Kinetic Theory, Thermodynamics

IIT-JAM 2005

Q1. The molar specific heat of a gas as given from the kinetic theory is $\frac{5}{2}R$. If it is not specified whether it is C_P or C_V, one could conclude that the molecules of the gas

(a) are definitely monatomic
(b) are definitely rigid diatomic
(c) are definitely non-rigid diatomic
(d) can be monatomic or rigid diatomic

Ans. : (d)

Solution: If molecule is mono atomic, then $C_p = \frac{5R}{2}$

And if molecule is rigid diatomic then $C_V = \frac{5R}{2}$

Q2. The value of entropy at absolute zero of temperature would be

(a) zero for all the materials
(b) finite for all the materials
(c) zero for some materials and non-zero for others
(d) unpredictable for any material

Ans. : (a)

Solution: If system will achieve absolute zero then it is perfectly ordered system then entropy will be zero.

Q3. Which of the following statements is ***INCORRECT***?

(a) Indistinguishable particles obey Maxwell-Boltzmann statistics
(b) All particles of an ideal Bose gas occupy a single energy state at $T = 0$
(c) The integral spin particles obey Bose-Einstein statistics
(d) Protons obey Fermi-Dirac statistics

Ans. : (a)

Solution: Distinguishable particles obey Maxwell-Boltzmann statistics.

IIT-JAM 2006

Q4. A solid melts into a liquid via first order phase transition. The relationship between the pressure P and the temperature T of the phase transition is $P=-2T+P_0$, where P_0 is a constant. The entropy change associated with the phase transition is $1.0\, J\, mole^{-1}K^{-1}$. The Clausius-Clapeyron equation for the latent heat is $L=T\left(\frac{dP}{dT}\right)\Delta v$. Here $\Delta v = v_{liquid}-v_{solid}$ is the change in molar volume at the phase transition. The correct statement relating the values of the volumes is

(a) $v_{liquid}=v_{solid}$ (b) $v_{liquid}=v_{solid}-1$

(c) $v_{liquid}=v_{solid}-\frac{1}{2}$ (d) $v_{quid}=v_{solid}+2$

Ans. : (c)

Solution: Since $P=-2T+P_0 \Rightarrow \frac{dP}{dT}=-2$

It is given $L=T\left(\frac{dP}{dT}\right)\Delta v \Rightarrow L=-2T\Delta v \Rightarrow \frac{dL}{dT}=-2\Delta v$

Since $dS=1.0\text{ Jmole}^{-1}\text{ K}^{-1}$, $dS=\frac{dQ}{T}=\frac{mdL}{dT}=1 \Rightarrow 1=-2\Delta v \Rightarrow \Delta v=-\frac{1}{2}$

IIT-JAM 2007

Q5. Experimental measurements of heat capacity per mole of Aluminum at low temperatures show that the data can be fitted to the formula, $C_V=aT+bT^3$, where $a=0.00135\, JK^{-2}mole^{-1}$, $b=2.48\times10^{-5}\, JK^{-4}mole^{-1}$ and T is the temperature in Kelvin. The entropy of a mole of Aluminum at such temperatures is given by the formula

(a) $aT+\frac{b}{3}T^3+c$, where $c>0$ is a constant (b) $\frac{aT}{2}+\frac{b}{4}T^3+c$, where $c>0$ is a constant

(c) $aT+\frac{b}{3}T^3$ (d) $\frac{aT}{2}+\frac{b}{4}T^3$

Ans. : (a)

Solution: $ds=\int\frac{c_V dT}{T} \Rightarrow s=\int\frac{aT+bT^3}{T}dT=aT+\frac{b}{3}T^3+c$

IIT-JAM 2008

Q6. The chemical potential of an ideal Bose gas at any temperature is

(a) necessarily negative (b) either zero or negative

(c) necessarily positive (d) either zero or positive

Ans. : (b)

Solution: The chemical potential of an ideal Bose gas at any temperature is either zero or negative.(zero in case of photon).

Q7. A thermodynamic system is maintained at constant temperature and pressure. In thermodynamic equilibrium, its

(a) Gibbs free energy is minimum (b) enthalpy is maximum

(c) Helmholtz free energy is minimum (d) internal energy is zero

Ans. : (a)

Solution: A thermodynamic system is maintained at constant temperature and pressure can be defined by Gibbs energy $dG = -SdT + VdP \leq 0$ i.e. Gibbs free energy is minimum.

IIT-JAM 2009

Q8. A box containing 2 moles of a diatomic ideal gas at temperature T_0 is connected to another identical box containing 2 moles of a monatomic ideal gas at temperature $5T_0$. There are no thermal losses and the heat capacity of the boxes is negligible. Find the final temperature of the mixture of gases (ignore the vibrational degrees of freedom for the diatomic molecules).

(a) T_0 (b) $1.5T_0$ (c) $2.5T_0$ (d) $3T_0$

Ans. : (c)

Solution: Internal energy of the system remains conserve i.e. $U_{monoatomic} + U_{diatomic} = U_{mixture}$

$$U_{monotomic} = n_1 C_{V_1} T_1, \quad U_{diatomic} = n_2 C_{V_2} T_2$$

$$C_{v_1} = \frac{5R}{2}, C_{v_2} = \frac{3R}{2}, \quad n_1 = n_2 = 2, T_1 = T_0, T_2 = 5T_0$$

Let the common temperature of mixture is T and specific heat is $C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$ and number of moles of mixture is $n = n_1 + n_2$, then

$$n_1 C_{V_1} T_1 + n_2 C_{V_2} T_2 = nC_V T \Rightarrow T = 2.5T_0$$

Q9. Isothermal compressibility k_T of a substance is defined as $k_T = \frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$. Its value for n moles of an ideal gas will be

(a) $\frac{1}{P}$ (b) $\frac{n}{P}$ (c) $-\frac{1}{P}$ (d) $-\frac{n}{P}$

Ans. : (c)

Solution: $PV = nRT$ and $k_T = \frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = -\frac{1}{P}$.

