ALL INDIA TEST SERIES

FOR

CSIR - JRF (PHYSICS) December 2017

Full Length Test – 01

PHYSICAL SCIENCES

TIME: 3 HOURS **MAXIMUM MARKS: 200**

Part 'A' This part shall carry 20 questions pertaining to *General Aptitude with emphasis, On logical reasoning, graphical, analysis, analytical and numerical ability, quantitative comparison, series formation, puzzles etc.* The candidates shall be required to answer any 15 questions. Each question shall be of two marks. The total marks allocated to this section shall be 30 out of 200.

Part 'B' This part shall contain 25 Multiple Choice Questions (MCQs) generally covering the topics given in the Part 'A' (CORE) of syllabus. All questions are compulsory. Each question shall be of 3.5 Marks. The total marks allocated to this section shall be 70 out of 200.

Part 'C' This part shall contain 30 questions from Part 'B'(Advanced) that are designed to test a candidate's knowledge of scientific concepts and/or application of the scientific concepts. The questions shall be of analytical nature where a candidate is expected to apply the scientific knowledge to arrive at the solution to the given scientific problem. A candidate shall be required to answer any 20. Each question shall be of 5 Marks. The total marks allocated to this section shall be 100 out of 200.

There will be negative marking @25% for each wrong answer.

PART A

ANSWER ANY 15 QUESTONS

Q1. If $\log\frac{a}{b}+\log\frac{b}{a}=\log(a+b)$, then

(a) $a+b=1$ (b) $a-b=1$ (c) $a=b$ (d) $a^2-b^2=1$

Q2. A tank is 25 meter long, 12 meter wide and 6 meter deep. The cost of plastering its walls and bottom at 75 paise per square meter is

(a) Rs. 465 (b) Rs. 458 (c) Rs. 558 (d) Rs. 568

Q3. A watch which gains 5 second, in 3 minutes was set right at 7 AM. In the afternoon of the same day, when the watch indicated quarter past 4'O clock the true time is

(a) $59\frac{7}{12}$ min past 3 (b) 4 P.M.

(c) $58\frac{7}{11}$ min past 3 (d) $2\frac{3}{11}$ min past 4

Q4. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at lest 3 men are there in the committee. In how many ways can it be done?

(a) 564 (b) 645 (c) 735 (d) 756

Q5. A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work, if he is assisted by B and C on every third day?

(a) 12 days (b) 15 days (c) 16 days (d)18 days

Q6. In a 500 m race, the ratio of the speeds of two contestants A and B is $3:4$. A has a start of 140 m. Then, A wins by

(a) 60 m (b) 40 m (c) 20 m (d) 10 m

Q7. The average of 20 numbers is zero. Of them, at the most, how many numbers may be greater than zero?

(a) 0 (b) 1 (c) 10 (d) 19

Q8. There are two examination rooms A and B. If 10 students are sent from A to B, then the number of students in each room is the same. If 20 candidates are sent from B to A, then the number of students in A is double the number of students in B. The number of students in room A is:

(a) 20 (b) 80 (c) 100 (d) 200

Q9. In a group of 40 people, 10 are healthy and every person of the remaining 30 people has either high blood pressure, a high level of cholesterol or both. If 15 have high blood pressure and 25 have high level of cholesterol, then how many people have both high blood pressure and a high level of cholesterol?

(a) 10 (b) 20 (c) 30 (d) 40

Q10. Perimeter of a triangle with integer sides is equal to 15. How many such triangles are possible?

(a) 7 (b) 6 (c) 8 (d) 5

Q11. Deepti and Pavitra walk up on escalator. Deepti takes 9 steps in the same time that Pavitra takes 16 steps. Deepti gets to the top of the escalator after having taken 30 steps while Pavitra takes 40 steps to reach the top. If the escalator was turned off, how many steps would they have to take to walk up?

(a) 100 (b) 90 (c) 80 (d) 70

Q12. Three numbers which are co-prime to each other are such that the product of first two is 551 and that of the last two is 1073. The sum of the three numbers is :

(a) 75 (b) 81 (c) 85 (d) 89

Q13. Three positive integers a, b and c are such that their average is 20 and $a \le b \le c$. If the median is $(a+11)$, what is the least possible value of c?

(a) 23 (b) 21 (c) 25 (d) 26

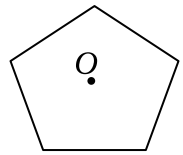
Q14. Six friends are sitting in a circle and are facing the centre of the circle. Deepa is between Prakash and Pankaj. Priti is between Mukesh and Lalit. Prakash and Mukesh are opposite to each other, who is sitting right to Prakash?

(a) Mukesh (b) Deepa (c) Pankaj (d) Lalit

Q15. At what time between 4 and 4:59 will the hands of a watch point in opposite directions?

(a) 45 minute past 4
(b) 40 minutes past 4
(c) $50\frac{4}{11}$ minute past 4
(d) $54\frac{6}{11}$ minute past 4

Q16. *ABCDE* is a regular pentagon. *O* is a point inside the pentagon such that *AOB* is an equilateral triangle. What is $\angle OEA$?



