ISM-JRF 2015-16

Section A (Questions carry 1 mark each)

Q1. Lagrangian for a compound pendulum is given by

(a) $\frac{1}{2}I\dot{\theta}^2 - mgl\cos\theta$ (b) $\frac{1}{2}I\dot{\theta}^2 + mgl\cos\theta$

(c) $\frac{1}{2}m\left(r^2 + r^2\dot{\theta}^2\right) + \frac{1}{2}mgl\cos\theta$ (d) $mgl\sin\theta$

Q2. The number of degrees of freedom of an isolated H_2O molecule moving in space is

(a) 3 (b) 4 (c) 6 (d) 1

Q3. For a motion under central force, which of the following is true?

(a) $\bar{V} = -2\vec{T}$ (b) $\bar{V} = 2\vec{T}$

(c) $\bar{V} = -\frac{1}{2}\vec{T}$ (d) $\bar{V} = \frac{1}{2}\vec{T}$

Q4. In a Rutherford Scattering process with scattering angle ϕ, the differential scattering cross-section is directly proportional to

(a) $\cot^4\frac{\phi}{2}$ (b) $\text{cosec}^4\frac{\phi}{2}$ (c) $\text{cosec}^2\phi$ (d) $\sin^4\frac{\phi}{2}$

Q5. For space like separation of two events along x-axis, which of the following holds true?

(a) $c^2t^2 = x^2$ (b) $c^2t^2 > x^2$ (c) $c^2t^2 < x^2$ (d) $c^2t^2 = 0$

Q6. If E' and p' are the energy and momentum defined in the frame S' which moves with a speed v w.r.t S then energy in the frames S is given by

(a) $E = \frac{\left(E' + vp'\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$ (b) $E = \frac{\left(E' - vp'\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$

(c) $E = \frac{E'}{\sqrt{1 - \frac{v^2}{c^2}}}$ (d) $E = \frac{E'v}{c}$

Q7. For three vectors $\vec{a},\vec{b}$, and $\vec{c}$ to be coplanar, which of the followings is satisfied?

(a) $a\times\left(\vec{b}\times\vec{c}\right)=0$ (b) $\vec{a}.\left(\vec{b}\times\vec{c}\right)=0$

(c) $\vec{a}\times\left(\vec{b}\times\vec{c}\right)\neq 0$ (d) $\vec{a}.\left(\vec{b}\times\vec{c}\right)\neq 0$

Q8. The determinant of 3×3 real symmetric matrix is 40. If two of its eigen values are 2 and 4, then the third eigen value is

(a) 34 (b) 2 (c) 46 (d) 5

Q9. The value of $\oint\frac{dz}{z-3}$ over the circle C in complex plane with $|z|=1$ is

(a) $3\pi i$ (b) 0 (c) $-3\pi i$ (d) 1

Q10. The independent solutions of the equation $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=0$ are

(a) e^{5x} and e^{3x} (b) e^{-5x} and e^{3x}

(c) e^{-5x} and e^{-3x} (d) $e^{\frac{5}{x}}$ and $e^{\frac{3}{x}}$

Q11. Which of the following conditions are true for an orthorhombic crystal?

(a) a=b=c and $\alpha=\beta=\gamma=\frac{\pi}{2}$ (b) $a=b=c$ and $\alpha=\beta=\gamma=\frac{2\pi}{3}$

(c) $a\neq b\neq c$ and $\alpha=\beta=\frac{\pi}{2}\neq\gamma$ (d) $a\neq b\neq c$ and $\alpha=\beta=\gamma=\frac{\pi}{2}$

Q12. In the vibration of tetra-atomic liner lattice, the number of optical phonon branches are

(a) 4 (b) 8 (c) 9 (d) 1

Q13. If thermal and electrical conductivities of a material are κ and σ respectively, then at constant temperature,

(a) $\kappa=\text{constant}\times\sigma$ (b) $\kappa=\text{constant}\times\sigma^2$