IIT-JAM 2010

Q10. A gas of molecules each having mass m is in thermal equilibrium at a temperature T. Let v_x, v_y, v_z be the Cartesian components of velocity, $\vec{v}$, of a molecules. The mean value of $(v_x - \alpha v_y + \beta v_z)^2$ is

(a) $(1+\alpha^2+\beta^2)\frac{k_B T}{m}$ (b) $(1-\alpha^2+\beta^2)\frac{k_B T}{m}$

(c) $(\beta^2-\alpha^2)\frac{k_B T}{m}$ (d) $(\alpha^2+\beta^2)\frac{k_B T}{m}$

Ans. : (a)

Solution: $(v_x - \alpha v_y + \beta v_z)^2 = v_x^2 + \alpha^2 v_y^2 + \beta^2 v_y^2 - 2\alpha v_x v_y + 2\beta v_z v_x - 2\beta\alpha v_y v_z$

$$\langle (v_x - \alpha v_y + \beta v_z)^2 \rangle = \langle v_x^2 \rangle + \alpha^2 \langle v_y^2 \rangle + \beta^2 \langle v_y^2 \rangle - 2\alpha \langle v_x \rangle \langle v_y \rangle + 2\beta \langle v_z \rangle \langle v_x \rangle - 2\beta\alpha \langle v_y \rangle \langle v_z \rangle$$

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{k_B T}{m} \text{ and } \langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0$$

$$\langle (v_x - \alpha v_y + \beta v_z)^2 \rangle = (1+\alpha^2+\beta^2)\frac{k_B T}{m}$$

Q11. A trapped air bubble of volume V_0 is released from a depth h measured from the water surface in a large water tank. The volume of the bubble grows to $2V_0$ as it reaches just below the surface. The temperature of the water and the pressure above the surface of water ($10^5 N/m^2$) remain constant throughout the process. If the density of water is $1000\,kg/m^3$ and the acceleration due to gravity is $10\,m/s^2$, then the depth h is

(a) $1m$ (b) $10\,m$ (c) $50\,m$ (d) $100\,m$

Ans. : (b)

Solution: At depth h pressure $P_1 = P_0 + \rho gh$ and volume $V_1 = V_0$

At surface pressure $P_2 = P_0$ and volume $V_2 = 2V_0$

Then, $P_1V_1 = P_2V_2$

$$(P_0 + \rho gh)V_0 = P_0 2V_0 \Rightarrow h = \frac{P_0V_0}{\rho g V_0} = \frac{P_0}{\rho g} = \frac{1\times 10^5}{10\times 10^3} = 10\ m$$

$P_0 = 10^5 N/m^2, g = 10m/s^2, \rho = 1000kg/m^3 \Rightarrow h = 10m$

IIT-JAM 2011

Q12. Consider free expansion of one mole of an ideal gas in an adiabatic container from volume V_1 to V_2. The entropy change of the gas, calculated by considering a reversible process between the original state (V_1, T) to the final state (V_2, T), where T is the temperature of the system is denoted by ΔS_1. The corresponding change in the entropy of the surrounding is ΔS_2. Which of the following combinations is correct?

(a) $\Delta S_1 = R\ln(V_1/V_2),\ \Delta S_2 = -R\ln(V_1/V_2)$

(b) $\Delta S_1 = -R\ln(V_1/V_2),\ \Delta S_2 = R\ln(V_1/V_2)$

(c) $\Delta S_1 = R\ln(V_2/V_1),\ \Delta S_2 = 0$

(d) $\Delta S_1 = -R\ln(V_2/V_1),\ \Delta S_2 = 0$

Ans. : (c)

Solution: Free expansion is irreversible process when gas expand V_1 to V_2 which can be explained by choosing any path between two state (because entropy is state function). So one can choose reversible isothermal process.

So, $\Delta S_1 = R\ln\frac{V_2}{V_1}$. Hence it is free expansion so entropy of surrounding is $\Delta S_2 = 0$.

Q13. A gas of molecular mass m is at temperature T. If the gas obeys Maxwell-Boltzmann velocity distribution, the average speed of molecules is given by

(a) $\sqrt{\frac{k_BT}{m}}$ (b) $\sqrt{\frac{2k_BT}{m}}$ (c) $\sqrt{\frac{2k_BT}{\pi m}}$ (d) $\sqrt{\frac{8k_BT}{\pi m}}$

Ans. : (d)

Solution: The velocity distribution function

$$f(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{-mv^2}{2kT} \cdot 4\pi v^2 dv, \quad 0 < v < \infty$$

Now,

$$\langle v\rangle = \int_0^\infty v \cdot \left(\frac{m}{2\pi kT}\right)^{3/2} e^{\frac{-mv^2}{2kT}}, 4\pi v^2 dv$$

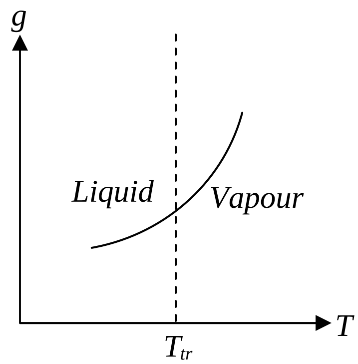
$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 e^{\frac{-mv^2}{2kT}} dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{2}\left(\frac{2kT}{m}\right)^2 \Gamma 2$$

$$= 4\pi\sqrt{\frac{m}{2\pi kT}}\left(\frac{m}{2\pi kT}\right) \times \frac{1}{2}\left(\frac{2kT}{m}\right)^2 \times 1 = 2\sqrt{\frac{m}{2\pi kT}} \cdot \left(\frac{2kT}{m}\right) = \sqrt{\frac{8kT}{\pi m}}$$

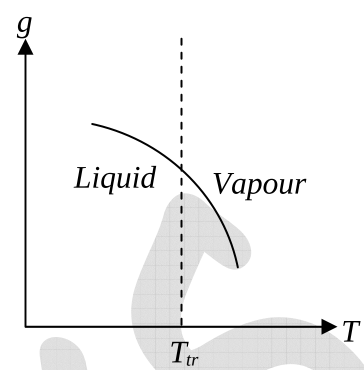
IIT-JAM 2012

Q14. For a liquid to vapour phase transition at T_{tr}, which of the following plots between specific Gibbs free energy g and temperature T is correct?