(a) 66^0
(b) 48^0
(c) 54^0
(d) 72^0

Q17. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

(a) $\frac{10}{21}$
(b) $\frac{11}{21}$
(c) $\frac{2}{7}$
(d) $\frac{5}{7}$

Q18. Find the next letter in a given series:

CMM, EOO, GQQ _____, KUU

(a) *GRR*
(b) *GSS*
(c) *ISS*
(d) *ITT*

Q19. A man covered a certain distance at some speed. Had he moved 3 kmph faster, he would have taken 40 minute less. If he had moved 2 kmph slower, he would have taken 40 minutes more. The distance (in kilometer) is

(a) 35
(b) $36\frac{2}{3}$
(c) $37\frac{1}{2}$
(d) 40

Q20. A vessel contains 20 liters of a mixture of milk and water in the ratio 3:2. 10 liters of the mixture are removed and replaced with an equal quantity of pure milk. If the process is repeated once more, find the ratio of milk and water in the final mixture obtained?

(a) 9:1
(b) 4:7
(c) 7:1
(d) 2:5

PART B

ANSWER ANY 20 QUESTONS

Q21. Let $x(t)$ be the solution of the differential equation $\frac{d^2x}{dt^2}+x=\delta(t-1)$ subjected to the initial condition $x(0)=0$ and $x(1)=1$. If $u(t-a)$ denotes the unit step function, then which of the of the following is correct. ?

(a) $x(t)=\sin t+u(t-1)\cos(t-1)$ (b) $x(t)=\sin t+u(t-1)\sin(t-1)$

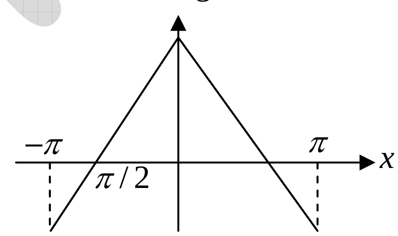
(c) $x(t)=\cos t+u(t-1)\cos(t-1)$ (d) $x(t)=\cos t+u(t-1)\sin(t-1)$

Q22. Let $y(t)$ be the solution of the differential equation $(x+1)\frac{dy}{dx}-2y=(x+1)^4$ subjected to the initial condition $y(0)=\frac{1}{3}$, then the value of $y(2)$ is

(a) 9 (b) 18 (c) 27 (d) 81

Q23. The graph of a periodic function $f(x)$ for one period is shown in the figure below. If the Fourier series of this function is written as

$$f(x)=a_0+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$



then which of the following options is incorrect?

(a) The value of a_0 is 0. (b) The value of $a_3=\frac{8}{9\pi^2}$.

(c) The sum of series $1+\frac{1}{3^2}+\frac{1}{5^2}+...$ is $\frac{\pi^2}{8}$ (d) The coefficient of $\cos 5x$ is $\frac{4}{25\pi^2}$.

Q24. The matrix $A=\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}$ has two equal eignvalues. If $B=A^3+2I$, then the trace of B^{-1} is

(a) $\frac{113}{372}$ (b) $\frac{145}{386}$ (c) $\frac{257}{381}$ (d) $\frac{340}{379}$

Q25. The Hamiltonian for a system described by the generalised coordinate x and generalised momentum p is

$$H = \alpha x^2 p + \frac{p^2}{2(1+2\beta x)} + \frac{1}{2}\omega^2 x^2$$

where α, β and ω are constants. Consider the following statements and choose the correct option.

Statement 1: The momentum is given by $p = \left(\dot{x} - ax^2\right)(1+2\beta x)$

Statement 2: The momentum is given by $p = \left(\dot{x} + ax^2\right)(1+2\beta x)$

Statement 3: The Lagrangian is given by $L = (1+2\beta x)\dfrac{\left(\dot{x} - \alpha x^2\right)^2}{2} - \dfrac{1}{2}\omega^2 x^2$

Statement 4: The Lagrangian is given by $L = (1+2\beta x)\dfrac{\left(\dot{x} + \alpha x^2\right)^2}{2} - \dfrac{1}{2}\omega^2 x^2$

(a) 1 and 3 are correct
(b) 2 and 3 are correct
(c) 1 and 4 are correct
(d) 2 and 4 are correct

Q26. An inertial observer sees two events E_1 and E_2 happening at the same location but $6\,\mu s$ apart in time. Another observer moving with a constant velocity v (with respect to the first one) sees the same events to be $9\,\mu s$ apart. Then find value of $\sqrt{1 - \dfrac{v^2}{c^2}}$

(a) $\frac{2}{3}$
(b) $\frac{1}{3}$
(c) $\frac{5}{9}$
(d) $\frac{2}{9}$

Q27. A particle of mass m is moving in $x-y$ plane. At any given time t, its position vector is given by $\vec{r}(t) = \left(A\cos\omega t\,\hat{i} + B\sin\omega t\,\hat{j}\right)$ where A, B and ω are constants with $A \neq B$.

