IMPORTANT NOTE FOR CANDIDATES

- **Geology Section: Q. Nos. 1-15 (Objective Questions) and Q. Nos. 46-52 (Subjective Questions).**
- **Physics Section: Q. Nos. 16-30 (Objective Questions) and Q. Nos. 53-59 (Subjective Questions).**
- **Mathematics Section: Q. Nos. 31-45 (Objective Questions) and Q. Nos. 60-66 (Subjective Questions).**
- **Select any *TWO* Sections.**
- **Attempt objective and subjective questions of the selected *TWO* sections.**
- **Questions 1-45 (objective questions) carry *three* marks each and questions 46-66 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 11 only.**

2012(GEO-PHYSICS)

GEOLOGY SECTION-(OBJECTIVE QUESTIONS)

Q1. Which one of the following landforms results exclusively from glacial melt waters?

(a) Roches moutonneés (b) Eskers

(c) Hanging valleys (d) Cirques

Q2. Wind-laid dust deposits consisting largely of silt are known as

(a) dunes (b) playas (c) pediments (d) loess

Q3. Coarse sediments accumulating at the inner-side of loops of meandering rivers are called

(a) point bars (b) spits (c) barrier islands (d) bay barriers

Q4. Match the minerals in **Group I** with their respective polymorphs in **Group II**.

Group I	**Group II**
P. Sillimanite	1. Calcite
Q. Tridymite	2. Pyrite
R. Aragonite	3. Kyanite
S. Troilite	4. Crisobalite

(a) P-3, Q-4, R-1, S-2 (b) P-3, Q-1, R-2, S-4

(c) P-4, Q-2, R-3, S-1 (d) P-1, Q-2, R-4, S-3

Q5. Match the geological units in **Group I** with the appropriate parts of the geological time-scale in **Group II**.

Group I	**Group II**
P. Siwalik Supergroup	1. Permian-Jurassic
Q. Cuddapah Supergroup	2. Cretaceous-Tertiary boundary
R. Gondwana Supergroup	3. Proterozoic
S. Deccan Trap	4. Miocene-Pleistocene

(a) P-1, Q-2, R-4, S-3 (b) P-2, Q-1, R-3, S-4

(c) P-4, Q-3, R-2, S-1 (d) P-4, Q-3, R-1, S-2

Q6. Match the time boundaries in **Group I** with the corresponding ages in **Group II**.

Group I	**Group II**
P. Archaean-Proterozoic boundary	1. ~ 550 Ma
Q. Cretaceous-Tertiary boundary	2. ~ 66.5 Ma
R. Precambrian-Cambrian boundary	3. ~ 104 Yrs
S. Pleistocene-Holocene boundary	4. ~ 2500 Ma

(a) P-4, Q-2, R-1, S-3 (b) P-1, Q-2, R-3, S-4

(c) P-1, Q-2, R-4, S-3 (d) P-4, Q-1, R-3, S-2

Q7. Diamond-bearing conglomerates are found along the contact between

(a) Bababudan Group and Chitradurga Group (b) Chari Formation and Katrol Formation

(c) Rewa Group and Bhander Group (d) Surma Group and Tipam Group

Q8. The Q-A-P-F double triangle in the IUGS classification scheme for the entire range of igneous rocks should be used when the volume of the ultramafic mineral component in the rock is

(a) > 90% (b) < 90% (c) > 66% (d) < 66%

Q9. A highly porous lithology

(a) always has high permeability (b) always has low permeability

(c) may or may not have high permeability (d) always has water inside the pore space

Q10. In rocks of which metamorphic facies would you expect to find micro-diamonds?

(a) amphibolite (b) granulite (c) greenschist (d) eclogite

Q11. In Bombay High, oil mainly occurs in limestone horizons of _____ age.

(a) Cretaceous (b) Palaeocene (c) Miocene (d) Pliocene

Q12. A circle of 12 cm diameter becomes an ellipse with a major axis of 36 cm after constant area homogeneous deformation. What is the length of its minor axis in cm?

