INDIAN INSTITUTE OF SCIENCE
BANGALORE - 560012

ENTRANCE TEST FOR ADMISSIONS - 2010

Program : Integrated Ph.D
Entrance Paper : Physical Sciences
Paper Code : PS

Day & Date
SUNDAY, 25TH APRIL 2010

Time
2.00 P.M. TO 5.00 P.M.

Integrated Ph.D. (Physical Sciences)

General Instructions

- This question paper consists only of multiple-choice questions.
- Answers are to be marked in the OMR sheet provided.
- For each question darken the appropriate bubble to indicate your answer. Use only HB pencils for bubbling answers.
- Mark only one bubble per question. If you mark more than one bubble, the question will be evaluated as incorrect.
- If you wish to change your answer, please erase the existing mark completely before marking the other bubble.

All 50 multiple-choice questions in this test should be answered. Each question carries **TWO MARKS**. For each of these questions, four answers are provided; of these only **ONE** is correct.

N.B. There is negative marking: 1/2 mark will be deducted for each incorrect answer.

Some information that may be useful in answering this test is given below:

Acceleration due to gravity	$g = 9.8$ m/s^2
Molar gas constant	$R = 8.314$ J/mol K
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ J/K
Planck's constant	$h = 6.626 \times 10^{-34}$ J s
Speed of light in vacuum	$c = 3 \times 10^8$ m/s
Gravitational constant	$G = 6.672 \times 10^{-11}$ N m^2/kg^2
Mass of electron	$m_e = 9.1095 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.6726 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.6749 \times 10^{-27}$ kg
Coulomb constant	$k = 1/(4\pi\epsilon_0) = 8.988 \times 10^9$ N m^2/C^2
Charge of electron	$e = 1.602 \times 10^{-19}$ C
Permittivity of vacuum	$\epsilon_0 = 8.854 \times 10^{-12}$ F/m
Permeability of vacuum	$\mu_0 = 4\pi \times 10^{-7}$ T m/A
Energy conversion (eV to J)	1 eV $= 1.602 \times 10^{-19}$ J

PHYSICAL SCIENCES

1. The series

$$\frac{1-(\pi/3)^2/2!+(\pi/3)^4/4!-....}{\pi/3-(\pi/3)^3/3!+(\pi/3)^5/5!-....}$$

sums to

(a) $1/2$ (b) $\sqrt{3}/2$ (c) $1/\sqrt{3}$ (d) $\sqrt{3}$

2. Consider the function $f(x)=ax^5-bx^2+c, a\neq 0$. The graph of this function

(a) Never crosses the x-axis

(b) Always crosses the x-axis at least once

(c) Always crosses the x-axis 2 times

(d) Always crosses the x-axis 5 times

3. Given the three matrices

$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

and $[\sigma_i,\sigma_j]\equiv\sigma_i\sigma_j-\sigma_j\sigma_i$, then $[\sigma_1,[\sigma_2,\sigma_3]]+[\sigma_2,[\sigma_3,\sigma_1]]+[\sigma_3,[\sigma_1,\sigma_2]]$ is

(a) $\sigma_1^2+\sigma_2^2+\sigma_3^2$ (b) $\sigma_1+\sigma_2+\sigma_3$

(c) 0 (d) Identity

4. The trace of a 3 × 3 matrix is 1 and its determinant is 0. Which of the following has to be true?

(a) One of the eigenvalues is 0 (b) One of the eigenvalues is 1

(c) Two of the eigenvalues are 0 (d) Two of the eigenvalues are real

5. The equation $|2x-1|=|x+3|$ for real x has

(a) no solution (b) one solution (c) two solution (d) three solution

6. The sum $S_n(x)=\cos x+\cos 2x+.....+\cos nx$ is the same as

(a) $\dfrac{\sin\frac{nx}{2}}{\sin\frac{x}{2}}\cos\dfrac{(n+1)x}{2}$ (b) $\dfrac{\sin\frac{nx}{2}}{\sin\frac{x}{2}}\cos\dfrac{(n-1)x}{2}$