Which of the following statements are correct?

(1) Orbit of the particle is an ellipse

(2) Speed of the particle is constant.

(3) At any given time t the particle experiences a force towards origin

(a) 1, 2, 3 are correct
(b) 2, 3 are correct ,
(c) 1, 3 are correct
(d) 1, 2 are correct.

Q28. A particle of mass m is bounded by potential $V(x) = kx^{2n}$ where $n = 1, 2, 3....$ The particle oscillates about $x = 0$. Given below are three statements. Study them and choose the correct option.

Statement 1: The particle has constant angular frequency for $n = 1, 2, 3...$

Statement 2: The particle has constant angular frequency for $n = 1$ and frequency will be dependent on energy for $n = 2, 3, 4...$

Statement 3: The particle has constant angular frequency for $n = 1$ and frequency will be dependent on amplitude for $n = 2, 3, 4...$

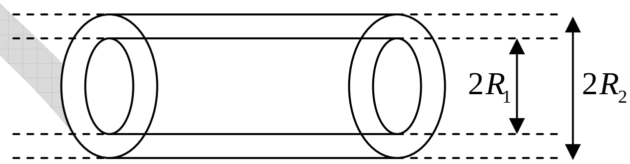
(a) Only 1 is correct (b) 1 and 2 are correct

(c) 2 and 3 are correct (d) 1 and 3 are correct

Q29. A block of mass $2.5\,kg$ is kept on a rough horizontal surface. It is found that the block does not slide if a horizontal force less than $15\,N$ is applied to it. Also it is found that it takes 5 seconds to slide through the first $10\,m$ if a horizontal force of $15\,N$ is applied and the block is gently pushed to start the motion. Take $g = 10\,m/s^2$. If μ_s and μ_k are coefficients of static and kinetic friction between the block and the surface respectively, then $\left(\frac{\mu_s}{\mu_k}\right)$ is given by

(a) 1 (b) 1.15 (c) 2.30 (c) 4.60

Q30. Two long hollow co-axial conducting cylinders of radii R_1 and R_2 $(R_1 < R_2)$ are placed in vacuum as shown in the figure below.



The inner cylinder carries a charge $+\lambda$ per unit length and the outer cylinder carries a charge $-\lambda$ per unit length. The electrostatic energy per unit length of this system is

(a) $\frac{\lambda^2}{\pi \in_0} \ln(R_2/R_1)$ (b) $\frac{\lambda^2}{4\pi \in_0}(R_2^2/R_1^2)$

(c) $\frac{\lambda^2}{2\pi \in_0} \ln(R_2/R_1)$ (d) $\frac{\lambda^2}{8\pi \in_0} \ln(R_2^2/R_1^2)$

Q31. A set of N concentric circular loops of wire, each carrying a steady current I in the same direction, is arranged in a plane. The radius of the first loop is $r_1 = a$ and the radius of the n^{th} loop is given by $r_n = nr_{n-1}$. The magnitude B of the magnetic field at the centre of the circles is

(a) $\frac{\mu_0 I}{2\pi a}\left(\sum_{n=1}^{N}\frac{1}{\lfloor\underline{n}}\right)$ (b) $\frac{\mu_0 I}{2\pi a}\left(\sum_{n=1}^{N}\frac{1}{\lfloor\underline{n+1}}\right)$

(c) $\frac{\mu_0 I}{2a}\left(\sum_{n=1}^{N}\frac{1}{\lfloor\underline{n}}\right)$ (d) $\frac{\mu_0 I}{2a}\left(\sum_{n=1}^{N}\frac{1}{\lfloor\underline{n+1}}\right)$

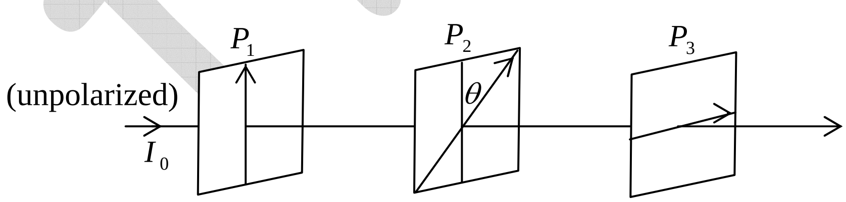
Q32. An electromagnetic wave (of wavelength λ_0 in free space) travels through an absorbing medium with dielectric permittivity given by $\varepsilon = \varepsilon_R + i\varepsilon_I$ where $\frac{\varepsilon_I}{\varepsilon_R} = \sqrt{3}$. If the skin depth is $\frac{\lambda_0}{4\pi}$, the ratio of the amplitude of electric field E to that of the magnetic field B, in the medium is (where c is speed of light in free space)