(a) 2 (b) 4 (c) 6 (d) 8

Q13. The sense of ductile shear should be interpreted from rock sections cut

(a) parallel to the main foliation and parallel to the stretching lineation

(b) perpendicular to the main foliation and perpendicular to the stretching lineation

(c) perpendicular to the stretching lineation and parallel to the main foliation

(d) perpendicular to the main foliation and parallel to the stretching lineation

Q14. About 80% of the coal reserves of India are in the

(a) Godavari valley (b) Wardha valley (c) Damodar valley (d) Mahanadi valley

Q15. In India, Tertiary coal is mainly found in the state of

(a) Kerala (b) Jammu & Kashmir

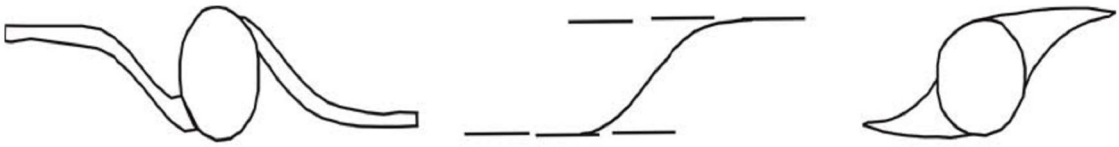
(c) Bihar (d) Uttar Pradesh

GEOLOGY SECTION-(SUBJECTIVE QUESTIONS)

Q16.(a) (i) Define extension (e), stretch (s) and quadratic elongation (λ). (ii) Calculate these parameters for a line that was initially 5 m long and has a final length of 10 m.

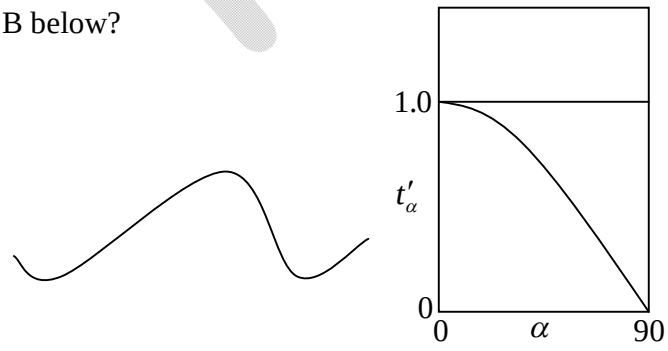
(6)

(b) (i) The figure below is a plan view of three ductile shear sense indicators. Name each and identify the sense of shear (dextral or sinistral) in each case.



A B C

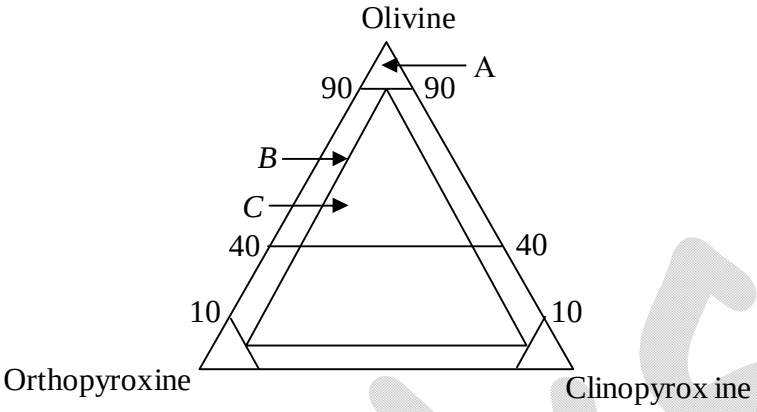
(ii) In the fold arc given in figure A below, mark one crest point, one through point and one point of inflection. Assume that its dip isogons are parallel to the axial trace. Name the fold according to Ramsay's classification scheme. Where do such folds plot in the given graph of t'_α (ratio of thickness measured at a location on the fold limb to thickness measured at the hinge of the fold) versus α (limb inclination) in figure B below? How is a *flattened parallel fold* designated in Ramsay's scheme, and in which field would it plot in figure B below?



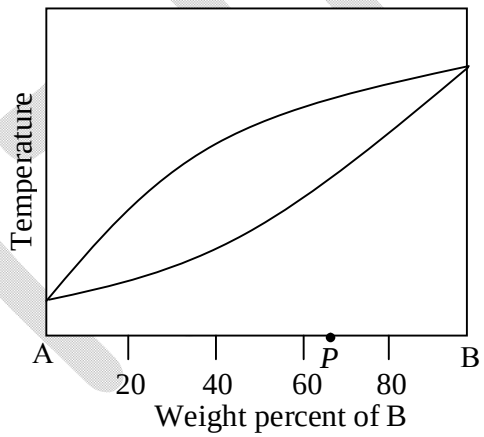
A B

(9)

Q17. (a) (i) In the following triangle for the classification of ultramafic rocks, name the rocks A, B and C.