(c) $\kappa=\text{constant}\times\sigma^{-1}$ (d) $\kappa=\text{constant}\times\sqrt{\sigma}$

Q14. If $g(E)$ is the density of state of two-dimensional semiconductor, then

(a) $g(E)\alpha E^{\frac{-1}{2}}$ (b) $g(E)\alpha E^{\frac{1}{2}}$

(c) $g(E)\alpha E^{0}$ (d) $g(E)\alpha E^{-1}$

Q15. If λ and ξ are penetration depth and coherent length respectively, then which of the followings is true for a type II superconductor?

(a) $\frac{\lambda}{\xi}>\frac{1}{\sqrt{2}}$ (b) $\frac{\lambda}{\xi}<\frac{1}{\sqrt{2}}$ (c) $\frac{\lambda}{\xi}=\frac{1}{\sqrt{2}}$ (d) $\frac{\lambda}{\xi}=1$

Q16. The de-Broglie wavelength of a thermal neutron at temperature T is given by the relation

(a) $\lambda=\frac{h}{2mk\sqrt{T}}$ (b) $\lambda=\frac{h}{\sqrt{3mkT}}$

(c) $\lambda=\frac{h}{\sqrt{mkT}}$ (d) $\lambda=\frac{h}{(2mkT)^2}$

Q17. In a double-slit experiment of classical bullets, the intensity, the resultant intensity can be written as

(a) $I_{12}=I_1+I_2+2\sqrt{I_1I_2}\cos\theta$ (b) $I_{12}=I_1+I_2-2\sqrt{I_1I_2}\cos\theta$

(c) $I_{12}=I_1+I_2$ (d) $I_{12}=I_1-I_2$

Q18. The position operator $\widehat{X}$ in momentum representation is defined as

(a) $-i\hbar\frac{\partial}{\partial p_x}$ (b) $i\hbar\frac{\partial}{\partial p_x}$ (c) $\frac{i}{\hbar}\frac{\partial}{\partial p_x}$ (d) $\frac{1}{i\hbar}\frac{\partial}{\partial p_x}$

Q19. if $\hat{a}^{\dagger}$ and $\hat{a}$ are creation and annihilatin operator respectively, then $\hat{a}^{\dagger}\hat{a}|4\rangle$ is equal to

(a) $2|4\rangle$ (b) $2\sqrt{2}|3\rangle$ (c) $\frac{1}{2\sqrt{2}}|4\rangle$ (d) $4|4\rangle$

Q20. The fifth-excited state energy of a particle in a cubical box of size L is $E_S=\frac{14h^2}{8mL^2}$. The degeneracy of this level is

(a) 5 (b) 0 (c) 6 (d) 3

Q21. At the same temperature, which gas exerts greatest pressure?

(a) Fermion gas (b) Boson gas

(c) Classical ideal gas (d) All exert same pressure

Q22. Fluctuation in number of particles occurs in

(a) Canonical ensemble (b) Micro-canonical ensemble

(c) Grand-canonical ensemble (d) None of these

Q23. If N be the particle density of a 3-D non-interacting Bose-gas, then for Bose-Einstein condensation, the critical temperature (T_c) can be stated as;

(a) $T_c \alpha N^{1/3}$ (b) $T_c \alpha N^2$ (c) $T_c \alpha N^{-1/3}$ (d) $T_c \alpha N^{2/3}$

Q24. Ground-state energy of a positronium (E_p) is related to that of the hydrogen atom (E_H) by

(a) $E_p = \frac{1}{2}E_H$ (b) $E_p = 2E_H$ (c) $E_p = E_H$ (d) $E_p = \frac{1}{4}E_H$

Q25. Which of the followings is the correct selection rule in $L-S$ coupling?

