ALL INDIA TEST SERIES

FOR

GATE (PHYSICS) January 2018

Full Length Test – 01

TIME: 3 HOURS **MAXIMUM MARKS: 100**

This question paper consists of **2 sections**, subject specific GATE paper for **85 marks** and General Aptitude (GA) for **15 marks**. Both these sections are compulsory.

The GA section consists of **10** questions. Question numbers 56 to 60 are of 1-Mark each, while question numbers 61 to 65 are of 2-Mark each.

The subject specific GATE paper section consists of **55** questions, out of which question numbers 1 to 25 are of 1-mark each, while question numbers 26 to 55 are of 2-mark each.

The question paper may consist of questions of **Multiple Choice Question Type** (MCQ) and **Numerical Answer Type (NAT)**.

Multiple choice type questions will have four choices against A, B, C, D, out of which only **ONE** is the correct answer.

For numerical answer type questions, each question will have a numerical answer and there will not be any choices**.**

All questions that are not attempted will result in zero marks. However, wrong answers for multiple choice type questions (MCQ) will result in **NEGATIVE** marks. For all MCQ questions a wrong answer will result in deduction of $\frac{1}{3}$ marks for a 1-mark question and $\frac{2}{3}$ marks for a 2-mark question.

There is **NO NEGATIVE MARKING** for questions of **NUMERICAL ANSWER TYPE**.

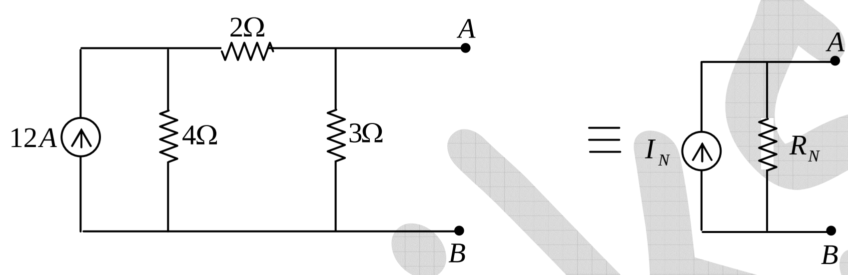
Non-programmable type Calculator is allowed. Charts, graph sheets, and mathematical tables are **NOT** allowed in the Examination Hall. You must use the Scribble pad provided to you at the examination centre for all your rough work. The Scribble Pad has to be returned at the end of the examination.

Q1-Q25 carrying 1 mark each.

Q1. A hollow sphere of mass M and radius a is rolling with a linear speed v on a flat surface without slipping. The magnitude of the angular momentum of the sphere on the surface is

(a) $\frac{2}{5}mav$ (b) $\frac{7}{5}mav$ (c) $\frac{5mva}{3}$ (d) $\frac{3}{2}mav$

Q2. In the circuit shown in the figure the Norton current I_N and Norton resistance R_N as seen by the load between points A and B are respectively



(a) $4A, 2\Omega$ (b) $8A, 2\Omega$ (c) $8A, 3\Omega$ (d) $4A, 3\Omega$

Q3. Two particles of identical mass move in circular orbits under a central potential $V(r) = kr$. Let l_1 and l_2 be the angular momenta and r_1, r_2 be the radii of the orbits respectively. If $\frac{l_1}{l_2} = 8$, the value of $\frac{r_1}{r_2}$ is: ………..

Q4. In a region of space, a time dependent magnetic field $B(t) = 0.4t$ tesla points vertically upwards. Consider a horizontal, circular loop of radius $1cm$ in this region. The magnitude of the electric field (in mV/m) induced in the loop is

(a) 1 (b) 2 (c) 3 (d) 4

Q5. If generalized coordinate transformation $Q = q^a p$ and $P = q^b$ are canonically transformed then value of a and b is given by

(a) $a = 2, b = 1$ (b) $a = 2, b = -1$ (c) $a = -2, b = 1$ (d) $a = -2, b = -1$

Q6. A circular disc of radius a on the xy plane has a surface charge density $\sigma = \sigma_0 \cos\theta$. The electric dipole moment of this charge distribution is

(a) $\frac{\sigma_0 \pi a^4}{4}\hat{x}$ (b) $\frac{\sigma_0 \pi a^3}{3}\hat{x}$ (c) $\frac{\sigma_0 \pi a^2}{3}\hat{x}$ (d) $\frac{\sigma_0 \pi a^2}{2}\hat{x}$

Q7. A high pass filter is formed by a resistance R and a capacitance C. At the cut-off angular frequency $\omega_C = \frac{1}{RC}$ the voltage gain is $\frac{1}{\alpha}$, then the value of α is...........