(c) $\dfrac{\cos\frac{nx}{2}}{\sin\frac{x}{2}}\sin\dfrac{(n+1)x}{2}$ (d) $\dfrac{\sin\frac{nx}{2}}{\cos\frac{x}{2}}\sin\dfrac{(n+1)x}{2}$

7. For $a > 1$,

$$\lim_{x\to\infty}\left[\frac{a^x - 1}{x(a-1)}\right]^{1/x}$$

is

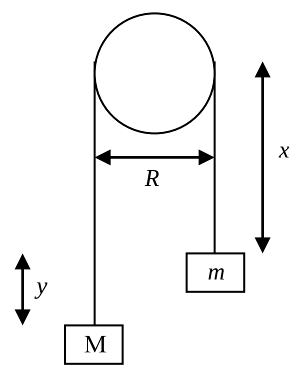
(a) e (b) e^a (c) a (d) 1

8. The motion of a particle moving in a line is described by the equation $x(t) = 6 + 4\left(\cos\omega t + \frac{1}{16}(\cos 2\omega t - 1)\right)$. Given that n is an integer, the maximum displacement occurs at

(a) $t = 2n\pi/\omega$ (b) $t = (2n+1)\pi/\omega$

(c) $t = \left(n + \frac{1}{2}\right)\pi/\omega$ (d) $t = \left(n + \frac{3}{4}\right)\pi/\omega$

9. A heavy uniform rope of length L and mass per unit length μ goes over a frictionless pulley of diameter R, and has two masses M and m attached to its two ends as shown. In terms of the distance x, the equilibrium position is given by



(a) $\frac{1}{2}\left(L - \frac{\mu L}{m+M} + \frac{\pi R}{2}\right)$ (b) $\frac{1}{2}\left(L + \frac{m+M}{\mu}\right)$

(c) $L - \frac{\pi R}{2}$ (d) $\frac{1}{2}\left(L - \frac{m-M}{\mu} - \frac{\pi R}{2}\right)$

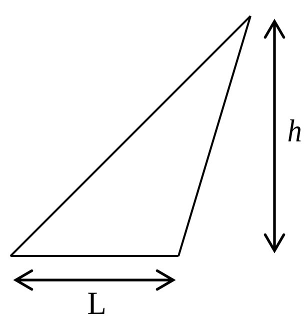
10. A triangle of uniform mass density of base L and height h is shown below. The centre of mass of the triangle lies at this distance above the base:

(a) $\frac{h}{2}$

(b) $\frac{2h}{3}$

(c) $\frac{h}{3}$

(d) $\frac{h}{6}$



11. A straight rod of length a is made of an unusual material having mass per unit length $\mu(x) = \lambda|x|$, where x is measured from the centre of the rod. The moment of inertia about an axis perpendicular to the rod and passing through one end of the rod is given by

(a) $\frac{\lambda a^4}{16}$ (b) $\frac{3\lambda a^4}{16}$ (c) $\frac{\lambda a^4}{32}$ (d) $\frac{3\lambda a^4}{32}$

12. A particle of mass m is located at a distance z along the axis of a uniform disk of mass M and radius R. The gravitational force felt by the mass m is given by

(a) $\frac{2GMm}{R^2}\left(\frac{z}{(z^2+R^2)^{1/2}}-1\right)$ (b) $\frac{2GMm}{R^2}\left(\frac{z}{(z^2-R^2)^{1/2}}-1\right)$

(c) $\frac{2GMm}{R^2}\left(\frac{z}{(z^2+R^2)^{1/2}}+1\right)$ (d) $\frac{2GMm}{R^2}\left(\frac{z}{(z^2+R^2)^{1/2}}\right)$

13. A pendulum consists of point bob of mass M hanging by a massless string. There is no friction anywhere in the system. For arbitrary angular displacements θ of the pendulum about the vertical, the time period of oscillation

