JNU-ENTRANCE EXAMINATION, 2008

Ph.d (Physical Science)

Maximum Marks: 70

PART-A

NOTE: Answer **all** questions. Each question carries 8 marks.

Q.1 The Bessel functions of integer order n are defined as

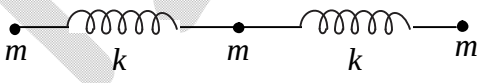
$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\sin\theta - n\theta)d\theta$$

(a) Use this to show that $J_n(x) = (-1)^n J_{-n}(x)$ and

$$\frac{dJ_n}{dx} = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$$

(b) If x_0 is the first zero of $J_0(x)$, compute the integral $\int_0^{x_0} J_1(x)dx$

Q.2 Obtain the frequencies of the normal modes of a one-dimensional system of three particles, each of mass m. The masses are connected by two identical massless springs, with spring constant k (see figure below). Comment on the physical nature of these normal modes.



Q.3 The wavefunction for the ground state of the hydrogen atom is

$$\psi(r) = \frac{1}{\pi^{1/2}a^{3/2}}e^{-r/a}$$

where a is the Bohr radius.

(a) What is the probability of finding the electron between r and $r + dr$? Sketch the radial probability density $P(r)$ versus the radius r.

(b) Find the average value of r and the distance r_m, where the probability density has a maximum.

Q.4 An ideal gas consists of identical non-interacting mono-atomic particles of mass m. Consider this gas in an ensemble with fixed temperature T, volume V and chemical potential μ. Identify the thermodynamic potential which is minimized in equilibrium. Obtain this potential explicitly as a function of T, V and μ.

Q.5 Consider a material which is a perfect conductor of electric current (resistivity is zero).

(a) Using Maxwell's equations and Ohm's law, show that the magnetic field inside a perfect conductor is constant in time.

(b) Using Maxwell's equations and Ohm's law, show that the magnetic flux through a perfectly conducting loop is a constant.

(c) A type-I superconductor is a perfect conductor with the additional property of complete expulsion of magnetic flux (Meissner effect). Show that, in the Meissner state, any current that flows to shield an external magnetic field must be confined to the surface.

PART–B

NOTE: Answer **all** questions. Each question carries 3 marks.

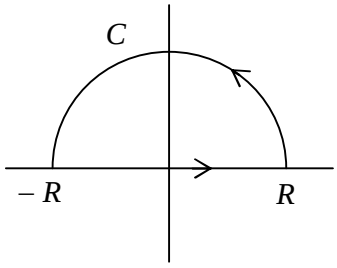
Q.1 Calculate the Fourier transform of the function

$$f(x) = \exp(-3|x|), \quad -\infty < x < \infty \ .$$

Q.2 Evaluate the contour integral

$$\oint_C \frac{dz}{1+z^4}$$

where C is a semi-circle in the upper half-plane, with radius $R \to \infty$ (see figure)



Q.3 Consider a point particle of mass m moving along the x-axis of a horizontal plane which is perfectly smooth for $x < 0$. The surface of the plane is rough for $x \geq 0$ and opposes the motion with a friction constant γ. If the particle enters the rough side of the plane with a speed v_0, what distance would it travel before coming to a stop?

Q.4 A particle of mass m moves in the (x, y) plane with the Hamiltonian $H = \frac{1}{2m}\left[(p_x - cy)^2 + (p_y + cx)^2\right]$ where c is a constant.

(a) Obtain the Hamiltonian equations of motion.

(b) Solve these equations to find the trajectory of the particle.

Q.5 A particle of mass m is constrained to move on a circle of radius R. What are the energy levels of the particle? What are the eigenfunctions corresponding to the first three energy levels?

Q.6 Consider a quantum system of four identical spins S_1, S_2, S_3 and S_4. Each of these takes values ½ and – ½ only. The Hamiltonian is given by

$$H = -J\,(S_1S_2 + S_2S_3 + S_3S_4 + S_4S_1)$$

The total spin is defined as $(S_1 + S_2 + S_3 + S_4)$. At temperature T, what is the probability that the total spin is zero?

Q.7 An ideal magnet consists of a set of N non-interacting spins $\{S_i\}$, where S_i can take values ½ and – ½. Obtain the partition function for this system at fixed temperature T, and magnetic field h.

Q.8 Suppose you are designing an experiment to study the electromagnetic response of silver at microwave frequency (~10^{10} Hz). Given that the resistivity of silver is 1.59 x 10^{-8} Ωm, how thick a silver coating would be needed? Assume that the permeability of silver is $\mu_0 \approx 4\,\pi$ x 10^{-7} henry/m.

Q.9 From X-ray data, Fe is found to have a cubic cell parameter of 0.287 nm. Its density and atomic mass are 7870 kg/m^3 and 55.85, respectively. Find out the type of Bravais lattice and the inter-atomic distance of Fe atoms.

Q.10 In accelerators electrons e^- are made to collide with their anti-particles, positrons e^+, at high energies to produce many different particles. In one such process, the pair annihilates to produce a muon μ^- and an anti-muon μ^+. The mass of a muon $m_\mu \approx 200\ m_e$, where m_e is the electrons rest mass. What is the minimum velocity to which $e^\pm$ must be accelerated so that $\mu^\pm$ may be produced? (You may give your answer as a friction of the speed of light c).