(a) $\frac{c}{2}$ (b) $\frac{c}{4}$ (c) $\frac{c}{6}$ (d) $\frac{c}{8}$

Q33. The vector potential $\vec{A} = ke^{-at}r\hat{r}$ (where a and k are constants) corresponding to an electromagnetic fild is changed to $\vec{A}' = -3ke^{-at}r\hat{r}$. This will be a gauge transformation if the corresponding change $\phi' - \phi$ in the scalar potential is

(a) akr^2e^{-at} (b) $2akr^2e^{-at}$ (c) $-akr^2e^{-at}$ (d) $-2akr^2e^{-at}$

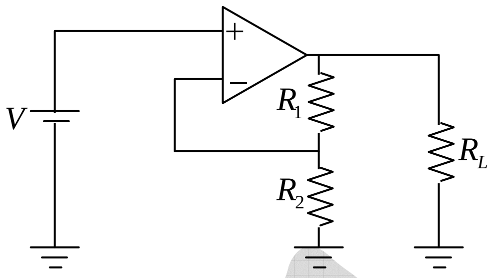
Q34. Consider three polarizer's P_1, P_2 and P_3 placed along an axis as shown in the figure.



The pass axis of P_1 and P_3 are at right angles to each other while the pass axis of P_2 makes an angle $\theta = 30^0$ with that of P_1. A beam of unpolarized light of intensity I_0 is incident on P_1 as shown. The intensity of light emerging from P_3 is approximately

(a) 0 (b) $\frac{I_0}{2}$ (c) $\frac{I_0}{8}$ (d) $\frac{I_0}{11}$

Q35. Consider an ideal operational amplifier as shown in the figure below with $R_1 = 5k\Omega, R_2 = 1k\Omega, R_L = 100k\Omega$. For an applied input voltage $V = 20\,mV$, the current passing through R_2 is



(a) $10mA$

(b) $20mA$

(c) $10\mu A$

(d) $20\mu A$

Q36. If the root-mean-squared momentum of a particle in the ground state $(n=0)$ of a one-dimensional simple harmonic potential is p_0, then its root-mean-squared momentum in the n^{th} excited state is

(a) $P_{rms} = \sqrt{\frac{m\omega\hbar}{2}}\sqrt{2n+1}$

(b) $P_{rms} = \sqrt{\frac{m\omega\hbar}{2}}\sqrt{2n}$

(c) $P_{rms} = \sqrt{\frac{m\omega\hbar}{2}}\sqrt{n+1}$

(d) $P_{rms} = \sqrt{\frac{m\omega\hbar}{2}}\sqrt{n}$

Q37. Consider a potential barrier A of height V_0 and width b, and another potential barrier B of height $4V_0$ and the same width b. The ratio T_A/T_B of tunnelling probabilities T_A and T_B, through barriers A and B respectively, for a particle of energy $V_0/10$ is best approximated by

(a) $\exp\left[\left(\sqrt{1.99}-\sqrt{0.99}\right)\right]$

(b) $\exp\left[\left(\sqrt{4.9}-\sqrt{0.9}\right)\right]$

(c) $\exp\left[\left(\sqrt{3.9}-\sqrt{0.9}\right)\right]$

(d) $\exp\left[\left(\sqrt{2.9}-\sqrt{0.9}\right)\right]$

Q38. If $\langle m|n\rangle = \delta_{m,n}$, the two vectors $|\phi_1\rangle = a|n\rangle$ and $|\phi_2\rangle = b|n\rangle + c|m\rangle$ are orthonormal if

(a) $a=\pm1,\ b=\pm1/\sqrt{2},\ c=\pm1/\sqrt{2}$

(b) $a=\pm1,\ b=\pm1,\ c=0$

(c) $a=\pm1,\ b=0,\ c=\pm1$

(d) $a=\pm1,\ b=\pm1/2,\ c=1/2$

Q39. The Coulomb potential $V(r) = -e^2/r$ of a hydrogen atom is perturbed by adding $H' = bx$ (where b is a constant) to the Hamiltonian. The first order correction to the ground state energy is

(The ground state wavefunction is $\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$)

(a) 0 (b) ba_0 (c) $\frac{ba_0}{2}$ (d) $\sqrt{2}ba_0$

Q40. A phosphorous doped silicon semiconductor (doping density: $10^{17}/\text{cm}^3$) is heated from 100°C to 200°C. Which one of the following statements is CORRECT?