(a) is independent of M and θ (b) depends on M but not θ

(c) depends on θ but not M (d) depends on both M and θ

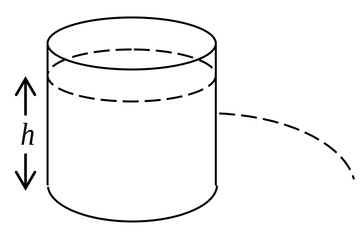
14. An observer O is at the origin of an inertial frame of reference I and rotating with angular velocity $\omega\hat{z}$ relative to it. A satellite of mass M is in a circular orbit of radius R about the origin with angular velocity $-\omega\hat{z}$ as seen from I. The total force $M\ddot{r}$ on the satellite in O's rest frame is

(a) $M\omega^2 R\hat{r}$ (b) $-M\omega^2 R\hat{r}$ (c) $4M\omega^2 R\hat{r}$ (d) $-4M\omega^2 R\hat{r}$

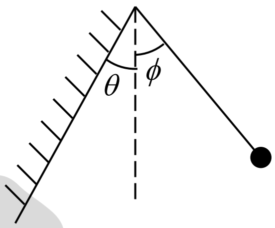
15. A large cylindrical container filled with water up to a height h rests on a table. Neglecting the effect of viscosity, at what height from the bottom of the container should a hole be made such that the resulting jet of water hits the surface of the table at the maximum distance?

(a) $h/\sqrt{2}$ (b) $h/2$

(c) $h/\sqrt{3}$ (d) $h/3$



16. A simple pendulum of length l is suspended from a hook mounted on a slanted wall. The wall makes a small angle θ with the vertical. The pendulum is displaced from the vertical by a small angle $\phi(\phi > \theta)$ and released. Assuming that the collision of the bob is elastic, the time period of oscillation is



(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{l}{g}}\left[\pi/2+\sin^{-1}\frac{\theta}{\phi}\right]$

(c) $2\pi\sqrt{\frac{l}{g}\frac{\theta}{\phi}}$ (d) $2\pi\sqrt{\frac{l}{g}}\left[\pi/2+\cos^{-1}\frac{\theta}{\phi}\right]$

17. Consider two blocks of constant specific heat capacity, one of which is at temperature T_1 and the other at T_2. A reversible engine transfers heat from the hotter block to the colder block until they reach the same temperature, which is

(a) $\sqrt{T_1T_2}$ (b) $\frac{T_1+T_2}{2}$ (c) $\frac{T_1T_2}{T_1+T_2}$ (d) T_2

18. A and B are both classical ideal gases of diatomic molecules. The point-like atoms in A are bonded rigidly to form diatomic molecules while in B they are connected by bonds of finite stiffness. The ratio of the specific heat per molecule at constant volume of gas A to gas B is

(a) 1 (b) 5/7 (c) 6/5 (d) 3/4

19. N molecules of an ideal gas are in a container of volume V_0. If a molecule has the same probability of being at any point in the container, the probability of M of them being in a smaller volume V in the container is

(a) M/N (b) $\frac{V^M(V_0-V)^{N-M}}{V_0^N}$

(c) $\frac{N!}{(N-M)!M!}\frac{V^M(V_0-V)^{N-M}}{V_0^N}$ (d) $\frac{N!}{(N-M)!M!}\frac{V_0^M(V_0-V)^{N-M}}{V^N}$

20. A mixture contains the same number of moles of two ideal gases A and B, with adiabatic constants γ_A nd γ_B respectively. The adiabatic constant γ of the mixture satisfies

(a) $\frac{1}{\gamma-1}=\frac{1}{2}\left(\frac{1}{\gamma_A-1}+\frac{1}{\gamma_B-1}\right)$ (b) $\frac{1}{\gamma-1}=\frac{1}{2}\left(\frac{1}{\gamma_A-1}-\frac{1}{\gamma_B-1}\right)$

(c) $\frac{1}{\gamma+1}=\frac{1}{2}\left(\frac{1}{\gamma_A+1}+\frac{1}{\gamma_B+1}\right)$ (d) $\frac{1}{\gamma+1}=\frac{1}{2}\left(\frac{1}{\gamma_A+1}-\frac{1}{\gamma_B+1}\right)$

21. A thermally insulated cubical box has two chambers of equal volume. Initially, one mole of a mono atomic ideal gas is placed in one of the chambers while the other chamber is kept empty. The gas slowly leaks through a small hole and eventually occupies the whole box. The change in entropy of the gas is given by

