{1}{f}$

(i) if $u=\infty, v'=f, v=\infty$

(ii) if $u>2f, v'<2f, v<2f$

(iii) if $u=2f, v'=2f$ No image

(iv) if $u<2f, v'>2f, v>2f$

(v) if $u=f, v'=\infty, v=\infty$

(vi) $u<f, v'=-ve$, No image

Thus, V cannot be $2f$. The correct options are (b),(c) and (d)

Q37. Let the electric field in some region R be given by $E=e^{-y^2}\hat{i}+e^{-x^2}\hat{j}$. From this we may conclude that

(a) R has a non-uniform charge distribution

(b) R has no charge distribution

(c) R has a time dependent magnetic field.

(d) The energy flux in R is zero everywhere.

Ans.: (b), (c)

Solution: $\because \vec{\nabla}\cdot\vec{E}=0$ and $\vec{\nabla}\times\vec{E}\neq 0$,

Thus R has no charge distribution and R has a time dependent magnetic field.

Q38. In presence of a magnetic field $B\hat{j}$ and an electric field $(-E)\hat{k}$, a particle moves undeflected . Which of the following statements is (are) correct?

(a) The particle has positive charge, velocity $=-\frac{E}{B}\hat{i}$

(b) The particle has positive charge, velocity $=\frac{E}{B}\hat{i}$

(c) The particle has negative charge, velocity $=-\frac{E}{B}\hat{i}$

(d) The particle has negative charge, velocity $=-\frac{E}{B}\hat{i}$

Ans.: (b), (d)

Solution: $\because \vec{F}=q\left[\vec{E}+\left(\vec{v}\times\vec{B}\right)\right]=0 \quad \Rightarrow |v|=\frac{E}{B}$

For $+ve$ charge: $\vec{a}\rightarrow -\hat{k} \Rightarrow v=\frac{E}{B}\hat{x}$

For $-ve$ charge: $\vec{a}\rightarrow \hat{k} \Rightarrow v=\frac{E}{B}\hat{x}$

Q39. In a pn junction, dopant concentration on the p-side is higher than that on the n-side. Which of the following statements is (are) correct, when the junction is unbiased?

(a) The width of the depletion layer is larger on the n-side.

(b) At thermal equilibrium the Fermi energy is higher on the p-side.

(c) In the depletion region, number of negative charges per unit area on the p-side is equal to number of positive charges per unit area on the n-side

(d) The value of the built-in potential barrier depends on the dopant concentration.

Ans. : (a), (c) and (d)

Q40. Which of the combinations of crystal structure and their coordination number is (are) correct?

(a) body centered cubic -8

(b) face centred cubic -6

(c) diamond -4

(d) hexagonal closed packed -12

Ans. : (a), (c), (d)

Solution: Co-ordination number in different crystal structure are

(i) Body central cubic -8

(ii) Face central cubic -12

(iii) Diamond -4

(iv) Hexagonal closed packed -12

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q41. – Q50. carry one mark each.

Q41. The coefficient of x^3 in the Taylor expansion of $\sin(\sin x)$ around $x=0$ is ______.

(Specify your answer upto two digits after the decimal point)

Ans.: 0.33

Solution:- Let $f(x)=\sin(\sin x)$

$$f'(x)=\cos(\sin x).\cos x$$

$$f''(x)=-\sin(\sin x).\cos x.\cos x-\sin x.\cos(\sin x)$$

$$=-\cos^2 x.\sin(\sin x)-\sin x.\cos(\sin x)$$

$$f'''(x)=-\left[-(2\cos x\sin x)\sin(\sin x)+\cos^3 x\cos(\sin x)+\cos x\cos(\sin x)+\sin x(-\sin(\sin x)\cos x)\right]$$

$$=\left[\sin 2x.\sin(\sin x)-\cos^3 x.\cos(\sin x)-\cos x.\cos(\sin x)+\frac{1}{2}\sin 2x.\sin(\sin x)\right]$$

at $x=0$,

$$f'''(0)=-1-1=-2$$

Hence,

$$f(x)=f(x_0)+\frac{(x-x_1)f'(x_0)}{\lfloor 1}+\frac{(x-x_1)^2 f''(x_0)}{\lfloor 2}+\frac{(x-x_0)^3}{\lfloor 3}f'''(x_1)+.....$$

Hence, coefficient of x^3 is _____

$$=\frac{1}{\lfloor 3}(-2)=\frac{-2}{3\times2\times1}=\left(-\frac{1}{3}\right)=-0.33$$

Q42. A particle of mass m is moving along the positive x direction under a potential $V(x)=\frac{1}{2}kx^2+\frac{\lambda}{2x^2}$ (k and λ are positive constants). If the particle is slightly displaced from its equilibrium position, it oscillates with an angular frequency (ω)______.