(a) Position of Fermi level moves towards conduction band

(b) Position of dopant level moves towards conduction band

(c) Position of Fermi level moves towards middle of energy gap

(d) Position of dopant level moves towards middle of energy gap

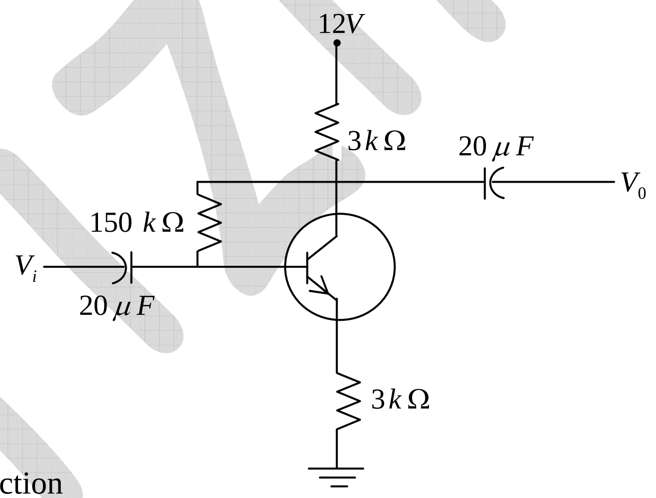
Q41. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current I_C is

(a) 0.6 mA

(b) 0.9 mA

(c) 1.6 mA

(d) 2.6 mA



Q42. A thermodynamic function

$$G(T, P, N) = U - TS + PV$$

is given in terms of the internal energy U, temperature T, entropy S, pressure P, volume V and the number of particles N. Which of the following relations is true? (In the following μ is the chemical potential.)

(a) $S = \left.\frac{\partial G}{\partial T}\right|_{N,P}$ (b) $P = \left.\frac{\partial G}{\partial V}\right|_{N,T}$ (c) $V = \left.\frac{\partial G}{\partial P}\right|_{N,T}$ (d) $\mu = -\left.\frac{\partial G}{\partial N}\right|_{P,T}$

Q43. A box, separated by a movable wall, has two compartments filled by a monoatomic gas of $\frac{C_P}{C_V}=\gamma$. Initially the volumes of the two compartments are equal, but the pressures are $2P_0$ and P_0 respectively. When the wall is allowed to move, the final pressures in the two compartments become equal. The final pressure is

(a) $\left(\frac{2}{3}\right)^{\gamma} P_0$ (b) $3\left(\frac{2}{3}\right)^{\gamma} P_0$ (c) $\frac{P_0}{2^{\gamma-1}}\left(1+2^{1/\gamma}\right)^{\gamma}$ (d) $\left(\frac{2^{1/\gamma}}{1+2^{1/\gamma}}\right)^{\gamma} P_0$

Q44. A gas of photons inside a cavity of volume V is in equilibrium at temperature T. If the temperature of the cavity is changed to $\frac{T}{2}$, the radiation pressure will change by a factor of

(a) $\frac{1}{16}$ (b) 16 (c) $\frac{1}{4}$ (d) 4

Q45. In a measurement of the viscous drag force experienced by spherical particles in a liquid, the force is found to be proportional to $V^{1/3}$ where V is the measured volume of each particle. If V is measured to be $30\,mm^3$, with an uncertainty of 19.4 mm^3, the resulting relative percentage uncertainty in the measured force is

(a) 2.08 (b) 0.09 (c) 0.66 (d) 0.33

PART C

ANSWER ANY 20 QUESTONS

Q46. The value of integral $\int_{-\infty}^{+\infty} \frac{x}{(x^2+1)(x^2+4)} dx$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

Q47. The Fourier transform of $f(x) = \begin{cases} -1 & -1 < x < 0 \\ 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ is

(a) $\frac{i}{\omega}\sqrt{\frac{2}{\pi}}(\cos\omega + 1)$ (b) $\frac{i}{\omega}\sqrt{\frac{2}{\pi}}(\cos\omega - 1)$

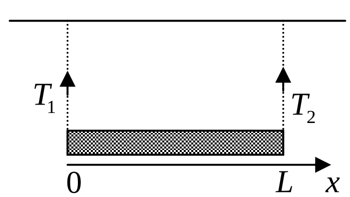
(c) $-\frac{i}{\omega}\sqrt{\frac{2}{\pi}}(1 + \cos\omega)$ (d) $\frac{i}{\omega}\sqrt{\frac{2}{\pi}}(1 - \cos\omega)$

Q48. The Lagrangian of a free relativistic particle (in one dimension) of mass m is given by $L = -m\sqrt{1-\dot{x}^2}$ where $\dot{x} = dx/dt$. If such a particle is acted upon by a constant force F in the direction of its motion, the equation of phase space trajectories obtained from the corresponding Hamiltonian are

(a) $(Fx + mc^2)^2 - P^2 = m^2c^4$ (b) $(Fx + mc^2)^2 + P^2 = m^2c^4$

(c) $(Fx - mc^2)^2 - P^2 = m^2c^4$ (d) $(Fx - mc^2)^2 + P^2 = m^2c^4$

Q49. The linear mass density of a rod of length L varies from one end to the other as $\lambda_0\left(1 + \frac{x^2}{L^2}\right)$, where x is the distance from one end with tensions T_1 and T_2 in them (see figure), and λ_0 is a constant. The rod is suspended from a ceiling by two massless strings. Then, which of the following statement(s) is **Incorrect**?