(ii) In the given binary diagram of compounds 'A' and 'B', a solid of composition 'P' is progressively heated. What will be the composition of the first melt formed, and why?



(6)

(b) (i) Under what conditions are an arkose and a quartz arenite expected to form?

(ii) What is 'dedolomitization'? What causes it?

(9)

Q18. (a) (i) What is the 'Principle of Uniformitarianism'? What are its merits and demerits?

(ii) What characterizes a litho-unit as a 'Formation'?

(9)

(b) Arrange the following stratigraphic units in order of decreasing age:

(i) Talchir Boulder Beds, Dihing Group, Bhuj Formation, Vaikrita Group, Older Metamorphic Group;

(ii) Peninsular Gneissic Complex, Chitradurga Group, Bababudan Group, Sargur Schist

(6)

Q19. (a) On what physical principle is the concept of isostasy based? Using Pratt's hypothesis, explain why continents are elevated with respect to oceans.

(9)

(b) The P-wave and S-wave velocities (V_P and V_S, respectively) within the Earth are given by the following expressions:

$$V_P = \sqrt{\frac{K + 4\mu/3}{\rho}} \text{ and } V_S = \sqrt{\frac{\mu}{\rho}}$$

Based on the above, explain

(ii) why V_P and V_S increase with depth in the same compositional layer;

(ii) why S-waves do not pass through the outer core.

(6)

Q20. (a) The reaction sets given below have relevance for the genesis of a particular category of copper deposits.

Set-B: $2FeS_2 + 15O + 8H_2O + CO_2 = 2Fe(OH)_3 + 4H_2SO_4 + H_2CO_3$

$2CuFeS_2 + 17O + 6H_2O + CO_2 = 2Fe(OH)_3 + 2CuSO_4 + H_2CO_3$

Set-B: $PbS + CuSO_4 = CuS + PbSO_4$

$5FeS_2 + 14CuSO_4 + 12H_2O = 7Cu_2S + 5FeSO_4 + 12H_2SO_4$

$CuFeS_2 + CuSO_4 = 2CuS + FeSO_4$

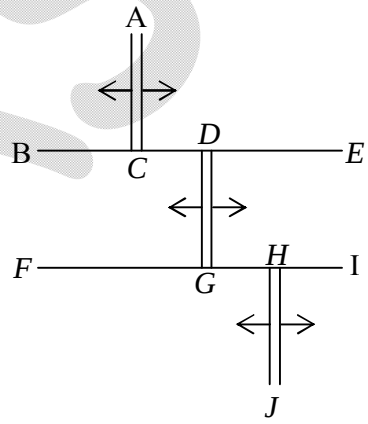
(i) What is the role of the reactions in Set-A for the formation and exploration of these copper deposits? (ii) What is the importance of the reactions in Set-B for the generation of these copper deposits?

(9)

(b) Which geophysical methods are best suited for the exploration of economic deposits of (i) chromite and (ii) magnetite, and why?

(6)

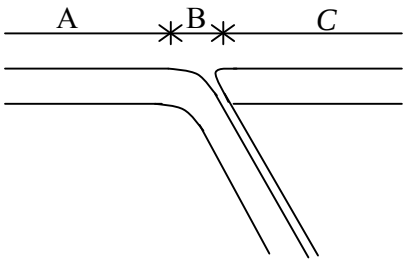
Q21. (a) The figure given is a plan view showing a segment of the boundary zone between two plates. The arrows represent the velocity of movement across parts of the boundary. BE and FI are fault scarps that apparently displace the segments AC, DG and HJ.



What type of plate boundary is represented by the segments AC, DG and HJ, and why? What other type of motion occurs along the plate boundary in this figure, and in which segments? Which parts of the fault scarps BE and FI experience no earthquakes and why?

(9)

(b) The figure shown is a cross-section through a subduction zone, with the plate to the left subducting below the plate to the right. In which zones (A, B or C) are the trench and volcanic arc located? Which of these zones would be characterized by the highest and the lowest heat flow, and why?



(6)

Q22. (a) Minerals A, B and C are all anhydrous silicates composed of Na, Al, Si and O. A and C are framework silicates, while B is a chain silicate. A is triclinic, B is monoclinic and C is hexagonal.