(Specify your answer in units of $\sqrt{\frac{k}{m}}$ as an integer.)

Ans. : 2

Solution: $V(x)=\frac{kx^2}{2}+\frac{\lambda}{2x^2}$

$$\frac{\partial V}{\partial x}=kx-\frac{\lambda}{x^3}=0 \Rightarrow x^4=+\left(\frac{\lambda}{x}\right) \Rightarrow x_0=\pm\left(\frac{\lambda}{x}\right)^{1/4}$$

$$\frac{\partial^2 V}{\partial x^2}=k+\frac{3\lambda}{x^4}=4k \quad =\omega=\sqrt{\frac{\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=x_0}}{m}}=\sqrt{\frac{4k}{m}}=2\omega=2$$

Q43. A planet has average density same as that of the earth but it has only $\frac{1}{8}$ of the mass of the Earth. If the acceleration due to gravity at the surface is g_p and g_e for the planet and Earth, respectively, then $\frac{g_p}{g_e}=$____________.

(Specific your answer upto one digit after the decimal point.)

Ans. : 0.5

Solution: $g=\frac{GM}{R^2}$ and $\frac{M}{V}=\rho$

$$V=\frac{M}{\rho} \Rightarrow R^3 \propto \frac{M}{\rho} \Rightarrow R \propto \left(\frac{M}{\rho}\right)^{1/3}$$

$$g \propto \frac{M}{(M/\rho)^{2/3}} \Rightarrow g \propto M^{1/3}$$

$$\frac{g_p}{g_e}=\left(\frac{M_p}{M_e}\right)^{1/3}=\left(\frac{M_e}{8M_e}\right)^{1/3}=\frac{1}{2}=0.5=\left(\frac{1}{2}\right)=0.5$$

Q44. In a grating with grating constant $d = a + b$, where a is the slit width and b is the separation between the slits, the diffraction pattern has the fourth order missing. The value of $\frac{b}{a}$ is______. (Specific your answer as an integer.)

Ans. : 3

Solution: $n = m\frac{d}{a} = m\left(\frac{a+b}{a}\right) = m\left(1+\frac{b}{a}\right)$

For, $m = 1, n = 4$ $\quad\quad \therefore 4 = 1 + \frac{b}{a} \Rightarrow \frac{b}{a} = 3$

Q45. Consider an electromagnetic plane wave $\vec{E} = E_0\left(\hat{i} + b\hat{j}\right)\cos\left[\frac{2\pi}{\lambda}\left\{ct - \left(x - \sqrt{3}y\right)\right\}\right]$, where λ is the wavelength, c is the speed of light and b is a constant. The value of b is _________. (Specific your answer upto two digits after the decimal point)

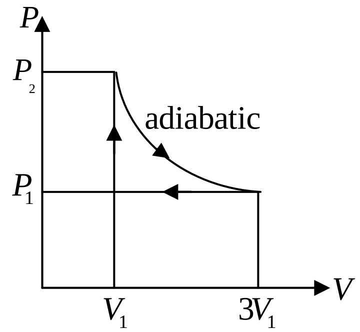
Ans. : 0.577

Solution: $\vec{E} = E_0\hat{n}\cos\left[\omega t - \hat{k}\cdot\vec{r}\right] \Rightarrow \hat{n} = \left(\hat{i} + b\hat{j}\right)$

$$\hat{k} = \frac{2\pi}{\lambda}\left(\hat{i} - \sqrt{3}\hat{j}\right)$$

$$\because \hat{k}\cdot\hat{n} = 0 \Rightarrow \frac{2\pi}{\lambda}\left(1 - b\sqrt{3}\right) = 0 \Rightarrow b = \frac{1}{\sqrt{3}} = 0.577$$

Q46. Consider a monoatomic ideal gas operating in a closed cycle as shown in the $P - V$ diagram given below. The ratio $\frac{P_1}{P_2}$ is _______. (Specific your answer upto two digits after the decimal point)