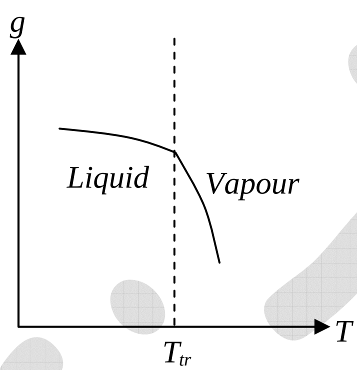
(a)



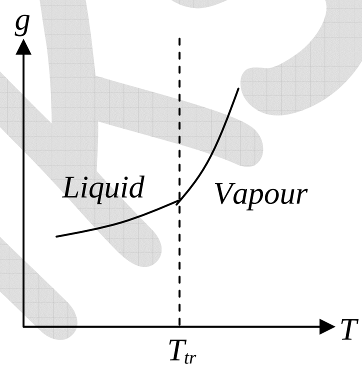
(b)



(c)



(d)



Ans. : (a)

Solution: $dG = (VdP - SdT)$

Q15. A tiny dust particle of mass $1.4\times10^{-11}\,kg$ is floating in air at $300\,K$. Ignoring gravity, its rms speed (in $\mu m/s$) due to random collisions with air molecules will be closest to

(a) 0.3 (b) 3 (c) 30 (d) 300

Ans. : (c)

Solution: $v_{rms} = \sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3\times1.38\times10^{-23}\times300}{1.4\times10^{-11}}} = 30\times10^{-6}\,m/s$

Q16. When the temperature of a blackbody is doubled, the maximum value of its spectral energy density, with respect to that at initial temperature, would become

(a) $\dfrac{1}{16}$ times (b) 8 times (c) 16 times (d) 32 times

Ans. : (c)

Solution: $U \propto T^4 \Rightarrow \frac{U_1}{U_2} = \frac{T_1^4}{T_2^4} \Rightarrow U_2 = U_1 \frac{T_2^4}{T_1^4} = 16U_1$

IIT-JAM 2013

Q17. A blackbody at temperature T emits radiation at a peak wavelength λ. If the temperature of the blackbody becomes $4T$, the new peak wavelength is

(a) $\frac{1}{256}\lambda$ (b) $\frac{1}{64}\lambda$ (c) $\frac{1}{16}\lambda$ (d) $\frac{1}{4}\lambda$

Ans. : (d)

Solution: From wein Law, $\lambda_{1\max}T_1 = \lambda_{2\max}T_2 \Rightarrow \lambda_{2\max} = \frac{\lambda_{1\max}T_1}{T_2} = \frac{\lambda T}{4T} = \frac{\lambda}{4}$

Q18. Let N_{MB}, N_{BE}, N_{FD} denote the number of ways in which two particles can be distributed in two energy states according to Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics respectively. Then $N_{MB} : N_{BE} : N_{FD}$ is

(a) $4:3:1$ (b) $4:2:3$ (c) $4:3:3$ (d) $4:3:2$

Ans. : (a)

Solution: $N = 2,\ g = 2,\ n = 2$

For Maxwell, Boltzmann, $W = \frac{\lfloor N}{\lfloor n} g^n = \frac{\lfloor 2}{\lfloor 2} 2^2 = 4$

For Boson, $N = 2, g = 2, n = 2$; $W = \frac{\lfloor n+g-1}{\lfloor n \lfloor g-1} = \frac{\lfloor 2+2-1}{\lfloor 2 \lfloor 2-1} = 3$

For Fermion, $N = 2,\ g = 2,\ n = 2$; $W = \frac{\lfloor g}{\lfloor g \lfloor g-n} = \frac{\lfloor 2}{\lfloor 2 \lfloor 1} = 1$

Q19. Two thermally isolated identical systems have heat capacities which vary as $C_v = \beta T^3$ (where $\beta > 0$). Initially one system is at $300\,K$ and the other at $400\,K$. The systems are then brought into thermal contact and the combined system is allowed to reach thermal equilibrium. The final temperature of the combined system is…………

Ans. : $357\,K \le T \le 360\,K$

Solution: There is not a unique value of temperature rather range of temperature

The maximum temperature when work done is zero so $dQ_1 + dQ_2 = 0$

$$m\beta \int_{300}^{T_{max}} T^3 dT + m\beta \int_{400}^{T_{max}} T^3 dT = 0 \Rightarrow 2\frac{T_{\max}^4}{4} - \frac{(300)^4}{4} - \frac{(400)^4}{4} = 0$$

$$\Rightarrow T_{\max}^4 = \frac{(300)^4 + (400)^4}{2} \Rightarrow T_{\max} = 360K$$

The minimum temperature of system, when process is reversible so change in entropy of system is zero

$$\Rightarrow \Delta S_1 + \Delta S_0 = 0$$

$$\int_{300}^{T_{min}} \frac{m\beta T^3}{T} dT + \int_{400}^{T_{min}} \frac{m\beta T^3}{T} dT = 0 \Rightarrow T_{\min}^3 = \frac{(300)^2 + (400)^2}{2} \Rightarrow T_{\min} = 357\,K$$

So $357\,K \le T \le 360\,K$

IIT-JAM 2014

Q20. In 1 - dimension, an ensemble of N classical particles has energy of the form $E = \frac{P_x^2}{2m} + \frac{1}{2}kx^2$. The average internal energy of the system at temperature T is

(a) $\frac{3}{2}Nk_BT$ (b) $\frac{1}{2}Nk_BT$ (c) $3Nk_BT$ (d) Nk_BT

Ans. : (d)

Solution: Since, $E = \left(\frac{P_n^2}{2m} + \frac{1}{2}kx^2\right)$

Now, $\langle E \rangle = \frac{1}{2m}\langle P_x^2 \rangle + \frac{1}{2}k\langle x^2 \rangle = \frac{kT}{2} + \frac{kT}{2} = kT$

And for N - classical particle, $\langle E \rangle = NkT$

Q21. A solid metallic cube of heat capacity S is at temperature $300\,K$. It is brought in contact with a reservoir at $600\,K$. If the heat transfer takes place only between the reservoir and the cube, the entropy change of the universe after reaching the thermal equilibrium is

(a) $0.69\,S$ (b) $0.54\,S$ (c) $0.27\,S$ (d) $0.19\,S$

Ans. : (d)

Solution: Heat taken by cube $= S(600-300) = 300\,S\;J$

entropy change of res $= -\dfrac{\partial Q}{T} = \dfrac{-300}{600}S = -0.5\,S$

entropy change of cube $= 5\ln\dfrac{600}{300} = 0.69\,S$

$\Delta S_{univ} = -0.5\,S + 0.69\,S = 0.19\,S$

Q22. A real gas has specific volume V at temperature T. Its coefficient of volume expansion and isothermal compressibility are α and k_T, respectively. Its molar specific heat at constant pressure C_p and molar specific heat at constant volume C_v are related as