(i) Identify A, B and C and write their chemical formulae.

(ii) Write down a set of optical properties by which A, B and C can be distinguished in thin section.

(9)

(b) (i) Give 3 ways in which the form {100} in the isometric system differs from the form {100} in the tetragonal system. (ii) How does the position of aluminium in the crystal structure of grossular garnet $\left(Ca_3Al_2Si_3O_{12}\right)$ differ from that in the crystal structure of anorthite $\left(CaAl_2Si_2O_3\right)$?

(6)

PHYSICS SECTION-(OBJECTIVE QUESTIONS)

Q23. The surface of metal is illuminated alternately with light waves of energies $E_1 = 4.0eV$ and $E_2 = 2.5eV$. The ratio of maximum velocities of the photoelectrons emitted in two cases is 2.0. The work function W of the metal in eV is

(a) 0.5 (b) 1.5 (c) 2.0 (d) 2.5

Q24. A satellite moves in an elliptical orbit around the earth. The minimum and the maximum distance of the satellite from the surface of the earth are $6.3\times10^5 m$ and $3.63\times10^6 m$, respectively. The radius of the earth is $6.37\times10^6 m$. The ratio of speed of the satellite at apogee to its speed at perigee is

(a) 0.3 (b)0.35 (c) 0.6 (d) 0.7

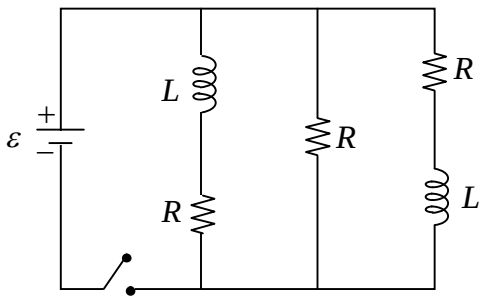
Q25. The dispesion relation for surface waves propagating in a find is given as $\omega^2 = \alpha k + \beta k^3$, where α and β are constant with appropriate units. The phase velocity V_P becomes equal to the group velocity V_g at $k = k_0$. The value of k_0 is

(a) $\sqrt{\alpha/2\beta}$ (b) $\sqrt{2\alpha/\beta}$ (c) $\sqrt{\alpha/\beta}$ (d) $\sqrt{3\alpha/\beta}$

Q26. The degree of polarization $P = (I_{max} - I_{min})/(I_{max} + I_{min})$ of mixture of unpolarized light and linearly polarized light is 0.2. The ratio of intensity I_P of the polarized component and the intensity I_u of the unpolarized component of this light is

(a) 0.55 (b) 0.5 (c) 0.3 (d) 0.25

Q27. The circuit given contains three identical resistors with resistance R, two identical inductors with inductance L and an ideal battery with emf ε. If i_1 and i_2 are value of the battery current just after the switch is closed and long after the switch is closed respectively, the ratio i_1 / i_2 is



(a) 1/3 (b)2/3 (c) 2 (d) 3

Q28. A photon ionizes a hydrogen atom which is in the ground state. The liberated electron recombines with a proton to form another hydrogen atom in the first excited state, and emits a new photon of energy 27 eV. The energy of the hydrogen atom in the n^{th} energy state is given as $E_n = -13.6/n^2\ eV$. The energy in eV of the original photon is

(a) 33.8 (b) 18.6 (c) 23.6 (d) 37.2

Q29. Some oxygen molecules are enclosed in a container at pressure P and temperature T such that their mean free path is 0.03 m and r.m.s. speed is 483 m/sec. The time interval in sec, during which half the molecules are unscattered, is

(a) 43×10^{-6} (b) 34×10^{-6} (c) 86×10^{-6} (d) 22×10^{-6}

Q30. The density ρ and the molar mass M of a bivalent sample of volume V = 4×10^{-6} m^3 are 1.7×10^3 kg/m^3 and 24.0×10^{-3} kg/mole, respectively. Avogadro's number, $N_A = 6.0 \times 10^{23}$/mol. The number of conduction electrons in the sample is

(a) 8.5×10^{22} (b) 1.7×10^{23} (c) 3.4×10^{23} (d) 4.2×10^{22}

Q31. A diffraction grating of length 2.5×10^{-2} m is illuminated by a light with two wavelengths 5997 Å and 6003 Å. The maximum size of the grating element d (in μm) required to resolve the two wavelengths in the first order is