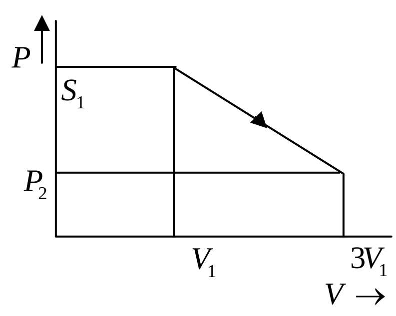


Ans. : 0.16

Solution: For monoatomic gas $r = \frac{5}{3}$

For adiabatic problems $P_1V_1^r = P_2V_2^r$

$$\Rightarrow \frac{P_1}{P_2} = \left(\frac{V_1}{V_1 \times 3}\right)^{5/3} = 3^{-\frac{5}{3}} = 0\cdot16$$



Q47. Consider the first order phase transition of the sublimation of zinc. Assume the vapour to be an ideal gas and the molar volume of solid to be negligible. Experimentally, it is found that $\log_{10}(P) = -\frac{c_1}{T} + c_2$ where P is the vapour pressure in Pascal, T is in $K, c_1 = 6790\ K$ and $c_2 = 9$. The latent heat of sublimation of zinc from the Clausius-Clapeyron equation is _________ $kJ/mole$. $(R = 8.314\ J/mole.K)$

(Specific your answer in kJ/mole upto one digit after the decimal point.)

Solution: $\log_{10} P = -\frac{c_1}{T} + c_2 = \frac{\log_e P}{2\cdot 303}$ (i)

$$\frac{dP}{dT} = \frac{L_{sub}}{T\left(V^f - V^i\right)} \quad \text{as} \quad V^f >> V^i$$

$$\frac{dP}{dT} = \frac{L_{sub}}{TV^f}, V^f = \frac{RT}{P} = \frac{L_{sub}}{T\times RT/P} = \frac{PL_{sub}}{RT^2} \Rightarrow \frac{dP}{dT} = \frac{PL_{sub}}{RT^2}$$

Differentiating equation - (1)

$$\frac{1}{2\cdot 303}\frac{1}{P}\frac{dP}{dT} = \frac{C_1}{T^2} \Rightarrow \frac{dP}{dT} = \frac{2.303\times Pc_1}{T^2} = \frac{PL_{sub}}{RT^2}$$

$$\Rightarrow L_{sub} = 2.303\times R\cdot c_1 = 2.303\times 8.314\times 6790 = 130\, kj/mol$$

Q48. A system of 8 non-interacting electrons is confined by a three dimensional potential $V(r) = \frac{1}{2}m\omega^2 r^2$. The ground state energy of the system in units of $\hbar\omega$ is ______

(Specify your answer as an integer.)

Ans. : 18

Solution: $n = 0$ is non degenerate so there will 2 electron in the ground state.

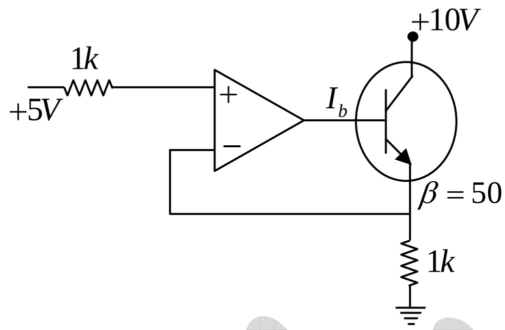
$n = 1$ is triple degenerate so there is 6 electron in the first excited state

$$E = 2\times\frac{3\hbar\omega}{2} + 6\times\frac{5\hbar\omega}{2} \Rightarrow 3\hbar\omega + 15\hbar\omega = 18\hbar\omega$$

Q49. For the given circuit, value of the base current (I_b) of the npn transistor will be ____mA.

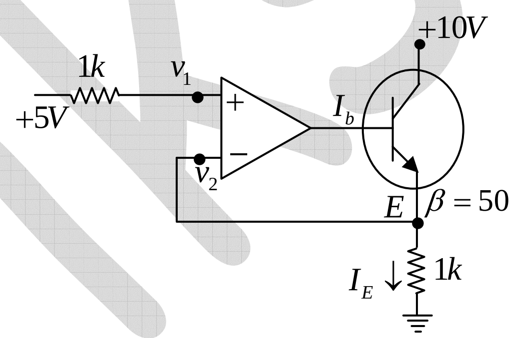
(β is the current gain and assume Op-Amp as ideal.)

(Specific your answer in mA upto two digits after the decimal point.)