Q8. Nuclei of a radioactive element A are being produced at a constant rate α. The element has decay constant λ. At time $t = 0$, there are N_0 nuclei of the element. Then the number N of nuclei of A at time

(a) $\frac{1}{\lambda}\left[\alpha - \alpha\left(1 - N_0\lambda\right)e^{-\lambda t}\right]$ (b) $\frac{1}{\lambda}\left[\left(\alpha - N_0\lambda\right)e^{-\lambda t}\right]$

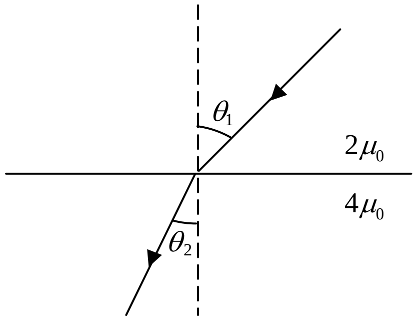
(c) $\frac{1}{\lambda}\left[\alpha - \left(\alpha - N_0\lambda\right)e^{-\lambda t}\right]$ (d) $\frac{1}{\lambda}\left[\alpha - \left(1 - N_0\lambda\right)e^{-\lambda t}\right]$

Q9. The work done in bringing a charge $+q$ from infinity in free space, to a position at a distance d in front of a infinite grounded metal surface is $\left(-\alpha \frac{q^2}{32\pi\varepsilon_0 d}\right)$. Then the value of α is..................

Q10. An alpha particle of energy $5\ MeV$ is scattered through 180^o by a fixed uranium nucleus. The distance of closest approach is of the order of

(a) $1\overset{0}{A}$ (b) $10^{-10}\ cm$ (c) $10^{-12}\ cm$ (d) $10^{-15}\ cm$

Q11. At the interface between two linear dielectrics (with permeability $2\mu_0$ and $4\mu_0$), the magnetic field lines bend, as shown in the figure. Assume that there is no free current at the interface. The ratio $\frac{\cot\theta_1}{\cot\theta_2}$ is...........



Q12. The nuclear reaction which results from the incidence of sufficiently energetic α-particles on nitrogen nuclei is

$$ {}_2^4He + {}_7^{14}N \rightarrow {}_8^{17}O + {}_1^1H .$$

The minimum α-particle kinetic energy required to initiate the above reaction is………… MeV

(Atomic masses in amu: ${}_1^1H = 1.0081$; ${}_2^4He = 4.0039$; ${}_7^{14}N = 14.0075$; ${}_8^{17}O = 17.0045$)

Q13. The inverse Laplace transform of $F(s) = \ln \frac{s^2 + w^2}{s^2}$ is

(a) $\frac{1}{t}(1-\cos wt)$ (b) $\frac{2}{t}(1-\cos wt)$ (c) $\frac{1}{t}(1+\cos wt)$ (d) $\frac{2}{t}(1+\cos wt)$

Q14. If the complex (exponential) Fourier series of the periodic function $f(x) = x,\ 0 < x < 2$ written as $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x}$ then it is found that $c_n = \frac{(-1)}{in\pi}$ for $n \neq 0$ and $c_n = 1$ for $n = 0$.

Using this fact the sum of series $\left[1 + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}\right]$ is ……

(Answer must be upto two digits after the decimal.)

