JNU-ENTRANCE EXAMINATION, 2007

Ph.d (Physical Science)

Maximum Marks: 70

PART-A

NOTE: Answer **all** questions. Questions Q1 and Q2 carry 8 marks each. Questions Q3 to Q6 carry 3 marks each.

Q.1 A particle of mass m moves in a one-dimensional box between $x = 0$ and $x = L$.

(a) Write down the ground state wave function for the particle.

(b) Suppose the wave function is given by $\psi(x) = Cx(L - x)$, where C is the constant of normalization. If you measure the energy of the particle, what is the probability that it will be the ground state energy?

Q.2 Show that, in two dimensions, the chemical potential of an ideal Fermi gas at temperature T is given by

$$\mu(T) = k_B T \ln\left[\exp\left(n\lambda_0^2\right) - 1\right]$$

where n is the number of particles per unit area and $\lambda_0 = h/\sqrt{2\pi m k_B T}$ is the thermal wavelength.

Q.3 Consider a set of N identical magnetic dipoles in a uniform external magnetic field of strength H. The projections of the dipole moments in the direction of the magnetic field have possible values of $\pm\mu$. Obtain the partition function of this system and calculate the entropy as a function of temperature. Assume that the dipoles do not interact among themselves.

Q.4 Suppose you are provided the values off a function $f(x)$ at $x = x_1$ and x_2 $(x_2 > x_1)$. Obtain a linear interpolation formula for approximating $f(x)$ in the interval $x_1 < x < x_2$. Show that the error associated with this approximation is not larger in magnitude than $M\,(x_2 - x_1)^2$, where M is the maximum value of $|f''(x)|$ in the interval $[x_1,\ x_2]$

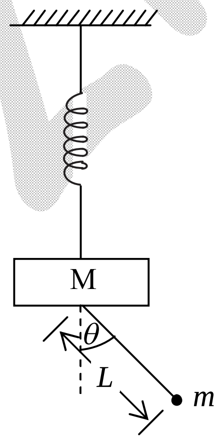
Q.5 The energy levels of a three-dimensional isotropic harmonic oscillator are given by $\hbar\omega(m+3/2)$, where m = 0, 1, 2, …. What is the degree of degeneracy of the energy level corresponding to m = 2?

Q.6 Write down all possible term symbols (in the form $^{2S+1}L_J$) of a carbon atom (Z = 6) whose electronic configuration is $1s^2 2s^2 2p^1 3d^1$.

PART – B

NOTE: Answer **all** questions. Questions Q1, Q2 and Q3 carry 8 marks each. Questions Q4 to Q9 carry 3 marks each.

Q.1 A block of mass M is suspended vertically from the ceiling by a spring of constant k. A pendulum of mass m is attached to the bottom of this block by a massless rod of length L (as shown in the attached figure). Assume that the block can move only vertically and that the motion of the pendulum takes place in a fixed vertical plane.



(a) Choose suitable generalized coordinates to describe the motion and write down the Lagrangian of the system.

(b) Derive the equations of motion for the generalized coordinates.

Q.2 Write down the four Maxwell equations of electromagnetism. The rate of work done by the electromagnetic field on charges is given by $W = \int_v \vec{J} \cdot \vec{E} dv$, where $\vec{J}$ is the current density. Using the Maxwell equations, derive the following relation.

$$\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} = -\vec{J} \cdot \vec{E}$$

where the energy density and the energy current are given by $u = \left(\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H}\right)/2$ and $\vec{S} = \vec{E} \times \vec{H}$, respectively. The symbols for the various electromagnetic fields $\left(\vec{E}, \vec{D}, \vec{H} \text{ and } \vec{B}\right)$ have their usual meanings.

Q.3 Use the residue theorem to evaluate the integral

$$I = \int_{-\infty}^{+\infty} \frac{x \sin(x)}{x^4 + 1} dx$$

Q.4 Four point charges are located on the *x*-*y* plane as described in the following:

(a) Two charges, each of strength *q*, at the points (0, *a*) and (0, 2*a*)

(b) Two charges, each of strength –*q*, at the points (*a*, 0) and (-*a*, 0)

Calculate the potential at a point on the *x*-*y* plane sufficiently far away from all the charges (keeping only the first non-vanishing term in the multipole expansion).

Q.5 A beam of blue-green light (λ = 5000 ´), with energy flux of 100 watt/m^2, is completely absorbed by the cathode of a phototube. The cathode has an area of 10^{-4} m^2 and its work function is 2 *eV*. What will be the magnitude of the saturated current when the anode potential is positive and sufficiently large?

Q.6 A point on the earth's surface receives solar energy at the rate of 1.4 kW/m^2 when the sun is directly overhead. The average radius of the earth's orbit around the sun is 1.5 x 10^{11} m and the sun's radius is 7 x 10^8 m. Estimate the surface temperature of the sun (assuming that it radiates like a black body).

Q.7 A space traveler is moving directly away from a light source at a constant speed βc. If the light has a frequency ν_0 in the rest frame of its source, what will be the frequency measured by the space traveler (in terms of ν_0 and β)?

Q.8 What is the terminal velocity of a steel ball of radius 1 *mm* falling in a tank of glycerine? The densities of steel and glycerine are 8.5 gm/cm^3 and 1.32 gm/cm^3, respectively. The viscosity of glycerine is 8.3 poise.

Q.9 Prove that

$$exp\,(i\theta\Delta) = \cos(\theta)\,I + i\sin(\theta)\Delta$$

where I is the (2 x 2) unit matrix, θ is a real number and Δ is one of the three Pauli matrices. The Pauli matrices are given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ and } \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$