Ans. 49: 0.1

Solution: $v_2 = 5V = V_E$



$$I_E = \frac{5}{1} = 5\,mA$$

$$\beta I_B = 5\,mA \Rightarrow I_B = \frac{5}{50}\,mA = 0.1\,mA$$

Q50. The lattice constant of unit cell of $NaCl$ crystal is $0.563\,nm$. X-rays of wavelength $0.141\,nm$ are diffracted by this crystal. The angle at which the first order maximum occurs is ________degrees.

Ans.: 12.5

Solution: $2d\sin\theta = \lambda$

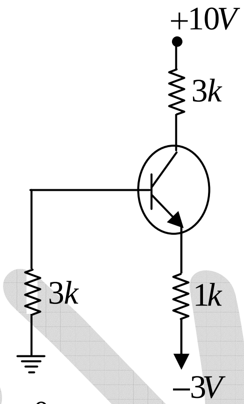
$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{\sqrt{3}\lambda}{2a}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{3}\times 0.141}{2\times 0.563}\right) = \sin^{-1}(0.217) = 12.53^0$$

Q51. – Q60. carry two marks each.

Q51. For the following circuit, the collector voltage with respect to ground will be _________ V. (Emitter diode voltage is $0.7V$ and β_{DC} of the transistor is large)

(Specify your answer in volts upto one digit after the decimal point).



Ans. : 51: 3.1

Solution: $V_{BE} = 0.7V$ and $\beta_{dc} \rightarrow$ large so $I_B \approx 0$

From input section;

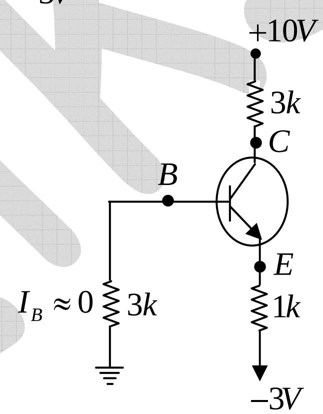
$0 + 0.7 + I_E \times 1 - 3 = 0$

$\Rightarrow I_E = 2.3mA$

From output section;

$-10 + 3 \times 2.3 + V_C = 0$

$\Rightarrow V_C = 10 - 6.9 = 3.1V$



Q52. A body of mass 1 kg is moving under a central force in an elliptic orbit with semi major axis 1000 m and semi minor axis $100\,m$. The orbital angular momentum of the body is $100\ kg\, m^2 s^{-1}$. The time period of motion of the body is _________ hours.

(Specific your answer upto two digits after the decimal point)

Ans. : 1.74

Solution: $\dfrac{dA}{dt} = \dfrac{L}{2m} \Rightarrow T = \pi ab \cdot \dfrac{2m}{L}$

$$= 3.14 \times 1000 \times 100 \times \frac{2 \times 1}{100} = 6 \cdot 28 \times 1000 \ = \frac{6280}{3600} \ = 1 \cdot 744\,hr$$

Q53. The moon moves around the earth in a circular orbit with a period of 27 days. The radius of the earth (R) is $6.4\times10^{6}\ m$ and the acceleration due to gravity on the earth surface is $9.8\ ms^{-2}$. If D is the distance of the moon from the center of the earth, the value of $\frac{D}{R}$ will be _________. (Specific your answer upto two digits after the decimal point)

Ans. : 59.6

Solution: For circular orbit $m\omega^2 D = \frac{GMm}{D^2}$

$$D^3 = \frac{GM}{\omega^2} = \frac{GMT^2}{(2\pi)^2}\frac{R^2}{R^2}$$

$$D^3 = \frac{GM}{R^2}\frac{T^2}{4\pi^2}R^2 = \frac{gT^2R^2\pi R}{4\pi^2}$$

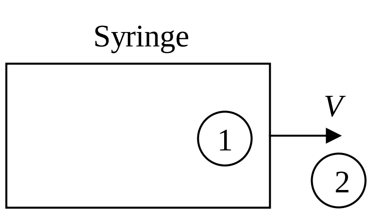
$$\left(\frac{D}{R}\right)^3 = \frac{gT^2}{4\pi^2 R} = \frac{9.8\times(27\times24\times60\times60)^2}{4\pi^2\times6.4\times10^6}$$

$$\Rightarrow \frac{D}{R} = \left(\frac{5.34\times10^{13}}{4\pi^2\times6.4\times10^6}\right)^{\frac{1}{3}} = 59.5$$

