JNU Ph.D (2017)

PART-A

A1. Consider the Lagrangian, $L=\frac{1}{2}\dot{x}^2-\frac{1}{2}\omega^2x^2+\alpha\log x$, where α and ω are positive constants. Let $x>0$.

(a) Find the equilibrium value of x.

(b) Write down the Lagrangian for small oscillations about the equilibrium.

(c) Work out the frequency of normal mode.

A2. Show that $u(x,y)=\sin x\cosh y+2\cos x\sinh y$ satisfies Laplace's equation in dimensions. If u is taken to be the real part of a complex analytic function $f(z)=u+iv$ that vanishes at $z=0$ then find $f(z)$

A3. The wave function of a particle of mass m, moving in a one-dimensional box lying between $x=0$ and $x=L$, is given at an instant by $\psi(x)=Cx(L-x)$, where C is normalization constant. If you measure the energy of the particle at this instant, what is the probability that the particle will be found in its ground state?

A4. The equation of state for an ideal gas is given by $p=nk_BT$, where $n=\frac{N}{V}$ is the number of particle per unit volume

(a) Find the entropy, S, of the ideal gas ass a function of p and V (up in the undetermined constant)

(b) Show that the pressure of a gas is given by $p=n\frac{\partial f}{\partial n}=-f$, where $f=\frac{F}{V}$ is the free energy per unit volume. Does it require the gas to be ideal?

A5. Aluminium has three valence electrons per atom, an atomic weight of $0.02698\ kg/mol$ a density of $2700\ kg/m^3$ and a conductivity of $3.54\times10^7\ S/m$. Assume that all three valence electrons of every atom participate in electrical conduction

(a) Calculate the carrier concentration, n.

(b) Calculate the mobility, μ in aluminium.

A6. For scalar potential, $\phi = 0$, and vector potential $A = A_0 \sin(kx - \omega t)\hat{y}$, find the corresponding electric field,, E, and magnetic field, B. Here A_0, ω and k are constants.

(b) Under what condition, will all the four Maxwell's equations be satisfied for these E and B?

PART - B

B1. Using dimensional analysis, show that $\frac{e^2}{\hbar}$ is the quantum unit of electrical conductance

B2. Calculate the density of states, $D(\varepsilon)$, for the free particle dispersion, $\varepsilon_k = \frac{\hbar^2 k^2}{2m}$ in two dimensions. Sketch $D(\varepsilon)$ vs. ε

B3. What is the difference between coherence length, ξ and penetration depth, λ of a superconductor? Which of the two is bigger in a type -1 superconductor?

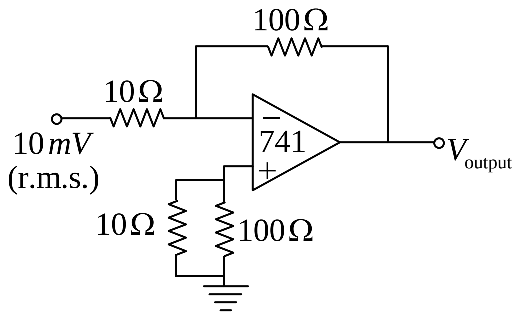
B4. In its rest frame, a muon (mass $= 106\ MeV$) has a lifetime of 2.2×10^{-6} sec. what is the lifetime in the laboratory frame if the muon has a kinetic energy of $500\ MeV$?

B5. A collection of free spin $\frac{1}{2}$ particles at temperature, T, is placed in a magnetic field of $2Wb/m^2$. If the number of spins parallel to the magnetic field is twice as large as the number of antiparallel spins, then what is the temperature, T?

B6. Find the general solution of the ordinary differential equation, $\frac{d^2x}{dt^2} + x = t^2$. Find also the solution with the initial condition $x(0) = 0$ and $\frac{dx}{dt}(0) = 0$

B7. For a diatomic molecule of identical atoms with atomic mass $40 \times 10^{27}\ kg$, and bond length of $2 \times 10^{-10}\ m$, calculate (using rigid rotator model) the frequency of rotational transition from the angular momentum state $J = 1$ and $J = 2$.

B8. In the circuit given below, what is the peak output voltage?



B9. The Doppler width of the orange line of krypton at $\lambda = 6058\,\overset{0}{A}$ is $\Delta\lambda = 0.0055\,\overset{0}{A}$. What is the bandwidth, Δv, of this line?

B10. A spin $-\frac{1}{2}$ particle is given to be in the normalized eigenstate, $|\uparrow\rangle$, of the operator $\hat{S}_z$ such that $\hat{S}_z|\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$. Calculate the uncertainty, $\langle \hat{S}_x^2 \rangle - \langle \hat{S}_x \rangle^2$, of the measurement of $\hat{S}_x$ in this state.