(a) $C_p = C_v + R$

(b) $C_p = C_v + \dfrac{Tv\alpha}{k_T}$

(c) $C_p = C_v + \dfrac{Tv\alpha^2}{k_T}$

(d) $C_p = C_v$

Ans. : (c)

Q23. At atmospheric pressure $(=10^5\text{ Pa})$, aluminium melts at $550\,K$. As it melts, its density decreases from $3\times10^3\text{ kg/m}^3$ to $2.9\times10^3\text{ kg/m}^3$. Latent heat of fusion of aluminium is 24×10^3 J/kg. The melting point of aluminium at a pressure of 10^7 Pa is closest to

(a) $551.3\,K$ (b) $552.6\,K$ (c) $558.7\,K$ (d) $547.4\,K$

Ans. : (a)

Solution: $\dfrac{dp}{dT} = \dfrac{L_{fu}}{T\left(V^f - V_i\right)} = \dfrac{24000}{T\left(\dfrac{1}{2900} - \dfrac{1}{3000}\right)}$

As $V = \dfrac{1}{\rho}$

$\dfrac{dp}{dT} = \dfrac{2088\times10^6}{T} \Rightarrow dp = 2088\times10^6\dfrac{dT}{T}$

or $p = 2088\times10^6\ln T + C$

At $10^5\,Pa$, $T = 550\,K$

$\Rightarrow 10^5 = 2088\times10^6\ln 550 + C$

$\Rightarrow C = -13.18 \times 10^9$

At $10^7\, Pa$, $10^7 = 2088 \times 10^6 \ln T - 13.18 \times 10^9$

$T = \text{antilog}\,(6.31)$

On solving, $T = 550.1\, K$

IIT-JAM 2015

Q24. A system consists of N number of particles, $N >> 1$. Each particle can have only one of the two energies E_1 or $E_1 + \varepsilon(\varepsilon > 0)$. If the system is in equilibrium at a temperature T, the average number of particles with energy E_1 is

(a) $\frac{N}{2}$ (b) $\frac{N}{e^{\varepsilon/kT} + 1}$ (c) $\frac{N}{e^{-\varepsilon/kT} + 1}$ (d) $Ne^{-\varepsilon/kT}$

Ans. : (d)

Solution: $\langle N \rangle = Ne^{\frac{-(E_2 - E_1)}{kT}} = Ne^{\frac{-[(E_1+\varepsilon) - E_1]}{kT}} \Rightarrow \langle N \rangle = Ne^{\frac{-\varepsilon}{kT}}$

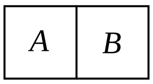
Q25. A rigid and thermally isolated tank is divided into two compartments of equal volume V, separated by a thin membrane. One compartment contains one mole of an ideal gas A and the other compartment contains one mole of a different ideal gas B. The two gases are in thermal equilibrium at a temperature T. If the membrane ruptures, the two gases mix. Assume that the gases are chemically inert. The change in the total entropy of the gases on mixing is

(a) 0 (b) $R\ln 2$ (c) $\frac{3}{2}R\ln 2$ (d) $2R\ln 2$

Ans. : (d)

Solution: For A, number of microstate after mixing is 2

For A, number of microstate before mixing is 1



$\Rightarrow \Delta S_A = R\ln 2 - R\ln 1 = R\ln 2$

Similarly, for $B \Rightarrow \Delta S_B = R\ln 2 \Rightarrow \Delta S = \Delta S_A + \Delta S_B = 2R\ln 2$

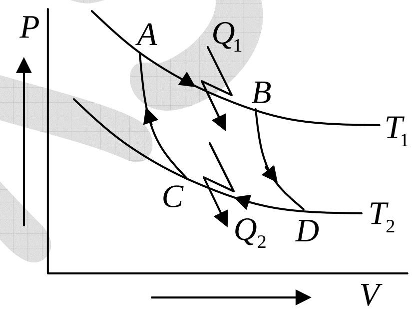
Q26. A rigid triangular molecule consists of three non-collinear atoms joined by rigid rods. The constant pressure molar specific heat (C_p) of an ideal gas consisting of such molecules is

(a) $6R$ (b) $5R$ (c) $4R$ (d) $3R$

Ans. : (c)

Solution: D.O.F $=6 \Rightarrow U=\frac{6RT}{2} \Rightarrow C_V=\left(\frac{\partial U}{\partial T}\right)_V=3R \Rightarrow C_P=C_V+R=4R$

Q27. As shown in the $P-V$ diagram AB and CD are two isotherms at temperatures T_1 and T_2, respectively $(T_1>T_2)$. AC and BD are two reversible adiabats. In this Carnot cycle, which of the following statements are true?



(a) $\frac{Q_1}{T_1}=\frac{Q_2}{T_2}$

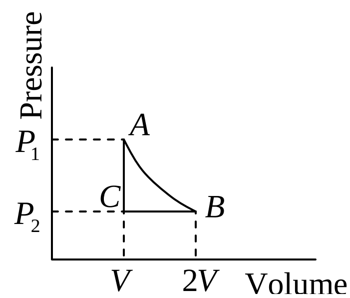
(b) The entropy of the source decreases

(c) The entropy of the system increases

(d) Work done by the system $W=Q_1-Q_2$

Ans. : (a), (b) and (d)

Q28. In the thermodynamic cycle shown in the figure, one mole of a monatomic ideal gas is taken through a cycle. AB is a reversible isothermal expansion at a temperature of $800K$ in which the volume of the gas is doubled. BC is an isobaric contraction to the original volume in which the temperature is reduced to $300K$. CA is a constant volume process in which the pressure and temperature return to their initial values. The net amount of heat (in Joules) absorbed by the gas in one complete cycle is...............



Ans. : 452

Solution: Process $A\to B$ is isothermal expansion

$$T_A=800K,\ V_A,\ P_A \text{ and } T_B=800K,\ V_B=2V_A,\ P_B=\frac{P_A}{2},\ R=8.314\ J/K$$

Process $B\to C$ is isobaric

$P_C = P_B = \frac{P_A}{2}$, $V_C = V_A$, $T_C = 300K$, $n = 1$, $\gamma = \frac{5}{2}$

$C \to A$ is Isochoric

For process $A \to B$, $\Delta Q_1 = nRT_A \ln\left(\frac{V_B}{V_A}\right) = 4610\, J$

$$\Delta Q_2 = nC_P \Delta T = \frac{n\gamma R\Delta T}{(\gamma - 1)} = \left(\frac{\gamma}{\gamma - 1}\right) R(300 - 800) = -10392\, J$$

$$\Delta Q_3 = \frac{R}{(\gamma - 1)}(800 - 300) = \frac{R}{(\gamma - 1)} \times 500 = 6235.5\, J$$

Total heat exchange is $Q_1 + Q_2 + Q_3 = 452$

Q29. One gram of ice at $0^o C$ is melted and heated to water at $39^o C$. Assume that the specific heat remains constant over the entire process. The latent heat of fusion of ice is 80 Calories/gm. The entropy change in the process (in Calories per degree) is............