Q54. A syringe is used to exert 1.5 atmospheric pressure to release water horizontally. The speed of water immediately after ejection is________ (take 1 atmospheric pressure $=10^5$ Pascal, density of water $=10^3 kg\ m^{-3}$) (Specify your answer in ms^{-1} as an integer)

Ans: 10

Solution: Apply Bernoulli's equation,



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = \rho_2 + \frac{1}{2}e v_2^2 + \rho g h_2$$

$$h_1 = h_2, v_1 \approx 0$$

$$P_1 = 1.5 \text{ atm}, P_2 = 1 \text{ atm}$$

$$\Rightarrow v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}} = \sqrt{\frac{2\times(1\cdot5-0.5)\times10^5}{1000}}$$

$$v_2 = 10\ m/s$$

Q55. Consider a slit of width $18\,\mu m$ which is being illuminated simultaneously with light of orange color (wavelength $600\,nm$) and of blue color (wavelength $450\,nm$). The diffraction pattern is observed on a screen kept at a distance in front of the slit. The smallest angle at which only the orange color is observed is θ_1 and the smallest angle at which only the blue color is observed is θ_2. The angular difference $\theta_2 - \theta_1$ (in degrees) is____________

(Specify your answer upto two digits after the decimal point)

Ans.: 0.48

Solution: $d\sin\theta = \lambda$

$$\theta_1 = \sin^{-1}\left(\frac{\lambda_1}{d}\right) = \sin^{-1}\left(\frac{600\times10^{-9}}{18\times10^{-6}}\right) = 1.91^0$$

$$\theta_2 = \sin^{-1}\left(\frac{\lambda_2}{d}\right) = \sin^{-1}\left(\frac{450\times10^{-9}}{18\times10^{-6}}\right) = 1.43^0$$

$$\therefore \theta_2 - \theta_1 = 0.48^0.$$

Q56. A particle of mass m is moving in a circular orbit given by $x = R\cos\omega t$; $y = R\sin(\omega t)$, as observed in an inertial frame S_1. Another inertial frame S_2 moves with uniform velocity $\vec{v} = \omega R\hat{i}$ with respect to $S_1.S_2$ are related by Galilean transformation, such that the origins coincide at $t = 0$. The magnitude of the angular momentum of the particle at $t = \frac{2\pi}{\omega}$, as observed in S_2 about its origin is expressed as $(mR^2\omega)x$. Then x is _______.

(Specify your answer upto two digits after the decimal point)

Ans. : 5.28

Solution: From S_2 Frame

$$\vec{r}^{\,'} = (x - vt)\hat{i} + y\hat{i} = (R\cos\omega t - vt)\hat{i} + R\sin\omega t\hat{i}$$

$$= \vec{v}' = (\dot{x} - v)\hat{i} + \dot{y}\hat{j} = (-R\omega\sin t - v)\hat{i} + R\omega\cos\omega t\hat{j}$$

$$\vec{L} = \vec{r}' \times \vec{P} = m\left[(R\cos\omega t - vt)\hat{i} + R\sin\omega t\hat{i}\right] \times \left[(-R\omega\sin t - v)\hat{i} + R\omega\cos\omega t\hat{j}\right]$$

$$= m\left(\omega R^2 \cos^2 \omega t - vtR\cos\omega t + R^2\omega \sin^2 \omega t + vR\sin\omega t\right)\hat{k}$$

$$= m\left(R^2\omega - vtR\omega\cos\omega t + vR\sin\omega t\right)$$

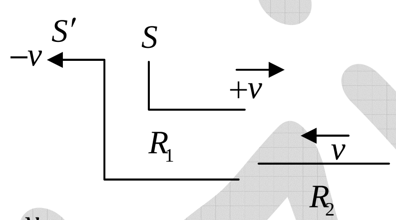
at $t = \frac{2\pi}{\omega}$, $\vec{L} = m\omega R^2\left(1-2\pi\right) = -5.28 m\omega R^2 \Rightarrow \left|L\right| = 5.28\ m\omega R^2$

Q57. Rod R_1 has a rest length $1m$ and rod R_2 has a rest length of $2m$. R_1 and R_2 are moving with respect to the laboratory frame with velocities $+v\hat{i}$ and $-v\hat{i}$, respectively. If R_2 has a length of $1m$ in the rest frame of R_1, $\frac{v}{c}$ is given by__________