(a) 50 (b) 25 (c) 75 (d) 100

Q32. The temperature in K at which Oxygen molecules have the same r.m.s. speed as that of Helium atoms at 300 K is

(a) 2100 (b) 2400 (c) 1225 (d) 1873

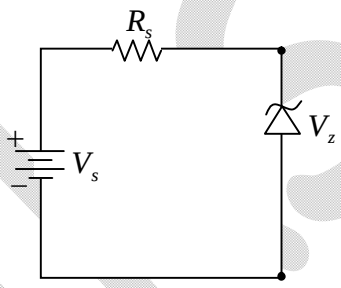
Q33. A light beam from a laser pointer, on normal incidence, creates a circular spot of diameter 2×10^{-3} m on a perfectly reflecting surface. If the radiation pressure P on the surface due to totally reflected beam is $(2/3) \times 10^{-5}$ N/m^2, the time averaged power of the laser beam (in mW) is

(a) 4π (b) $\pi/2$ (c) 2π (d) π

Q34. A train is travelling on straight rails with a speed of 49 m/sec. Its whistle emits a sound at frequency 480 Hz. A car is moving with a speed of 28 m/sec on a nearby road parallel to the rails in the opposite direction. The velocity of sound in air is 343 m/sec. The difference in frequencies (in Hz) heard by the car driver when the car approaches the train and the car moves away from the train is

(a) 110 (b) 220 (c) 330 (d) 440

Q35. The Zener diode, as shown in the figure below, has Zener voltage V_z = 15 V and power rating of 0.5 W. If V_s = 40 V, the minimum value of R_s in Ω that prevents the Zener diode from being destroyed is



(a) 750

(b) 1050

(c) 540

(d) 250

Q36. The electric field of an electromagnetic wave is $\vec{E}(\vec{r},t)=\sqrt{2}E_0\hat{z}\cos(x-y-\omega t)$. If c stands for the velocity of the wave, the magnetic field $\vec{B}(\vec{r},t)$ is

(a) $\vec{B}(\vec{r},t)=-\frac{1}{c\sqrt{2}}(\hat{x}+\hat{y})E_0\cos(x-y-\omega t)$

(b) $\vec{B}(\vec{r},t)=\frac{1}{c}(\hat{x}-\hat{y})E_0\cos(x-y-\omega t)$

(c) $\vec{B}(\vec{r},t)=-\frac{1}{c}(\hat{x}+\hat{y})E_0\cos(x-y-\omega t)$

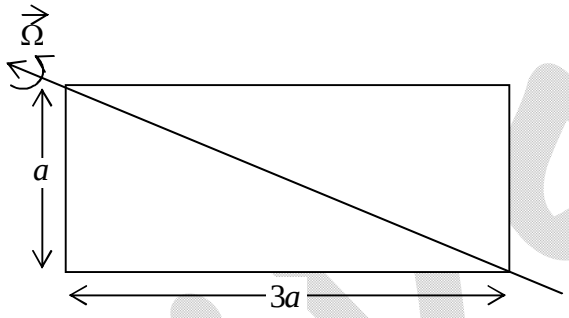
(d) $\vec{B}(\vec{r},t)=\frac{1}{c\sqrt{2}}(\hat{x}-\hat{y})E_0\cos(x-y-\omega t)$

Q37. X-rays are diffracted from a set of planes with Miller indices (111) in a NaCl crystal at Bragg angle of 30°. If the lattice constant of the crystal is 5.65 Å, the wavelength λ of the X-rays is

(a) 3.25 Å (b) 1.26 Å (c) 6.23 Å (d) 2.62 Å

PHYSICS SECTION-(SUBJECTIVE QUESTIONS)

Q38.(a) A uniform rectangular plate of length $3a$ and width a has mass M. Ignoring gravity, determine the magnitude and direction of the torque $\vec{\tau}$ required to rotate the plate with uniform angular velocity $\vec{\Omega}$ in counter clock-wise direction about the diagonal shown below.



(9)

(b) A smaller rectangular plate of length $3a/2$ and width $a/2$ is now cut from the plate shown above. Find the torque $\vec{\tau}'$ to rotate the new plate along the diagonal with the same angular velocity $\vec{\Omega}$.

(6)

Q39. The stream function $\Psi(x, y)$ of a two-dimensional flow is given by $\Psi(x, y) = 7x^2 - by^2$.