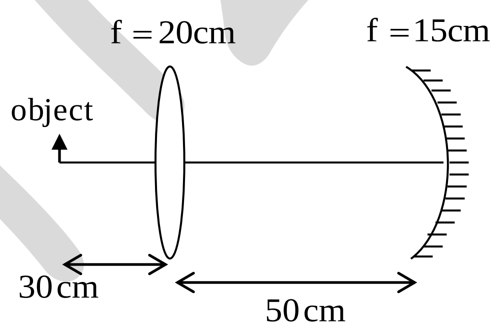
(a) $R/\ln 2$ (b) $R\ln 2$ (c) zero (d) $2R$

22. One mole of an ideal monoatomic gas is kept in a volume V and is at temperature T. If the volume and temperature of the gas are respectively changed to V' and T', in such a way that the entropy of the gas is unchanged. Then which of the following is true?

(a) $V' = 2V$, $T' = T/2$ (b) $V' = 8V$, $T' = T/4$

(c) $V' = 4V$, $T' = 8T$ (d) $V' = V/2$, $T' = 2T$

23. A thin convex lens L and concave mirror M with focal lengths 20 cm and 15 cm respectively are separated by a distance of 50 cm along their axis as shown below:



An object is placed at a distance of 30 cm before the lens. The location of the final image with respect to the mirror is

(a) 6 cm (b) 12 cm (c) 15 cm (d) 30 cm

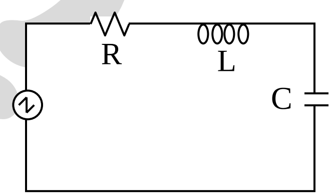
24. In a Michelson interferometer operating with monochromatic light of wavelength 2 μm, the mirror positions are initially adjusted to get maximum intensity at the detector. A thin glass plate of refractive index 1.5 is then inserted in the path of one of the interfering beams. If the detector intensity continues to be a maximum, the thickness of the plate is (neglecting reflections at the surfaces of the plate):

(a) 65 μm (b) 86 μm (c) 92 μm (d) 133 μm

25. A very thick piece of glass with refractive index n has a convex surface with radius of curvature R. For paraxial light rays incident on this surface from vacuum, the glass acts like a lens with focal length f measured from the surface, where f is given by

(a) $\frac{nR}{n-1}$ (b) $\frac{(n-1)R}{n}$ (c) $\frac{(n-1)R}{n+1}$ (d) $\frac{(n-1)R}{n-1}$

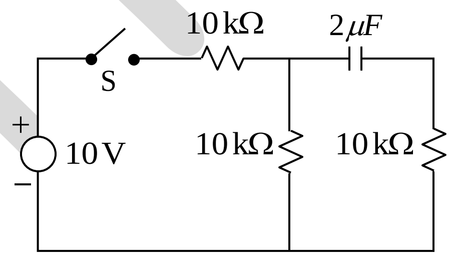
26. The LCR circuit shown below is driven by an ideal AC voltage source. The angular frequency ω_0 at which the voltage amplitude across the capacitor is a maximum is,



(a) $\sqrt{\frac{1}{LC}}$ (b) $\sqrt{\frac{1}{LC}\left(1-\frac{R^2C}{2L}\right)}$

(c) $\sqrt{\frac{1}{LC}\left(1-\frac{R^2C}{2L}\right)^2}$ (d) $\sqrt{\frac{1}{LC}\left(1-\frac{R^2C}{L}\right)}$

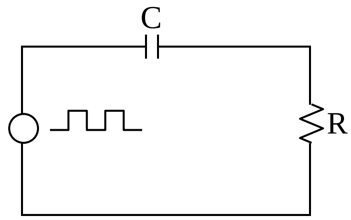
27. An RC circuit is connected to a DC voltage source through a switch as shown below.



The switch is initially open and the capacitor uncharged. At time t = 0, the switch is closed. The voltage across the capacitor at t = 30 μs is

(a) 5 mV (b) 7.5 mV (c) 10 mV (d) 20 mV

28. The RC circuit shown below is driven by a continuous square wave.



Which of the following figures is a correct representation of the output wave-form across the resistor?