(Specify your answer upto two digits after the decimal point)

Ans. : 0.48

Solution:



$V = -v, u_x' = -v$

$$u_x = \frac{u_x' + V}{1 + \frac{u_x' V}{c^2}} = \frac{-2v}{1 + \frac{v^2}{c^2}}$$

$$l = l_0\sqrt{1 - \frac{u_x^{\ 2}}{c^2}}$$

$$1 = 2\sqrt{1 - \frac{u_x^{\ 2}}{c^2}}$$

$$\frac{1}{4} = 1 - \frac{\left(\frac{4v^2}{1+v^2/c^2}\right)}{c^2} \Rightarrow \frac{4v^2/c^2}{1+\frac{v^2}{c^2}} = \frac{3}{4} \Rightarrow 4\left(\frac{v^2}{c^2}\right) = \frac{3}{4} + \frac{3}{4}\left(\frac{v^2}{c^2}\right)$$

$$\frac{13}{4}\left(\frac{v}{c}\right)^2 = \frac{3}{4} \Rightarrow \left(\frac{v}{c}\right)^2 = \frac{12}{52} \Rightarrow \frac{v}{c} = \sqrt{\frac{12}{52}} \Rightarrow \frac{v}{c} = 0.479 = 0.48.$$

Q58. Two events E_1 and E_2 take place in an inertial frame S with respective time space coordinates (in SI units): $E_1\left(t_1=0,\vec{r}_1=0\right)$ and $E_2\left(t_2=0,x_2=10^8,y_z=0,z_2=0\right)$. Another inertial frame S' is moving with respect to S with a velocity $\vec{v}=0.8c\hat{i}$. The time difference $\left(t_2'-t_1'\right)$ as observed in S' is ___________ s.

(Specify your answer in seconds upto two digits after the decimal point)

Ans. : 0.44

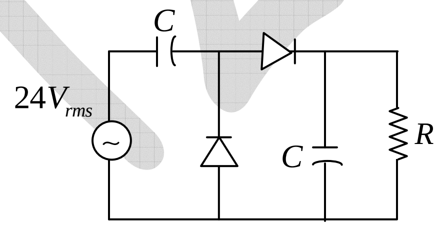
Solution: $t_2-t_1=0$ and $x_2-x_1=10^8$

$$t_2'-t_1'=\frac{\left(t_2-t_1\right)}{\sqrt{1-v^2/c_2}}-\left(\frac{x_2-x_1}{\sqrt{1-v^2/c_2}}\right)\frac{v}{c_2}$$

$$=-\frac{\left(x_2-x_1\right)\frac{v}{c^2}}{\sqrt{1-v^2/c^2}}=-\frac{\left(x_2-x_1\right)\frac{v}{c^2}}{\sqrt{1-v^2/c^2}}=-\frac{10^8\times\frac{0.8c}{c^2}}{\sqrt{1-0.64}}=\frac{\cdot 8\times 10^8}{\cdot 6\times 3\times 10^8}=\frac{8}{18}=0.44\,\text{sec}.$$

Q59. In the following circuit, the time constant RC is much greater than the period of the input signal. Assume diode as ideal and resistance R to be large. The dc output voltage across resistance R will be ____________ V.

(Specify your answer in volts upto one digit after the decimal point)



Ans. : 68

Solution: It's a voltage doubler circuit

$$V_R=2V_m=2\left(\sqrt{2}\,V_{rms}\right)=2\left(\sqrt{2}\times 24\right)$$

$$\Rightarrow V_R\approx 68\ V$$

Q60. For a metal, the electron density is $6.4\times10^{28}\,m^{-3}$. The Fermi energy is __________ eV.

($h = 6.626\times10^{-34}\ J\,s,\ m_e = 9.11\times10^{-31}kg,\ 1\ eV = 1.6\times10^{-19}\,J$)

(Specify your answer in electron volts (eV) upto one digit after the decimal point)

Ans. : 5.84

Solution: $E_F = \dfrac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} = \dfrac{\left(1.05\times10^{-34}\right)^2}{2\times9.11\times10^{-31}}\left(3\pi^2\times6.4\times10^{28}\right)^{2/3}$

$= 6.1\times10^{-39}\left(1.53\times10^{20}\right) = 9.34\times10^{-19}\,J$

$= \dfrac{9.34\times10^{-19}}{1.6\times10^{-19}}eV = 5.84eV$.