(a) Find the value of b for which the flow field is irrotational. Determine the velocity potential $\Theta(x, y)$ corresponding to the irrotational flow.

(9)

(b) Show that lines of constant $\Psi(x, y)$ are orthogonal to lines of constant $\Theta(x, y)$.

(6)

Q40. The motion of a one-dimensional damped oscillator is described by the differential equation: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$, where ω_0 is the natural angular frequency of the oscillator and β is the damping parameter. The oscillator is given an initial velocity V_0 at time t = 0.

(a) Find an expression for the displacement $x(t)$ of the oscillator.

(6)

(b) If $V_0 = 0.13$ m/sec, $\omega_0 = 13\text{sec}^{-1}$ and $\beta = 5\text{sec}^{-1}$,

(i) find the value of the 'amplitude' A_0 of the damped oscillatory motion at t = 0,

(ii) calculate the time t_1 at which the displacement first reaches its maximum X_1,

(iii) compute the maximum displacement X_1.

[Use data: $\tan^{-1}(2.4) \approx 1.176, e^{-0.49} \approx 0.613$ and $\sin(1.176) \approx 0.923$]

(9)

Q41. The convex surface of a plano-convex lens of glass with radius of curvature R = 0.5 m is placed on a flat glass plate, and illuminated from above with a monochromatic light of wavelength λ. The radius of the 30th dark, after the central spot, in the reflected light is r. Watching this ring, the lens is moved vertically upward by a distance h = 5×10^{-6} m. The new radius r' of the ring is 10^{-3} m smaller than the old radius r.

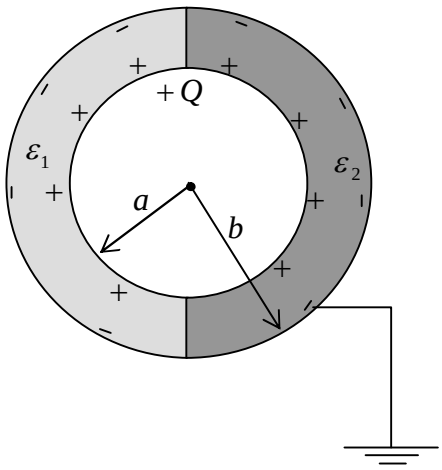
(a) Determine both the radii r and r'

(9)

(b) Compute the wavelength λ of the light used to form the rings.

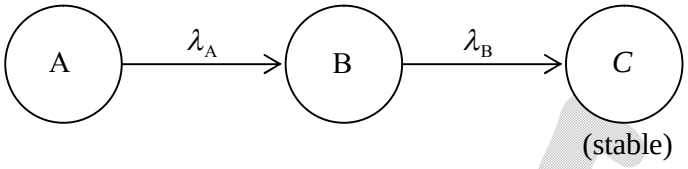
(6)

Q42. One half of the region between two thin concentric metallic spherical shells of radii a and b ($b > a$) is filled with a dielectric of permittivity ε_1 and the other half is filled with another dielectric of permittivity ε_2 . A charge $+Q$ is placed on the inner shell and the outer shell is grounded (see the figure below). Find the capacitance C of the system.



(15)

Q43. Nuclei of type A are being produced at a constant rate Q in a nuclear reaction. The unstable nuclei A decay to stable nuclei of type C through intermediate unstable nuclei of type B, as shown in the figure. If λ_A and λ_B are the decay constants of the nuclei A and B respectively, calculate the activity of the sample at time τ after the reaction process has started.

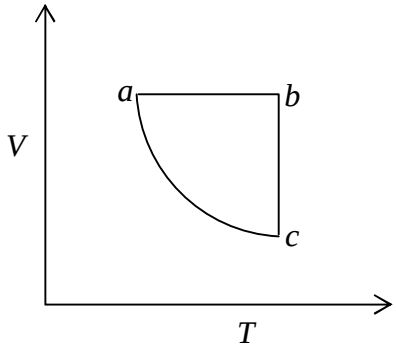


(15)

Q44.(a) A Carnot engine operating between two temperatures 727° C and 27° C is supplied heat energy at the rate of 500 Joule/cycle. Sixty percent of the work output is used to drive a refrigerator, which rejects heat to the surrounding at 27° C. If the refrigerator removes 1050 Joule of heat per cycle from the low temperature reservoir, determine the temperature of the reservoir.