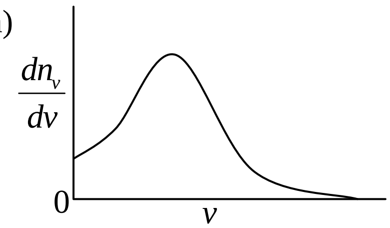
Ans. : 0.39

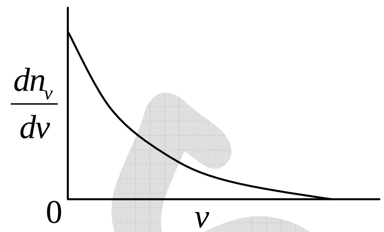
Solution: $\Delta S_1 = \frac{ML}{T} = \frac{1 \times 80}{273}$, $\Delta S_2 = MC\int_{273}^{302} \frac{dT}{T} \Rightarrow \Delta S_2 = 1 \times 1 \ln\frac{302}{273}$

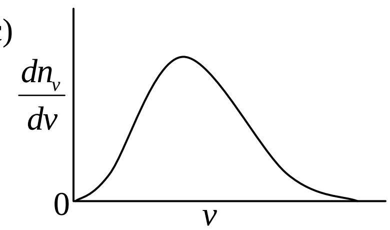
$$\Rightarrow \Delta S = \Delta S_1 + \Delta S_2 \Rightarrow \Delta S = \frac{80}{273} + 1.1 \ln\frac{302}{273} = 0.29 + 0.1 = 0.39$$

IIT-JAM 2016

Q30. A spherical closed container with smooth inner wall contains a monoatomic ideal gas. If the collisions between the wall and the atoms are elastic, then the Maxwell speed-distribution function $\left(\frac{dn_v}{dv}\right)$ for the atoms is best represented by:

(a)


(b)


(c)


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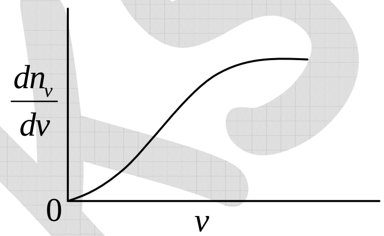
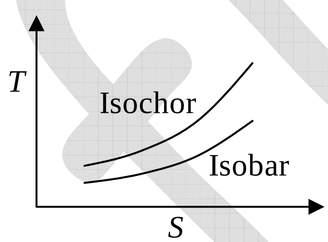


Ans. : (c)

Solution: $\frac{dn}{dv} \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right)$

Q31. For an ideal gas, which one of the following T-S diagram is valid?

(a)


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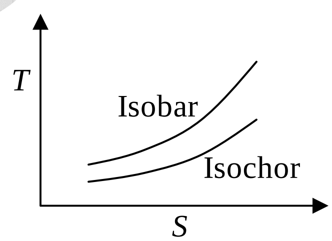

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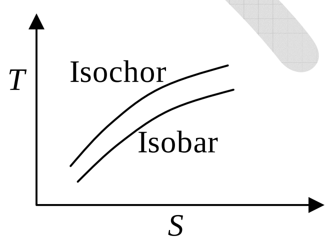

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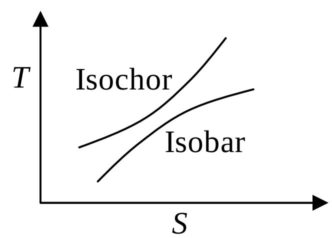


Ans. : (a)

Solution: $C_V = T\left(\frac{\partial S}{\partial T}\right)_V, C_P = T\left(\frac{\partial S}{\partial T}\right)_P$, $C_P > C_V$ from the slope of T-S diagram one can plot isochoric and isobaric plot.

Q32. If U, F, H and G represent internal energy, Helmholtz free energy, enthalpy and Gibbs free energy respectively, then which one of the following is a correct thermodynamic relation?

(a) $dU = PdV - TdS$ (b) $dH = VdP + TdS$

(c) $dF = -PdV + SdT$ (d) $dG = VdP + SdT$

Ans. : (b)

Solution: $H = U + PV \Rightarrow dH = dU + PdV + VdP = TdS - PdV + PdV + VdP \Rightarrow dH = VdP + TdS$

Q33. One mole of an ideal gas with average molecular speed v_0 is kept in a container of fixed volume. If the temperature of the gas is increased such that the average speed gets doubled, then

(a) the mean free path of the gas molecule will increase

(b) the mean free path of the gas molecule will not change

(c) the mean free path of the gas molecule will decrease

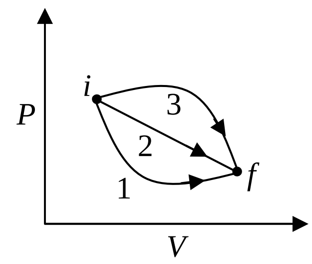
(d) the collision frequency of the gas molecule with wall of the container remains unchanged.

Ans. : (b)

Solution: For fixed volume if temperature is increased then pressure is also increased by same amount so mean free path will not change.

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$$

Q34. The $P-V$ diagram below shows three possible paths for an ideal gas to reach the final state f from an initial state i. Which among the following statements (s) is (are) correct?



(a) The work done by the gas is maximum along path - 3.

(b) Minimum change in the internal energy occurs along path - 2.

(c) Maximum heat transfer is for path - 1

(d) Heat transfer is path independent

Ans. : (a)

Solution: Work done is area under curve so it is maximum in path 3 Hence change in internal energy is same in all path so heat exchange is also maximum in path 3

Q35. A cylinder contains $16\,g$ of O_2. The work done when the gas is compressed to 75% of the original volume at constant temperature of $27^0 C$ is.................. J.