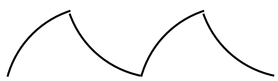


Figure 1

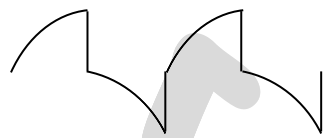


Figure 2

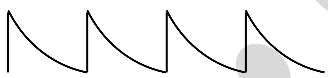


Figure 3

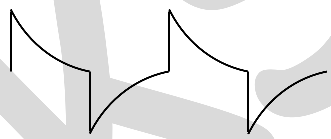


Figure 4

(a) Figure 1 (b) Figure 2 (c) Figure 3 (d) Figure 4

29. In an experiment the voltage across a resistor is measured repeatedly and found to obey a Gaussian distribution with mean 5V and standard deviation 1V. In any given measurement, if P_A is the probability of obtaining 3 ± 0.1 V and P_B the probability of obtaining 8 ± 0.1 V, then

(a) $P_A = P_B = 0$ (b) $P_A = P_B \neq 0$

(c) $P_A > P_B$ (d) $P_A < P_B$

30. A point charge Q is brought without any acceleration from infinity to a distance d from an infinite plane conducting sheet. The work done on the charge is given by

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4d}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2d}$ (c) $-\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4d}$ (d) $-\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2d}$

31. The electrostatic potential due to a uniformly charged circular disc is

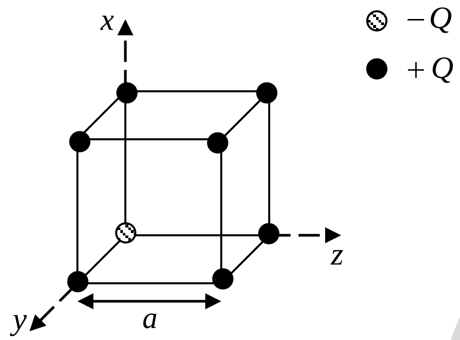
(a) the same at all points on the disc

(b) is larger at the centre than at the edge

(c) is larger at the edge than at the centre

(d) has a maximum value at half the radius

32. A cube of side a has point charges $+Q$ located at each of its vertices except for the origin where the charge is $-Q$ as shown below.



The electric field at the centre is:

(a) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$ (b) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(c) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$ (d) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

33. Two metallic cubes of side a and $2a$ respectively are separated by a large distance D, ($D >> a$). Initially, the smaller cube carries a charge Q while the larger one is uncharged. If a thin metallic wire is connected between the two cubes, the ratio of the charges on the smaller cube to the larger one will be:

(a) 1/4 (b) 1/2 (c) 2/3 (d) 4/9

34. A metallic ring of area 1 cm^2 and resistance 10 Ω is placed in a perpendicular time-varying magnetic field which has the following form

$$B(t) = 2e^{-0.5t}\cos(2\pi t)$$

where B is in Tesla and t is in seconds. The net charge that flows past any point in the ring from $t = 0$ to $t = \infty$ is

(a) 1 μC (b) 3 μC (c) 5 μC (d) 20 μC

35. Which of the following is NOT a solution of the Maxwell equations for the magnitude of the electric field $E(x, t)$ in free space? (ω and k are constants)

(a) $E(x,t) = e^{i(kx-\omega t)} + e^{i(kx+\omega t)}$ (b) $E(x,t) = e^{i(kx-\omega t)} + \sqrt{2}e^{t(kx+\omega t)}$

(c) $E(x,t) = e^{i(kx-\omega t)} + e^{i\left(\frac{kx}{\sqrt{2}}-\sqrt{2}\omega t\right)}$ (d) $E(x,t) = e^{i(kx-2\omega t)} + e^{i\left(\frac{kx}{\sqrt{2}}-\sqrt{2}\omega t\right)}$

36. A particle of charge q moves at a constant speed v parallel to an infinite line charge of density λ, maintaining a constant distance d from the line charge. If $\gamma = 1/\sqrt{1-v^2/c^2}$, then the magnitude of the force on the particle in the rest frame of the line charge is given by