(a) $\Delta L \neq 0, \pm 1; \Delta S = 0; \Delta J = 0, \pm (0 \not\leftrightarrow 0)$

(b) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta J = 0, \pm (0 \not\leftrightarrow 0)$

(c) $\Delta L \neq 0; \Delta S = 0; \Delta J = 0, \pm (0 \not\leftrightarrow 0)$

(d) $\Delta L = 0, \pm 1; \Delta S = 0;\ \Delta J \neq 0,\ \pm 1$

Q26. If the population in excited state of a 3-level system is given by $N_{ex} = gN_0 e^{\frac{-\Delta E}{kT}}$, where N_0 is the population in the ground-state, then lasing action occurs for the case,

(a) $\frac{N_{ex}}{N_0} > 1$ (b) $\frac{N_{ex}}{N_0} < 1$ (c) $\frac{N_{ex}}{N_0} = 1$ (d) $\frac{N_{ex}}{N_0} \leq 1$

Q27. Degeneracy of the spectral term ^3D is

(a) 3 (b) 7 (c) 10 (d) 15

Q28. The ratio of the size of ${}^{208}_{82}Pb$ and ${}^{26}_{12}Mg$ nuclei is approximately

(a) $(3)^{\frac{2}{3}}$ (b) $\sqrt{3}$ (c) 2 (d) $(2)^{\frac{2}{3}}$

Q29. Which of the following reactions is allowed?

(a) $p \rightarrow n + e^{+}$ (b) $p \rightarrow e^{+} v_{e}$

(c) $\bar{p} + n \rightarrow \pi^{-} + \pi^{0}$ (d) $p \rightarrow \pi^{+} + \gamma$

Q30. According to shell model which is a sequence of magic numbers?

(a) $1, 4, 9, 16, 25$ (b) $2, 8, 20, 28, 50$

(c) $2, 8, 18, 16, 32$ (d) $1, 2, 3, 4, 5$

Section B (Questions carry 2 marks each)

Q31. The generating function F of the transformation $q = PQ^2,\ p = Q^{-1}$ is

(a) $\frac{p}{q}$ (b) $\frac{P}{q}$ (c) qp (d) $p^2 q$

Q32. The speed v of a particle moving in an elliptical path in an inverse square field is equal to

(a) $\sqrt{\frac{k}{m}\left(\frac{2}{r}+\frac{1}{2a}\right)}$ (b) $\sqrt{\frac{k}{m}\left(\frac{2}{r}-\frac{1}{a}\right)}$

(c) $\sqrt{\frac{k}{m}\left(\frac{2}{r}+\frac{1}{a}\right)}$ (d) $\sqrt{\frac{k}{m}\left(\frac{2}{r}-\frac{a}{2}\right)}$

Q33. With a given angular momentum L, the particle moves in a potential, $V(r) = -\frac{c}{3r^3}$, maximum value of effective potential is given by

(a) $V_{eff}^{\max} = \frac{L^6}{3m^3C^2}$ (b) $V_{eff}^{\max} = \frac{6L^6}{m^3C^2}$

(c) $V_{eff}^{\max} = \frac{L^6}{6m^3C^2}$ (d) 0

Q34. A square of length l_0 moves parallel to one of its side with a speed $0.5c$. Relative change in area of the square is

(a) 0 (b) $\frac{\sqrt{3}}{2}$ (c) $1+\frac{\sqrt{3}}{2}$ (d) $1-\frac{\sqrt{3}}{2}$

Q35. Unit normal to the surface $x^2 + 2y^2 + 3z^2 = 5$ at the point $(0,\ 3,\ -2)$ is

(a) $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$ (b) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{2}}$

(c) $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{2}}$

Q36. Laplace transform of $F(t)=\sin\alpha t$

(a) $\frac{1}{s-\alpha}$ (b) $\frac{1}{s+\alpha}$ (c) $\frac{\alpha}{s^2+\alpha^2}$ (d) $\frac{s}{s^2+\alpha^2}$

Q37. If $P_n(x)$ is the Legendre polynomial of n^{th} order, then which of the following correctly represents the equation $3x^2-5x+1$?