Q15. Three identical non-interacting particles, each of spin ½ and mass m, are moving in a one-dimensional infinite potential well given by,

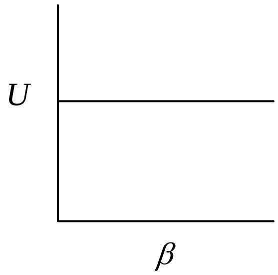
$$V(x) = \begin{cases} 0 & \text{for } -a < x < a \\ \infty & otherwise \end{cases}$$

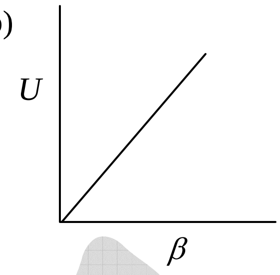
The energy of the lowest energy state of the system is

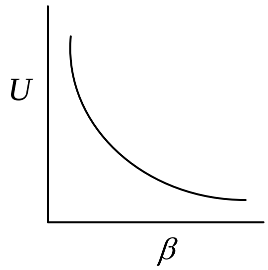
(a) $\frac{3\pi^2\hbar^2}{ma^2}$ (b) $\frac{7\pi^2\hbar^2}{4ma^2}$ (c) $\frac{3\pi^2\hbar^2}{4ma^2}$ (d) $\frac{5\pi^2\hbar^2}{2ma^2}$

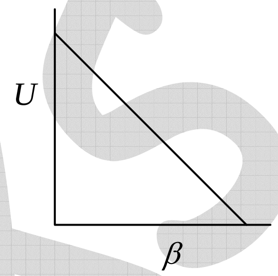
Q16. Suppose the spin degrees of freedom of a 2 - particle system with spin $s_1 = \frac{1}{2}$ and $s_2 = \frac{3}{2}$ can be described by a n - dimensional Hilbert subspace. Then value of n is ……..

Q17. Consider one dimensional classical harmonic oscillator at temperature T. Which of the following is the correct plot between energy and β where $\beta = \frac{1}{kT}$?

(a) 

(b) 

(c) 

(d) 

Q18. In a Young's double slit experiment using light, the apparatus has two slits of unequal widths. When only slit-1 is open, the maximum observed intensity on the screen is $9I_0$. When only slit-2 is open, the maximum observed intensity is I_0. When both the slits are open, an interference pattern appears on the screen. The ratio of the intensity of the principal maximum to that of the nearest minimum is…………..

Q19. A rigid and thermally isolated tank is divided into two compartments of equal volume V, separated by a thin membrane. One compartment contains one mole of an ideal gas A and the other compartment contains one mole of a different ideal gas B. The two gases are in thermal equilibrium at a temperature T. If the membrane ruptures, the two gases mix. Assume that the gases are chemically inert. The change in the total entropy of the gases on mixing is $\alpha R \ln 2$ then value of α is ………

Q20. In a crystalline solid, the energy band structure (E-K relation) for an electron of mass m is given by, $E = \frac{\hbar^2}{2m} k(3k - 2)$. The ratio of effective mass of the electron in the crystal to its rest mass is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

Q21. A $^{200}Hg^{35}Cl$ molecule emits a $4.4\,cm$ photon when it undergoes a rotational transition from $J=0$ to $J=1$. The interatomic distance in this molecule is

(a) $2.2\,\overset{0}{A}$ (b) $2.9\,\overset{0}{A}$ (c) $1.12\,\overset{0}{A}$ (d) $1.5\,\overset{0}{A}$

Q22. Fermi energies of metals A and B are $7\cdot05eV$ and $3\cdot2eV$ respectively. In the presence of external magnetic field $0\cdot1T$, the ratio of hall resistance $\frac{P_A}{P_B}$ is……

Q23. Which of the following transition is allowed for radiative transitions?

(a) $He:(1s)(2p):^3P_1\rightarrow(1s)^2:{}^1s_0$

(b) $C:(1s)^2(2s)^2(2p)(3s):{}^3P_0\rightarrow(1s)^2(2s)^2(2p)^2:{}^3P_0$

(c) $Na:(1s)^2(2s)^2(2p)^6(4d):{}^2D_{5/2}\rightarrow(1s)^2(2s)^2(2p)^6(3p):{}^2P_{1/2}$

(d) $C:(1s)^2(2s)^2(2p)(3s):{}^3P_1\rightarrow(1s)^2(2s)^2(2p)^2:{}^3P_0$

Q24. The lowest vibrational states of $^{22}Na^{35}Cl$ molecule are $0.063eV$ apart. The approximate force constant of this molecule is $\left[\hbar=6.58\times10^{-16}eV\cdot s\right]$

(a) $200\,N/m$ (b) $180\,N/m$ (c) $240\,N/m$ (d) $280\,N/m$

Q25. The ratio of $np_{3/2}\rightarrow n's_{1/2}$ and $np_{1/2}\rightarrow n's_{1/2}$ is

(a) $3:1$ (b) $3:2$ (c) $2:1$ (d) $1:3$

Q26-Q55 carrying 2 marks each.