(a) $\frac{\lambda q}{2\pi\varepsilon_0 d}$ (b) $\frac{\lambda q}{2\pi\varepsilon_0 d\gamma}$ (c) $\frac{\lambda q\gamma}{2\pi\varepsilon_0 d}$ (d) $\frac{\lambda q\gamma^2}{2\pi\varepsilon_0 d}$

37. An inertial frame O has a non-zero constant electric field $\vec{E}$, and the magnetic field $\vec{B}$ is zero. An observer in a frame O' is moving with a velocity $\vec{v}$ with respect to O and $\vec{E}$ and $\vec{v}$ are not parallel to each other. If $\perp$ denotes the perpendicular component with respect to the velocity of the frame, them in frame O' the electric field $\vec{E}'$ and magnetic field $\vec{B}'$ satisfy

(a) $\vec{B}' = 0$ (b) $\vec{E}' \cdot \vec{B}' = 0$

(c) $\left(\vec{E}'\right)^2 - c^2\left(\vec{B}'\right)^2 = 0$ (d) $B'_\perp = 0$

38. A light source moves with constant speed along a line relative to an observer standing in its path. The observer measures the wavelength of the light to be 365 nm as it moves away from her and 1096 nm as it moves away from her. The wavelength of the light in the rest frame of its source is about

(a) 596 nm (b) 632 nm (c) 730 nm (d) 836 nm

39. Consider a beam of relativistic particles of kinetic energy K at normal incidence upon a perfectly absorbing surface. The particle flux (number of particles per unit area per unit time) is J and each particle has rest mass m_0. The pressure on the surface is

(a) $\frac{JK}{c}$ (b) $\frac{J\sqrt{K\left(K+m_0c^2\right)}}{c}$

(c) $\frac{J\left(K+m_0c^2\right)}{c}$ (d) $\frac{J\sqrt{K\left(K+2m_0c^2\right)}}{c}$

40. An electron and a proton have kinetic energy 1 keV each. The ratio of their de Broglie wavelength is approximately

(a) 1 (b) 43 (c) 930 (d) 1860

41. Electrons of energy E are incident on a small hole of radius R and produce an interference pattern with number density I_0 at the centre of a screen a large distance away from the hole. If the energy is increased to $4E$ and the radius is increased to $2R$ then which of the following statements is true?

(a) The central peak becomes narrower and the number density becomes $4I_0$

(b) The central peak remains unchanged and the number density becomes $4I_0$

(c) The central peak becomes wider and the number density becomes $I_0/4$

(d) The central peak remains unchanged and the number density becomes $I_0/4$

42. In quantum mechanics, the waveform $\psi(x,y,z)$ of a particle of mass m has the dimensions (where M=Mass, L=Lenghth, T =Time)

(a) $L^{-3/2}$ (b) L^{-3} (c) $M^{-1}L^{-3/2}$ (d) $L^{-3}T^{-1}$

43. If $\psi(x,t)=A\sin(xp/\hbar)\exp(Et/\hbar)$ is the wavefunction of a particle of mass m in one dimension, where p and E are the momentum and energy of the particle respectively then which of the following can not be an outcome of a measurement?

(a) The momentum of a particle is p (b) The energy of the particle is $p^2/2m$

(c) momentum is $-p$ (d) momentum is zero

44. If the normalized wavefunction for a particle in a one dimensional box is given by

$$\psi(x)=A(1-x^2) \quad for \ -1\le x\le 1$$

then what is the value of A?

(a) $\sqrt{15/16}$ (b) $\sqrt{16/15}$ (c) $15/16$ (d) $16/15$

45. A particle ρ of mass 770 MeV/c^2 decays into two pions each of rest mass 140 MeV/c^2. The momentum of each pion is about

(a) 359 MeV/c (b) 455 MeV/c (c) 490 MeV/c (d) 630 MeV/c

46. A neutron decays at rest in about 9 minutes. The maximum kinetic energy of the electron is about

(a) 780 eV (b) 780 keV (c) 780 MeV (d) 780 GeV