[Universal gas constant $R = 8.31\, J/(\text{mole}\, K)$]

$$n = \frac{16}{32} = 0.5, T = 300K$$

Ans. : 358

Solution: $W = PdV = nRT \int_{V}^{.75V} \frac{dV}{V} = nRT \ln \frac{75}{100} = \frac{1}{2} \times 8.31 \times 300 \times \ln 0.75 = -358\, J$

Q36. An aluminum plate of mass $0.1\, kg$ at $95^0 C$ is immersed in 0.5 litre of water at $20^0 C$ kept inside an insulating container and is then removed. If the temperature of the water is found to be $23^0 C$, then the temperature of the aluminum plate is........... $^0 C$

(The specific heat of water and aluminum are $4200\; J/kg-K$ and $900\; J/kg-K$ respectively, the density of water is $1000\; kg/m^3$).

Ans. : 94.36

Solution: $-M_a S_a \left(T_{a_f} - T_{a_i}\right) = M_w S_w \left(T_{w_f} - T_{w_i}\right) \Rightarrow -0.1 \times 4200\left(T_{af} - 368\right) = 0.5 \times 900\left(296 - 293\right)$

$$\Rightarrow -2100\left(T_{af} - 368\right) = 450 \times 3 \Rightarrow \left(T_{af} - 368\right) = \frac{450}{700} = -0.64$$

$$\Rightarrow T_{af} = 368 - .64 = 367.36 = 367.36 - 273 = 94.36$$

Q37. If there is a 10% decrease in the atmospheric pressure at a hill compared to the pressure at sea level, then the change in the boiling point of water is.................. $^0 C$

(Take latent heat of vaporisation of water as $2270\; kJ/kg$ and the change in the specific volume at the boiling point to be $1.2\; m^3/kg$)

Ans. : 2

Solution: $\frac{dP}{dT} = \frac{1}{T} \frac{L}{V_2 - V_1} \Rightarrow dT = dP \times T \times \frac{V_2 - V_1}{L} = \frac{0.1 \times 1.01 \times 10^5 \times 373 \times 1.2}{2270 \times 10^3} = 0.02 \times 10^2 = 2^0 C$

IIT-JAM 2017

Q38. Consider a system of N particles obeying classical statistics, each of which can have an energy 0 or E. The system is in thermal contact with a reservoir maintained at a temperature T. Let k denote the Boltzmann constant. Which one of the following statements regarding the total energy U and the heat capacity C of the system is correct?

(a) $U=\frac{NE}{1+e^{E/kT}}$ and $C=k\frac{NE}{kT}\frac{e^{E/kT}}{\left(1+e^{E/kT}\right)^2}$

(b) $U=\frac{NE}{kT}\frac{E}{1+e^{E/kT}}$ and $C=k\frac{NE}{kT}\frac{e^{E/kT}}{\left(1+e^{-E/kT}\right)^2}$

(c) $U=\frac{NE}{1+e^{E/kT}}$ and $C=k\frac{NE^2}{(kT)^2}\frac{e^{E/kT}}{\left(1+e^{E/kT}\right)^2}$

(d) $U=\frac{NE}{1+e^{E/kT}}$ and $C=k\frac{NE^2}{(kT)^2}\frac{e^{E/kT}}{\left(1+e^{E/kT}\right)^2}$

Ans. : (c), (d)

Solution: Since, $E_1=0, E_2=E$, For one particle,

$$U=\frac{E_1\times\exp\left(-\frac{E_1}{kT}\right)+E_2\times\exp\left(-\frac{E_2}{kT}\right)}{\exp\left(-\frac{E_1}{kT}\right)+\exp\left(-\frac{E_2}{kT}\right)}=\frac{E\times\exp\left(-\frac{E}{kT}\right)}{1+\exp\left(-\frac{E}{kT}\right)}=\frac{E}{1+\exp\left(\frac{E}{kT}\right)}$$

So for N Particle $U=\frac{NE}{1+\exp\left(\frac{E}{kT}\right)}\Rightarrow C=\left(\frac{dU}{dT}\right)_V=k\frac{NE^2}{(kT)^2}\frac{e^{E/kT}}{\left(1+e^{E/kT}\right)^2}$

Q39. Consider two identical, finite, isolated systems of constant heat capacity C at temperature T_1 and $T_2\ (T_1>T_2)$. An engine works between them until their temperatures become equal. Taking into account that the work performed by the engine will be maximum $(=W_{\max})$ if the process is reversible (equivalently, the entropy change of the entire system is zero), the value of $W_{\max}$ is:

(a) $C(T_1-T_2)$ (b) $\frac{C(T_1-T_2)}{2}$ (c) $C\left(T_1+T_2-\sqrt{T_1T_2}\right)$ (d) $C\left(\sqrt{T_1}-\sqrt{T_2}\right)^2$

Ans. : (d)

Solution: For maximum, $dS = C\left(\ln\frac{T_f^2}{T_1T_2}\right) = 0 \Rightarrow T_f = \sqrt{T_1T_2}$

$$dW_{\max} = C\left(T_1 - T_f\right) + C\left(T_2 - T_f\right) = C\left(T_1 + T_2 - 2T_f\right)$$

$$= C\left(T_1 + T_2 - 2\sqrt{T_2T_1}\right) = C\left(\sqrt{T_1} - \sqrt{T_2}\right)^2$$

Q40. A white dwarf star has V and contains N electrons so that the density of electrons is $n = \frac{N}{V}$. Taking the temperature of the star to be $0\,K$, the average energy per electron in the star is $\varepsilon_0 = \frac{3\hbar^2}{10\,m}\left(3\pi^2 n\right)^{2/3}$, where m is the mass of the electron. The electronic pressure in the star is:

(a) $n\varepsilon_0$ (b) $2n\varepsilon_0$ (c) $\frac{1}{3}n\varepsilon_0$ (d) $\frac{2}{3}n\varepsilon_0$

Ans. : (d)

Solution: Since, $\varepsilon_0 = \frac{3\hbar^2}{10\,m}\left(3\pi^2 n\right)^{2/3} = \frac{3\hbar^2}{10m}\left(3N\pi^2\right)^{2/3}\frac{1}{V^{2/3}}$ and $U = N\varepsilon_0$

$$P = -\frac{dU}{dV} = -N\frac{d\varepsilon_0}{dV} = -N\frac{3\hbar^2}{10m}\left(3N\pi^2\right)^{2/3}\left(-\left(\frac{2}{3}\right)V^{-5/3}\right) = \frac{2}{3}n\varepsilon_0$$