(a) $2P_2+3P_1-P_0$ (b) $2P_2-35+2P_0$ (c) $P_2+3P_1+2P_0$ (d) $-2P_2+P_1-P_0$

Q38. If $S_i=\frac{\hbar\sigma_i}{2}, i=x,y,z$ and σ_i are the Pauli matrices, then expectation value of $S_xS_yS_z$ is

(a) $\frac{i\hbar^3}{8}$ (b) $\frac{\hbar^3}{8}$ (c) $i\hbar$ (d) 0

Q39. The value of $\frac{1}{2}(L_-L_+ + L_+L_-)$ is

(a) $\hbar L_z$ (b) L^2+L_z (c) L^2-L_z (d) $L^2-L_z^2$

Q40. If $a^\dagger=\sqrt{N+1}e^{-i\theta}$ and $a=\sqrt{N+1}e^{i\theta}$, then $\left[e^{-i\theta},N\right]$ is equal to

(a) $e^{i\theta}$ (b) $e^{-i\theta}$ (c) $-e^{-i\theta}$ (d) $-e^{i\theta}$

Q41. For system $\Psi(\theta,\phi)=\frac{1}{\sqrt{10}}\left(2Y_4^1-3Y_2^0+Y_2^1\right)$, where Y_l^m are spherical harmonics. The probability of finding the system in $m=1$ is

(a) $\frac{9}{10}$ (b) $\frac{3}{10}$ (c) $\frac{1}{2}$ (d) 1

Q42. The partition function of a rigid rotator is given by $8\pi^2 lkT$, where l is the moment of inertia. The specific heat C_v for such a system is

(a) k (b) $\frac{k}{8\pi^2}$ (c) 0 (d) $\frac{k}{8}$

Q43. The number of ways in which 6 indistinguishable particles can be distributed in 4 cells is

(a) 24 (b) 84 (c) 210 (d) 10

Q44. Miller Indices of a plane which is parallel to x - axis and cuts intercepts of 2 and $\frac{1}{2}$ along y and x - axes respectively is

(a) $(2\ 0\ 0)$ (b) $(0\ 1\ 4)$ (c) $(0\ 4\ 1)$ (d) $(0\ 1\ 1)$

Q45. If n_i is the intrinsic concentration in a semiconductor and N_d is number of donor atoms doped in it such that $N_d = \frac{4}{\sqrt{3}} n_i$ then electron concentration in the semiconductor is

(a) N_d (b) $\frac{1}{\sqrt{3}} N_d$ (c) $\frac{1}{2} N_d$ (d) n_i

Q46. Fermi energy of an electron gas at $T = 0\,K$ is $E_f(0) = AV^{-2/3}$, where V is the volume of the gas. Pressure of the gas is equal to

(a) $-\frac{2}{3V} E_f(0)$ (b) $-\frac{3}{5V} E_f(0)$ (c) $\frac{2}{5V} E_f(0)$ (d) $\frac{3}{5V} E_f(0)$

Q47. The Lande g factor of spectral tem $^3F_2$ is

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Q48. The ratio between the wavelengths of first line of Balmer series to that of Paschen series is

(a) $\frac{20}{7}$ (b) $\frac{7}{20}$ (c) $\frac{27}{5}$ (d) $\frac{5}{27}$

Q49. An atom $^{17}_{8}X$ captures an α - particle to result in a new atom along with a proton. The atomic number (Z) and mass number (A) of the resulting atom are

(a) $Z = 9, A = 18$ (b) $Z = 9, A = 20$ (c) $Z = 8, A = 19$ (d) $Z = 8, A = 20$

Q50. The element ^9A_x consists of 13 up quarks and 14 down quarks. What is the value of?