(a) The mass of the rod is $\frac{4\lambda_0 L}{3}$

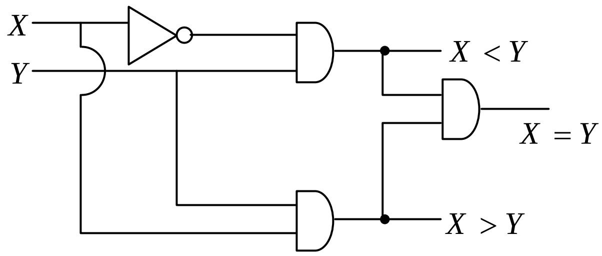
(b) The centre of gravity of the rod is located at $x = \frac{9L}{16}$

(c) The tension T_1 in the left string is $\frac{7\lambda_0 Lg}{12}$

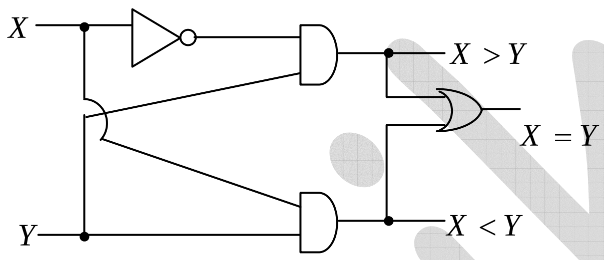
(d) The tension T_2 in the right string is $\frac{3\lambda_0 Lg}{2}$

Q50. In the figures below, X and Y are one bit inputs. The circuit which corresponds to a one bit comparator is

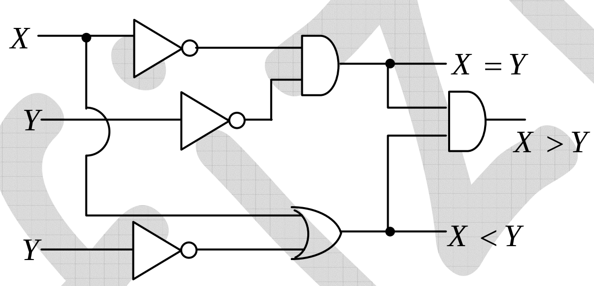
(a)



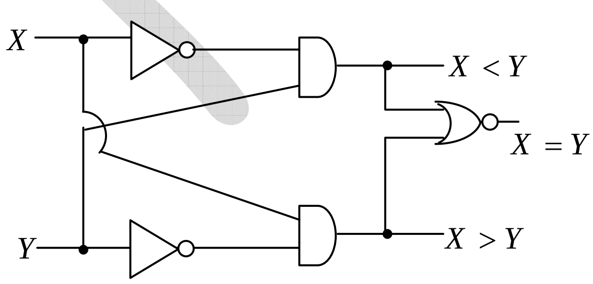
(b)



(c)



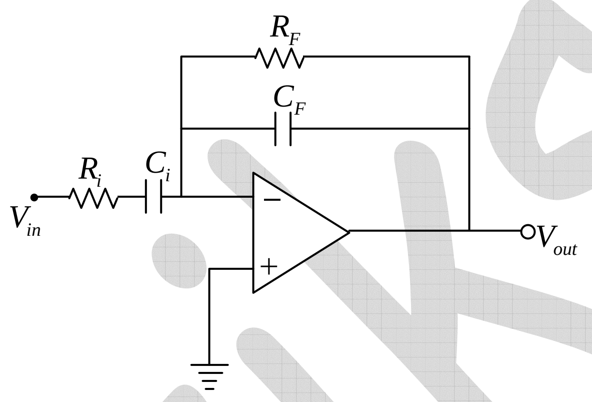
(d)



Q51. The energy of a one-dimensional system if force is given as $F(x) = -knx^{2n-1}$, where k and n are two positive constants, is E_0. The time period of oscillation τ satisfies (it is given as potential, $V = 0$ as $x = 0$)

(a) $\tau \propto E_0^{-\frac{1}{n}}$ (b) $\tau \propto E_0^{\frac{1-n}{2n}}$ (c) $\tau \propto E_0^{\frac{n-2}{2n}}$ (d) $\tau \propto E_0^{\frac{1+n}{2n}}$

Q52. In the following operational amplifier circuit $C_{in} = 10\,nF$, $R_{in} = 20\,k\Omega$, $R_F = 400\,k\Omega$ and $C_F = 50\,pF$.



The magnitude of the gain at a input signal frequency of $16\,kHz$ is

(a) 67 (b) 0.15 (c) 9 (d) 3.5

Q53. The active medium in a blue LED (light emitting diode) is a $Ga_xIn_{1-x}N$ alloy. The band gaps of GaN and InN are $3.5\,eV$ and $1.5\,eV$ respectively. If the band gap of $Ga_xIn_{1-x}N$ varies approximately linearly with x, the value of x required for the emission of light of wavelength $600\,nm$ is (take $hc \approx 1200\,eV\text{-}nm$)

(a) 0.95 (b) 0.75 (c) 0.50 (d) 0.25

Q54. In a thermodynamic system in equilibrium, each molecule can exist in three possible states with two of them has probabilities $1/2, 1/3$ respectively. The entropy per molecule is