Q41. In the radiation emitted by a black body, the ratio of the spectral densities at frequencies 2ν and ν will vary with ν as:

(a) $\left[e^{h\nu/k_BT} - 1\right]^{-1}$ (b) $\left[e^{h\nu/k_BT} + 1\right]^{-1}$ (c) $\left[e^{h\nu/k_BT} - 1\right]$ (d) $\left[e^{h\nu/k_BT} + 1\right]$

Ans. : (b)

Solution: The ratio of spectral density at the frequencies 2ν and ν

$$\frac{\dfrac{1}{\exp\frac{2h\nu}{kT} - 1}}{\dfrac{1}{\exp\frac{h\nu}{kT} - 1}} = \frac{\exp\frac{h\nu}{kT} - 1}{\exp\frac{2h\nu}{kT} - 1} = \frac{\exp\frac{h\nu}{kT} - 1}{\left(\exp\frac{h\nu}{kT} - 1\right)\left(\exp\frac{h\nu}{kT} + 1\right)} = \frac{1}{\left(\exp\frac{h\nu}{kT} + 1\right)} = \left[e^{h\nu/k_BT} + 1\right]^{-1}$$

Q42. An isolated box is divided into two equal compartments by a partition (see figure). One compartment contains a van der Waals gas while the other compartment is empty. The partition between the two compartments is now removed. After the gas has filled the entire box and equilibrium has been achieved, which of the following statement(s) is (are) correct?

(a) Internal energy of the gas has not changed

(b) Internal energy of the gas has decreased

(c) Temperature of the gas has increased

(d) Temperature of the gas has decreased

Ans. : (a) and (d)

Solution: It is the example of free expansion, so Internal energy of the gas has not changed

$$dU = C_V T - \frac{a}{V} dV$$

For van der Waal gas, $dU = C_V dT + \frac{a}{V^2} dV$

For keeping internal energy constant, if dV increases then dT must decrease

Q43. Consider a Carnot engine operating between temperature of $600\,K$ and $400\,K$. The engine performs $1000\,J$ of work per cycle. The heat (in Joules) extracted per cycle from the high temperature reservoir is…………..

(Specify your answer to two digits after the decimal point)

Ans. : 3000

Solution: $\eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} \Rightarrow 1 - \frac{400}{600} = \frac{1000}{Q_1} \Rightarrow \frac{1000}{Q_1} = \frac{2}{6} \Rightarrow Q_1 = 3000\,J$

IIT-JAM 2018

Q44. Two boxes A and B contain an equal number of molecules of the same gas. If the volumes are V_A and V_B and λ_A and λ_B denote respective mean free paths, then

(a) $\lambda_A = \lambda_B$ (b) $\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$ (c) $\frac{\lambda_A}{V_A^{1/2}} = \frac{\lambda_B}{V_B^{1/2}}$ (d) $\lambda_A V_A = \lambda_B V_B$

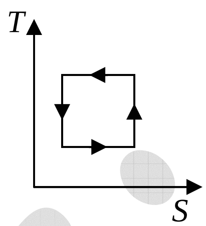
Ans. : (b)

Solution: $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P} = \frac{1}{\sqrt{2}\pi d^2 n} = \frac{V}{\sqrt{2}\pi d^2 N} \Rightarrow \lambda \propto V$, where $n = \frac{N}{V}$

$\lambda_A = KV_A,\ \lambda_B = KV_B$

$\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$

Q45. Which one of the figures correctly represents the $T-S$ diagram of a Carnot engine?

(a)


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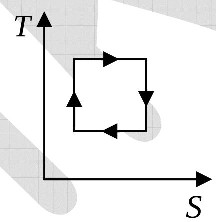
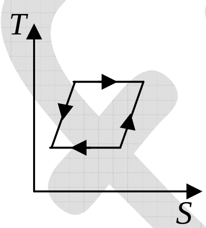
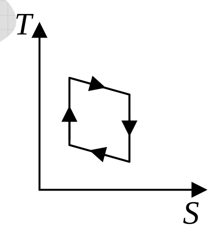
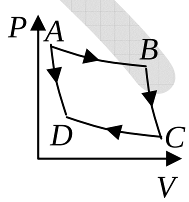


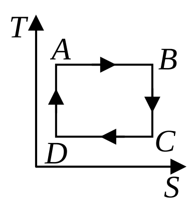
(c)


(d)


Ans. : (b)

Solution:





$A \rightarrow B$ Isothermal expansion

$B \rightarrow C$ Adiabatic expansion

$C \rightarrow D$ Isothermal compression

$D \rightarrow A$ Adiabatic compression

Q46. The equation of state for one mole of a non-ideal gas is given by $PV = A\left(1+\frac{B}{V}\right)$, where the coefficient A and B are temperature dependent. If the volume changes from V_1 to V_2 in an isothermal process, the work done by the gas is

(a) $AB\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$ (b) $AB\ln\left(\frac{V_2}{V_1}\right)$

(c) $A\ln\left(\frac{V_2}{V_1}\right)+AB\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$ (d) $A\ln\left(\frac{V_2-V_1}{V_1}\right)+B$

Ans. : (c)

Solution: $PV = A\left(1+\frac{B}{V}\right) \Rightarrow P = \left(\frac{A}{V}+\frac{AB}{V^2}\right)$

$$W = PdV = \int_{V_1}^{V_2}\frac{A}{V}dV + \int_{V_1}^{V_2}\frac{AB}{V^2}dV = A\ln\frac{V_2}{V_1} + AB\cdot\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$$

Q47. An ideal gas consists of three dimensional polyatomic molecules. The temperature is such that only one vibrational mode is excited. If R denotes the gas constant, then the specific heat at constant volume of one mole of the gas at this temperature is

(a) $3R$ (b) $\frac{7}{2}R$ (c) $4R$ (d) $\frac{9}{2}R$

Ans.: (c)

Solution: For a polyatomic gas

$$C_P = (4+f)R$$

$$C_V = (3+f)R$$

As, $f=1, C_V = 4R$

Q48. Consider an ensemble of thermodynamic systems each of which is characterized by the same number of particles, pressure and temperature. The thermodynamic function describing the ensemble is

(a) Enthalpy (b) Helmholtz free energy

(c) Gibbs free energy (d) Entropy

Ans. : (c)

Solution: The variable

$$G = H - TS$$

$$dG = dH - TdS - SdT$$

$$= TdS + VdP - TdS - SdT$$

$$dG = VdP - SdT$$

Q49. Which of the following relations is (are) true for thermodynamic variables?