(6)

(b) A system compressed along an adiabatic path $a \rightarrow c$ (see the figure below), requires 1000 Joule. Compressing the system along $b \rightarrow c$ requires 1500 Joule but 600 Joule of heat flows out of the system. Calculate (i) the change in the internal energy of the system for the path $a \rightarrow b$, and (ii) the total work done in the cycle $a \rightarrow b \rightarrow c \rightarrow a$.



(9)

MATHEMATICS SECTION-(OBJECTIVE QUESTIONS)

Q45. If a function $f:[0,1]\to \mathrm{R}$ is continuously differentiable, then there exists a continuous function $g:[0,1]\to$ R such that

(a) $g(x)=g(0)+\int_0^x f(t)dt$

(b) $f(x)=g(0)+\int_0^x g(t)dt$

(c) $f(x)=f(0)+\int_0^x g(t)dt$

(d) $g(x)=f(0)+\int_0^x f(t)dt$

Q46. Which of the following functions $f:\mathrm{C}\to\mathrm{C}$ is analytic?

(a) $f(z)=\mathrm{Re}z$

(b) $f(z)=z^2+\frac{\sin z}{2}$

(c) $f(z)=\bar{z}$

(d) $f(z)=1+i\,\mathrm{Im}\,z$

Q47. Let Q denote the set of all rational numbers. Then

(a) Q is an open set in R

(b) Q is a closed set in R

(c) The set of all limit points of Q is R

(d) Q is complete

Q48. Let A be an $m \times n$ real matrix and $x=[x_1\ x_2\ ....x_n]^T$. The system $Ax = 0$ has

(a) no solution

(b) infinitely many solutions if m is less than n

(c) infinitely many solutions if rank of A is n

(d) unique solution if rank of A is $\frac{n}{2}$

Q49. Let the function $f : \mathrm{R} \to \mathrm{R}$ be defined by $f(x) = x^3 - 15x^2 + 40$. The minimum value of f on [-1, 7] is

(a) 24 (b) 40 (c) -400 (d) -352

Q50. The radius of convergence of the series $1 + \frac{1}{2}\frac{x^2}{3} + \frac{1.3}{2.4}\frac{x^4}{5} + \frac{1.3.5}{2.4.6}\frac{x^6}{7} + \ldots..$ is

(a) 1/2 (b) 1 (c) 3/8 (d) $\sqrt{2}$

Q51. The value of $\lim_{n\to\infty} \sqrt{n^2 + n} - n$ is

(a) $\sqrt{2} - 1$ (b) ∞ (c) 1 (d) 1/2

Q52. Let $C[0, 1]$ be the set of all real valued continuous functions on [0, 1]. Which one of the following subsets of $C[0, 1]$ is linearly dependent?

(a) $\{1, \cos t, \sin t\}$ (b) $\{\tan^2 t, \cos^2 t, \sin^2 t\}$

(c) $\{1, \cos^2 t, \sin^2 t\}$ (d) $\{\tan t, \cos t, \sin t\}$

Q53. The differential equation for which $c^2x^2 + y^2 = c^2$ is the general solution, is

(a) $\frac{dy}{dx} = xy$ (b) $\frac{dy}{dx} = \frac{xy}{x^2 - 1}$ (c) $\frac{dy}{dx} = \frac{x^2 - y}{xy}$ (d) $\frac{dy}{dx} = xy(x^2 - 1)$

Q54. Which of the following is an exact differential equation?

(a) $2xydx + y^2dy = 0$ (b) $2xydx - y^2dy = 0$

(c) $2xydx + x^2dy = 0$ (d) $2xydx - x^2dy = 0$

Q55. If Δ and ∇ respectively stand for the forward and backward difference operators, then which of the following is true?

(a) $\Delta\nabla = 1$ (b) $\nabla\Delta = 1$

(c) $(1 + \Delta)(1 + \nabla) = 1$ (d) $(1 - \Delta)(1 - \nabla) = 1$

Q56. If the interval of differencing h is equal to 1, the factorial representation of the polynomial $x^3 - x$ is

(a) $x^{(3)} + 3x^{(2)}$ (b) $x^{(3)} - x^{(2)}$ (c) $x^{(3)} + x^{(2)}$ (d) $x^{(3)} + 3x^{(2)} - 2$

Q57. The curl of $\vec{v} = xyz\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$ is

(a) $x\left(z^2 - y^2\right)\hat{i} + y\left(x^2 - z^2\right)\hat{j} + z\left(y^2 - x^2\right)\hat{k}$

(b) $x\left(z^2 - y^2\right)\hat{i} - y\left(x^2 - z^2\right)\hat{j} + z\left(y^2 - x^2\right)\hat{k}$

(c) $y^2z^2\hat{i} + x^2z^2\hat{j} + x^2y^2\hat{k}$

(d) $y^2z^2\hat{i} - x^2z^2\hat{j} + x^2y^2\hat{k}$

Q58. A ball is drawn from a box containing 8 red balls, 6 white balls and 4 blue balls. What is the probability that it is not red?