(a) $k_B \ln 3$ (b) $\frac{2}{3}k_B \ln 2 + \frac{1}{2}k_B \ln 3$

(c) $\frac{1}{2}k_B \ln 2 + \frac{2}{3}k_B \ln 3$ (d) $\frac{1}{2}k_B \ln 2 + \frac{1}{6}k_B \ln 3$

Q55. The single particle energy levels of a non-interacting three-dimensional isotropic system, labelled by momentum k, are proportional to k^2. The ratio $\bar{P}/\in$ of the average pressure $\bar{P}$ to the energy density $\in$ at a fixed temperature, is

(a) $1/3$ (b) $2/3$ (c) 1 (d) 3

Q56. The Hamiltonian for three Ising spins S_0, S_1 and S_2, taking values ± 1, is

$$H = -JS_0\left(S_1 + S_2\right)$$

If the system is in equilibrium at temperature T, the Helmoltz energy of the system, in terms of $\beta = \left(k_B T\right)^{-1}$, is

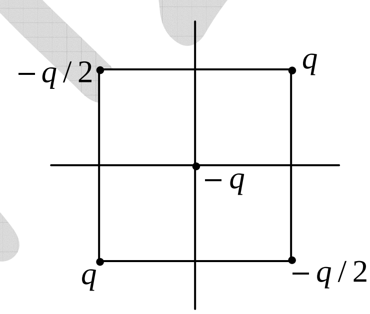
(a) $-2kT\ln 2 - kT\ln\left(1+\sinh 2\beta J\right)$ (b) $-2kT\ln 2 - kT\ln\left(1+\cosh 2\beta J\right)$

(c) $-2kT\ln 2 + kT\ln\left(1+\cosh 2\beta J\right)$ (d) $-2kT\ln 2 + kT\ln\left(1+\sinh 2\beta J\right)$

Q57. An electron is decelerated at a constant rate starting from an initial velocity u (where $u << c$) to $u/2$ during which it travels a distance s. The amount of energy radiated in time t is

(a) $\frac{3\mu_0 e^2 u^3}{96\pi cs}$ (b) $\frac{\mu_0 e^2 u^3}{12\pi cs}$ (c) $\frac{\mu_0 e^2 u^2}{6\pi c^2 s}$ (d) $\frac{\mu_0 e^2 u}{16\pi cs}$

Q58. Let four point charges q, $-q/2$, q and $-q/2$ be placed at the vertices of a square of side a. Let another point charge $-q$ be placed at the cnetre of the square (see the figure).



Let $V(r)$ be the electrostatic potential at a point P at a distance $r >> a$ from the centre of the square. Then $E(2r)/E(r)$ is

(a) $\frac{1}{16}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Q59. An electromagnetically-shielded room is designed so that at a frequency $\omega = 10^7$ rad/s the intensity of the external radiation that penetrates the room is 1% of the incident radiation. If $\sigma = \frac{1}{8\pi}\times 10^6 (\Omega m)^{-1}$ is the conductivity of the shielding material, its minimum thickness should be (given that $\ln 10 = 2.3$)

(a) 4.60 mm (b) 2.30 mm (c) 0.23 mm (d) 0.46 mm

Q60. A beam of unpolarized light in a medium with dielectric constant $\in_1$ is reflected from a plane interface formed with another medium of dielectric constant $\in_2 = 3\in_1$. The two media have identical magnetic permeability. If the angle of incidence is 60^0, then the reflected light

(a) has the same polarization as the incident light

(b) is plane polarized parallel to the plane of incidence

(c) is circularly polarized

(d) is plane polarized perpendicular to the plane of incidence

Q61. Consider the potential

$$V(\vec{r}) = \sum_i V_0 a^3 \delta^{(3)}(\vec{r} - \vec{r}_i)$$

where $\vec{r}_i$ are the position vectors of the vertices of a cube of length a centered at the origin and V_0 is a constant. If $V_0 a^2 << \frac{\hbar^2}{m}$, the differential scattering cross-section, in the low-energy limit, is

(a) $\frac{4m^2V_0^2a^6}{\pi^2\hbar^4}$ (b) $\frac{8m^2V_0^2a^6}{\pi^2\hbar^4}$

(c) $\frac{16m^2V_0^2a^6}{\pi^2\hbar^4}$ (d) $\frac{32m^2V_0^2a^6}{\pi^2\hbar^4}$

Q62. Using the trial function

$$\psi(x) = \begin{cases} A(a^2 - x^2), & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the ground state energy of a one-dimensional harmonic oscillator is

(a) $\hbar\omega$ (b) $\sqrt{\frac{5}{14}}\frac{\hbar\omega}{2}$ (c) $\frac{1}{2}\hbar\omega$ (d) $\frac{\hbar\omega}{2}\sqrt{\frac{10}{7}}$

Q63. A random variable n obeys Poisson statistics. The probability of finding $n=0$ is 10^{-6}. The variance value of n is nearest to