(a) 5 (b) 3 (c) 2 (d) 4

Section C (Question carry 3 marks each)

Q51. Value of $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ is

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{2\pi}{3}$ (c) $\frac{2\pi}{\sqrt{3}}$ (d) $\frac{\pi}{\sqrt{3}}$

Q52. Eigen values of the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 1 & 1\end{bmatrix}$ are

(a) $1,-1,0$ (b) $0,1,2$ (c) $1,2,3$ (d) $-1,\ 0,\ -2$

Q53. Kinetic and potential energies of a vibrating coupled system of two masses are given by $T=\frac{1}{2}\left(4\dot{x}_1+5\dot{x}_2\right)$ and $V=\frac{5}{2}\beta\left(x_2-x_1\right)^2$ respectively. Vibrational frequency (ω) of this system is

(a) $\frac{3}{2}\sqrt{\beta}$ (b) $\frac{1}{2}\left(\sqrt{5\beta}\right)$ (c) $\left(\sqrt{\frac{\beta}{5}}\right)$ (d) $\frac{\beta}{20}$

Q54. The inertia tensor of two particles of mass 1 gram and 2 grams located at $(1,\ -1,\ 0)$ and $(-1,\ 0,\ -2)$ respectively is

(a) $\begin{bmatrix}9 & -4 & 1\\ -4 & 4 & 0\\ 0 & 1 & 11\end{bmatrix}$ (b) $\begin{bmatrix}11 & -4 & 0\\ -4 & 9 & 0\\ 1 & 1 & 4\end{bmatrix}$ (c) $\begin{bmatrix}9 & 1 & -4\\ 1 & 11 & 0\\ -4 & 0 & 4\end{bmatrix}$ (d) $\begin{bmatrix}-9 & 1 & 4\\ 0 & 11 & 1\\ -4 & 0 & -4\end{bmatrix}$

Q55. CO molecule has a force constant of $11.4\times10^6\ dyne/cm$. Its vibrational frequency is approximately equal to

(a) $0.52\times10^9\ \sec^{-1}$ (b) $0.13\times10^{11}\ \sec^{-1}$ (c) $0.52\times10^7\ \sec^{-1}$ (d) $0.16\times10^9\ \sec^{-1}$

Q56. A system has two energy levels of energy 0 and 3 with degeneracy 1 and 3. The temperature at which the probability of finding the system at lower level is

(a) $\frac{1}{k\ln 2}$ (b) $\frac{3}{k\ln 3}$ (c) $\frac{\ln 2}{k}$ (d) $k\ln 3$

Q57. The value of $\left[x, p_x^5\right]$ is

(a) $5i\hbar p_x^4$ (b) $-5i\hbar p_x^4$ (c) $5\hbar p_x^4$ (d) $-5\hbar p_x^4$

Q58. For wave function $\psi = \frac{1}{r}\exp\left[2ir\right]$. The probability of current density is

(a) $\frac{2i\hbar}{mr}$ (b) $-\frac{2\hbar}{mr^2}$ (c) $\frac{2\hbar}{mr^2}$ (d) 0

Q59. The motion of an electron with mass m and momentum $\vec{p}$ in one dimension is confined to the right-half space $(x > 0)$, where the potential is $(x) = -\frac{e^2}{4x}$. If the ground-state wave function is of the form $\psi(x) = xe^{-2x}$, then the ground-state energy is

(a) $-\frac{4\hbar^2}{m}$ (b) $-\frac{\hbar^2}{2m}$ (c) $-\frac{2\hbar^2}{m}$ (d) $-\frac{\hbar^2}{4m}$

Q60. In a 1-D crystal, the electron energy as a function of wave vector (k) is given by $E = 3k^2 - 2k^3$, the effective mass at the edge of Brillouin zone is

(a) $\frac{\hbar^2}{6}$ (b) $\frac{-\hbar^2}{6}$ (c) $\frac{\hbar^2}{6k^4}$ (d) $\frac{-\hbar^2}{6k^4}$