(a) $TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$ (b) $TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$

(c) $dF = -SdT + PdV$ (d) $dG = -SdT + VdP$

Ans. : (b), (d)

Solution: $S = S(T,V) \Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$

$TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$ (a) is correct.

$$S = S(T,P) \Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

$$TdS = T\left(\frac{\partial S}{\partial T}\right)_P dT + T\left(\frac{\partial S}{\partial P}\right)_T dP$$

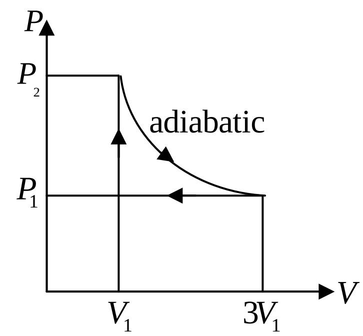
$= C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$, (b) is correct.

$dF = -SdT - PdV$ so (c) is incorrect

$dG = -SdT + PdV$ so (d) is correct

Q50. Consider a monoatomic ideal gas operating in a closed cycle as shown in the $P-V$ diagram given below. The ratio $\frac{P_1}{P_2}$ is ________.

(Specific your answer upto two digits after the decimal point)

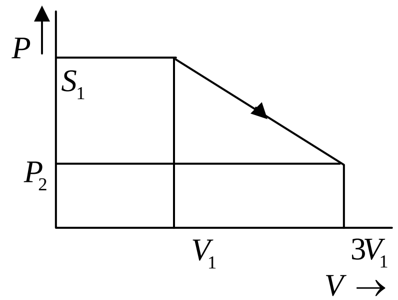


Ans. : 0.16

Solution: For monoatomic gas $r = \frac{5}{3}$

For adiabatic problems $P_1V_1^r = P_2V_2^r$

$$\Rightarrow \frac{P_1}{P_2} = \left(\frac{V_1}{V_1 \times 3}\right)^{5/3} = 3^{-\frac{5}{3}} = 0\cdot16$$



IIT-JAM 2019

Q51. The Fermi-Dirac distribution function $\left[n(\varepsilon)\right]$ is

(k_B is the Boltzmann constant, T is the temperature and ε_F is the Fermi energy)

(a) $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\varepsilon_F}{k_BT}} - 1}$

(b) $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon_F-\varepsilon}{k_BT}} - 1}$

(c) $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\varepsilon_F}{k_BT}} + 1}$

(d) $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon_F-\varepsilon}{k_BT}} + 1}$

Ans. : (c)

Q52. In a heat engine based on the Carnot cycle, heat is added to the working substance at constant

(a) Entropy

(b) Pressure

(c) Temperature

(d) Volume

Ans. : (c)

Q53. Isothermal compressibility is given by

(a) $\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$

(b) $\frac{1}{P}\left(\frac{\partial P}{\partial V}\right)_T$

(c) $-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$

(d) $-\frac{1}{P}\left(\frac{\partial P}{\partial V}\right)_T$

Ans. : (c)

Q54. A red star having radius r_R at a temperature T_R and a white star having radius r_w at a temperature T_w, radiate the same total power. If these stars radiate as perfect black bodies, then

(a) $r_R > r_w$ and $T_R > T_w$

(b) $r_R < r_w$ and $T_R > T_w$

(c) $r_R > r_w$ and $T_R < T_w$

(d) $r_R < r_w$ and $T_R < T_w$

Ans. : (c)

Solution: $E = \sigma AT^4 \left(\in = 1\right) \Rightarrow \sigma \times 4\pi r_W^2 T_W^4 = \sigma \times 4\pi \times r_R^2 \times T_R^4$ as $r_W < r_R$

$$T_W = T_R \times \left(\frac{r_R}{r_W}\right)^2 \; T_W > T_R$$

Q55. During free expansion of an ideal gas under adiabatic condition, the internal energy of the gas.

(a) Decreases (b) Initially decreases and then increases

(c) Increases (d) Remains constant

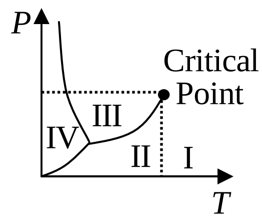
Ans. : (d)

Solution: As $W = \Delta U + Q$

$$Q = 0 \Rightarrow W = \Delta U$$

Work is done at the expense of internal energy.

Q56. In the given phase diagram for a pure substance regions I, II, III, IV, respectively represent



(a) Vapour, Gas, Solid, Liquid (b) Gas, Vapour, Liquid, solid

(c) Gas, Liquid, Vapour, solid (d) Vapour, Gas, Liquid, Solid

Ans. : (b)

Solution: IV – Solid

III – Liquid

II – Vapour

I – Gas (superheated dry vapour)

Q57. A thermodynamic system is described by the P, V, T coordinates. Choose the valid expression(s) for the system.

(a) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial P}{\partial T}\right)_V$ (b) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial P}{\partial T}\right)_V$

(c) $\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -\left(\frac{\partial V}{\partial P}\right)_T$ (d) $\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = \left(\frac{\partial V}{\partial P}\right)_T$

Ans. : (a), (c)

Q58. Two gases having molecular diameters D_1 and D_2 and mean free paths λ_1 and λ_2, respectively, are trapped separately in identical containers. If $D_2 = 2D_1$, then $\frac{\lambda_1}{\lambda_2} =$ ________.

(Assume there is no change in other thermodynamic parameters)

Ans. : 4

Solution: $x \propto \frac{1}{d^2} \Rightarrow \frac{x_1}{x_2} = \left(\frac{d_2}{d_1}\right)^2 = 4$

Q59. A di-atomic gas undergoes adiabatic expansion against the piston of a cylinder. As a result, the temperature of the gas drops from $1150\ K$ to $400\ K$. The number of moles of the gas required to obtain $2300\,J$ of work from the expansion is ______. (The gas constant $R = 8.314\,J\,\text{mol}^{-1}K^{-1}$.)

(Round off to 2 decimal places)

Ans. : 0.1475

Solution: $\gamma = \frac{7}{5}$

$$W = \frac{nR(T_2 - T_1)}{1 - V}$$

$$\Rightarrow 2300 = n \times 8.314 \times \frac{(400 - 1150)}{1 - 1.4} \Rightarrow n = 0.1475$$

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} \Rightarrow \lambda \propto \frac{1}{d^2}$$