(a) 14 (b) 10^6 (c) e (d) 10^2

Q64. For Dirac particle if x component of angular momentum is given by $L_z = xp_y - yp_x$ then the value of $\frac{dL_z}{dt}$ is

(a) $c\left(p_y\alpha_x - p_x\alpha_y\right)$ (b) $c\left(p_z\alpha_y - p_y\alpha_z\right)$

(c) $c\left(p_y\alpha_y - p_z\alpha_z\right)$ (d) $c\left(p_y\alpha_z - p_z\alpha_y\right)$

Q65. If potential is given by $V(x) = \begin{cases} 0, & x<0 \\ V_0 - \lambda x^2, & x>0 \end{cases}$

then using W.K.B approximation tunneling probability T is proportional to $T \propto \exp\left[-c\left(V_0 - E\right)^{\alpha}\right]$ then value of α is given by

(a) $\alpha = 1$ (b) $\alpha = 2$

(c) $\alpha = \frac{1}{2}$ (d) $\alpha = \frac{2}{3}$

Q66. The atom of lead vapor have the ground state configuration $6s^2 6p^2$. The total number of levels in the presence of weak magnetic field are

(a) 3 (b) 10 (c) 15 (d) 18

Q67. A particular spectral line corresponding to a $J=1 \rightarrow J=0$ transition is split in a magnetic field of 1000 Gauss into three components. The g - factor is one and zero field line occurs at $1849\,\overset{0}{\mathrm{A}}$. The minimum resolution required in spectrometer to resolve these three components is

(a) $0.16\,\overset{0}{\mathrm{A}}$ (b) $0.016\,\overset{0}{\mathrm{A}}$ (c) $0.0016\,\overset{0}{\mathrm{A}}$ (d) $0.00016\,\overset{0}{\mathrm{A}}$

Q68. If in a spontaneous α - decay of ${}^{232}_{92}U$ at rest, the total energy released in the reaction is $5.2\,MeV$, then the energy carried by the α - particle is

(a) $4.1\,MeV$ (b) $5.1\,MeV$ (c) $5.3\,MeV$ (d) $5.4\,MeV$

Q69. The range of the nuclear force between two nucleons due to the exchange of pions is $1.40\ fm$. If the mass of pion is $140\,MeV/c^2$ and the mass of the rho-meson is $1106\,MeV/c^2$, then the range of the force due to exchange of rho-mesons is

(a) 1.40 fm (b) 7.70 fm (c) 0.25 fm (d) 0.18 fm

Q70. In the β decay process, the transition $2^+ \rightarrow 3^+$, is

(a) allowed both by Fermi and Gamow-Teller selection rule

(b) allowed by Fermi but not by Gamow-Teller selection rule

(c) not allowed by Fermi but allowed by Gamow-Teller selection rule

(d) not allowed both by Fermi and Gamow-Teller selection rule

Q71. Gold (FCC) with lattice parameter $4.08\,\overset{0}{\mathrm{A}}$ has electrical resistivity $\rho = 2.2\,\mu\Omega$ cm at room temperature. Using a free electron model and assuming one valence electron per atom, the electric heat capacity at room temperature ($298\,K$) is (in units of eVK^{-1})

(a) 2×10^{-4} (b) 2×10^{-5} (c) 2×10^{-6} (d) 2×10^{-7}

Q72. Consider the normal modes of a linear chain in which the force constants between nearest neighbour atoms are alternatively c and $8c$. Assuming that the masses are equal and the nearest neighbour separation is $\frac{a}{2}$. The cut off frequency of acoustic branch is

(a) $\frac{\sqrt{16c}}{m}$ (b) $\frac{\sqrt{18c}}{m}$ (c) $\frac{\sqrt{4c}}{m}$ (d) $\frac{\sqrt{2c}}{m}$

Q73. A two-dimensional metal has one atom at valence one in a simple rectangular primitive cell of $a_1 = 2\,\overset{0}{\mathrm{A}}$ and $a_2 = 4\,\overset{0}{\mathrm{A}}$. The radius of the free electron Fermi sphere is

(a) $0.89\,\overset{0}{\mathrm{A}}$ (b) $0.28\,\overset{0}{\mathrm{A}}$ (c) $0.12\,\overset{0}{\mathrm{A}}$ (d) $0.09\,\overset{0}{\mathrm{A}}$

Q74. The generating function $F(x,t) = \sum_{n=0}^{\infty} P_n(x)t^n$ for the Legendre polynomials $P_n(x)$ is $F(x,t) = \left(1-2xt+t^2\right)^{-1/2}$. The value of $P_2(-1)$ is

(a) $5/2$ (b) $3/2$ (c) $+1$ (d) -1

Q75. From the Taylor's series for $y(x)$, find $y(0.1)$ correct to four decimal places if $y(x)$ satisfies

$$y'' - xy' - y = 0 \text{ with the conditions } y(0) = 1 \text{ and } y'(0) = 0.$$

(a) 1.0050 (b) 0.0150 (c) 0.0155 (d) 0.5025