(a) $\frac{1}{8}$ (b) $\frac{1}{18}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

Q59. A population consists of the four numbers 1, 4, 6, 9. Consider all possible samples of size two which can be drawn with replacement from this population. Then the standard deviation of the population is

(a) 8.5 (b) 8.15 (c) 7.5 (d) 5.0

MATHEMATICS SECTION-(SUBJECTIVE QUESTIONS)

Q60. (a) Let P be the vector space of all polynomials with real coefficients of degree less than or equal to three. Find the matrix representation of the linear transformation D:$P \rightarrow P$ defined as $Dp(x) = \frac{dp(x)}{dx}$ with respect to the basis $\{1, x, x^2, x^3\}$ of P. Determine all eigenvalues and eigenvectors of the matrix representation of D so obtained.

(9)

(b) Determine $g(x_1, x_2, x_3, x_4)$ where

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_1^2 & x_2^2 & x_3^2 & x_4^2 \\ x_1^3 & x_3^3 & x_3^3 & x_4^3 \end{vmatrix} = (x_4 - x_3)(x_4 - x_2)g(x_1, x_2, x_3, x_4)$$

(6)

Q61. (a) If f has a finite third derivative f''' in [a, b] and if $f(a) = f'(a) = f(b) = f'(b) = 0$, prove that $f'''(c) = 0$ for some c in (a, b).

(6)

(b) Suppose $\vec{v} = y\hat{i} - x\hat{j} + \hat{k}$. Let S be $x^2 + y^2 + z^2 = 4, x \geq 0, y \geq 0$ and let $\hat{n}$ denote the outer unit normal to S. Compute $\int\int_S \vec{v}.\hat{n}d\sigma$.

(9)

Q62.(a) Determine all the poles and their residues of $f(z) = \frac{1}{z^2 + z + 1}$

(6)

(b) Find the Laurent series of $\frac{1}{(z-1)(z-2)}$ with center 0 for

(i) the annulus $1 < |z| < 2$ and (ii) the region $|z| > 2$

(9)

Q63.(a) Using Green's theorem in the plane, evaluate $\oint_C y^2dx+\left(y^3+2x\right)dy$,where C is the boundary of the region R in the first quadrant bounded by the curves $y^2 = x$ and $x^2 = y$.

(9)

(b) Show that the function defined f : R $\rightarrow$ R defined by f(x)=(x+ 4)^3 is not uniformly continuous.

(6)

Q64.(a) Solve the differential equation $\frac{dy}{dx}=\left(x+y\right)^2$

(6)

(b) Solve the following initial value problem

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=0,\ y(0)=1,\ y'(0)=1$$

(9)

Q65.(a) Using Newton-Raphson method find the approximate value of the root of the equation $x^3+x^2-1=0$ upto the second iteration x_2, assuming the initial approximation x_0 as 1.

(6)

(b) Solve the following system of equations

$$4x_1-x_2+3x_3=2,\ \ 2x_1+5x_2-x_3=1,\ \ x_1+2x_2+5x_3=4$$

by using Gauss-Seidel method upto two iterations starting with the initial solution $x_1^{(0)}=x_2^{(0)}=x_3^{(0)}=0$ **(9)**

Q66.(a) If X and Y are independent random variables having density functions

$$f_1(x)=\begin{cases}e^{-x}, & x\geq 0\\ 0, & x<0\end{cases} \quad \text{and} \quad f_2(y)=\begin{cases}4e^{-4y}, & y\geq 0\\ 0, & y<0\end{cases}$$

respectively, then find the density function of their sum $U=X+Y$. **(9)**

(b) Find the characteristic function of a random variable X having density function

$$f(x)=\begin{cases}\frac{1}{2a}, & |x|\leq a\\ 0, & \text{otherwise}\end{cases}$$ **(6)**