JNU-ENTRANCE EXAMINATION, 2010

Ph.d (Physical Science)

Maximum Marks: 70

PART-A

NOTE: Answer **all** questions. Each question carries 6 marks.

Q.1 Find the general solution to the first-order differential equation $\frac{dx}{dt} = \lambda x(1-x)$

where λ is an arbitrary constant. How does x(t) behave as $t \to \infty$, when λ > 0 and $\lambda < 0$ respectively?

Q.2 In addition to the planets orbiting around the Sun, there are comets in the solar system. Of these, Halley's Comet has a periodic orbit with an average period of 75.3 years.

(a) Calculate the semi-major axis of its orbit in terms of the parameters of the Earth's orbit. (You may take the Earth's orbit to be approximately a circle; the radius of the circle is a convenient unit of distance called the *astronomical unit*, AU)

(b) If the minimum distance of the Comet to the Sun (distance to the perihelion) is 0.6 AU, what is the eccentricity of its orbit?

(c) What is the farthest point of the orbit from the Sun (distance to the aphelion)?

Q.3 Consider a uniform electric field ***E*** along the positive *z*-direction and a uniform magnetic field ***B*** along the positive *x*-direction in a right-handed coordinate system. An electrically charged particle (of mass *m* carrying a charge +*q*) is released with zero velocity at the origin.

(a) Find the trajectory of the charged particle.

(b) Make a qualitative plot of the trajectory.

Q.4 The energy levels of a two-dimensional quantum harmonic oscillator are given by $E_{n_1,n_2} = \hbar\omega(n_1 + n_2 + 1)$.

(a) What is the degeneracy (that is, the number of quantum states) of a given energy $\hbar\omega(k+1)$? (Here *k* is a fixed integer)

(b) Evaluate the partition function Z = Tr$e^{-\beta H}$ of the system and calculate its internal energy.

Q.5 A spin-orbit interaction in an atom adds a term of the following form to the Hamiltonian of an orbiting electron:

$$H_{SO} = \mathbf{L}.\mathbf{S} + \alpha(L_Z + S_Z)$$

where α is a real constant, **L** and **S** are the orbital and spin angular momentum operators, and L_Z and S_Z are their z-components respectively. Find all the eigenvalues of H_{SO} corresponding to $l = 1$ orbital state.

PART-B

NOTE: Answer **all** questions. Each question carries 4 marks.

Q.1 Find the condition for a 2x2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ (where a, b, c and d are complex numbers) to be Hermitian and unitary.

Q.2 A point charge $+q$ is placed at a distance h from an infinite conducting and grounded plane. Find the induced charge density on the surface of the conducting plane.

Q.3 Let R_π be the 3 x 3 rotation matrix corresponding to a rotation by an angle π about any axis. Consider the matrices

$$P_\pm = \frac{1}{2}(1 \pm R_\pi)$$

show that $P_\pm^2 = P_\pm$.

Write the explicit form of the matrix P_+ in a suitable coordinate system.

Q.4 The ground-state energy of a one-dimensional quantum harmonic oscillator is $\hbar\omega/2$. Consider a small anharmonic perturbation of the form λx^4 to the harmonic potential. Compute the correction to first order in λ to the ground-state energy. [You may want to use the relation $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{i}{\sqrt{2m\omega\hbar}}\hat{p}$ between the annihilation, the position and the momentum operators.]

Q.5 The equilibrium separation between the oxygen atoms in an O_2 molecule is 1.2×10^{-10} *m*. Estimate the separation between the rotational energy levels corresponding to $l = 1$ and $l = 2$.

Q.6 The electric field of radiation due to a current density $\mathbf{j}(\mathbf{r}, t)$ is

$$E_{\text{rad}}(\mathbf{r}, t) = \frac{\mu_0}{4\pi}\hat{r} \times \left(\frac{\hat{r}}{r} \times \int d^3r' \frac{\partial}{\partial t}\mathbf{j}(\mathbf{r}', t - r/c)\right)$$

where $\hat{r} = \mathbf{r}/r$ is the unit vector along $\mathbf{r}$. A point charge $+q$ is accelerating with its acceleration $\boldsymbol{a}$ perpendicular to $\mathbf{r}$. Calculate the power radiated by the charge. Assume that the radiation consists of transverse electromagnetic waves with $|\mathbf{E}| = c|\mathbf{B}|$.

Q.7 A monovalent metal has a face-centered cubic (FCC) lattice structure with a lattice constant a. Show that the radius k_F of the free-electron Fermi surface is $4.90/a$.

Q.8 A one-dimensional polymer is composed of N monomers, each of length a, that may be oriented along the positive or negative x-direction. Show that the force required to increase the length of the polymer by a small amount Δx at a temperature T is $-k_B T(\Delta x)/Na^2$.

Q.9 Estimate the number of electrons that would be thermally excited when a metal is heated to a temperature T K. Compute the electronic heat capacity.

Q.10 A relativistic neutron is travelling at half the speed of light. How much energy is required to increase its speed to $0.6c$? Compare this with the answer that you would get using non-relativistic (Newtonian) mechanics.