Q26. If the Lagrangian of a dynamical system in two dimensions is $L=\frac{1}{2}m\dot{x}^2-m\dot{x}\dot{y}$, then its Hamiltonian is

(a) $H=-\frac{1}{m}p_xp_y-\frac{1}{2m}p_y^2$ (b) $H=-\frac{1}{2m}p_xp_y-\frac{1}{2m}p_y^2$

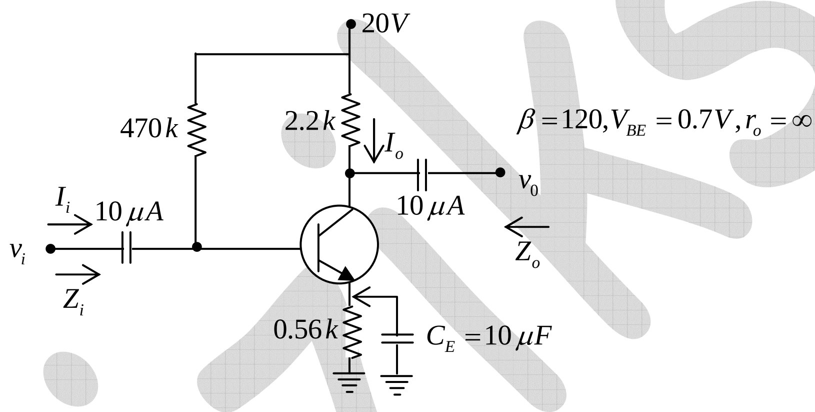
(c) $H=-\frac{1}{2m}p_xp_y-\frac{1}{2m}p_x^2$ (d) $H=-\frac{1}{m}p_xp_y-\frac{1}{2m}p_x^2$

Q27. Consider the motion of a classical particle with $m = 1kg$ in a one dimensional double-well potential $V(x) = \frac{1}{4}(x^2 - 2)^2$. If the particle is displaced infinitesimally from the minimum on the x-axis (and friction is neglected), then time period is $\alpha\pi$. The value of α is ………..

Q28. A material has conductivity of 10^{-2} mho/m and relative permittivity of 4. The frequency at which the conduction current in the medium is equal to the displacement current is

(a) 45 MHz (b) 90 MHz (c) 450 MHz (d) 900 MHz

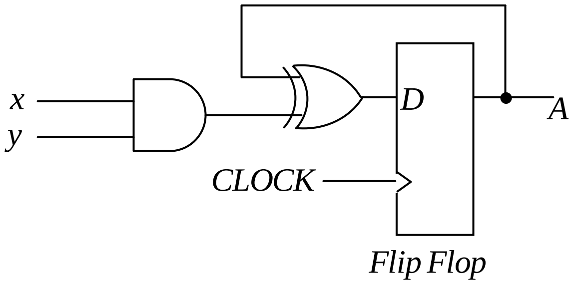
Q29. For the network shown in figure, the value of A_i is: (use $r_e = 6\Omega$)

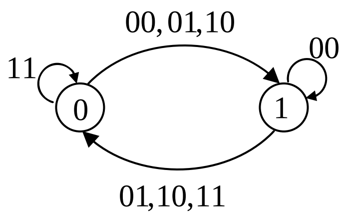


(a) ≈ 80 (b) ≈ 90 (c) ≈ 100 (d) ≈ 120

Q30. A space crew has a life support system that can last only for 2000 hours. If the minimum speed required for safe travel of the crew between two space stations separated by a fixed distance of 2.16 x 10^{12} km is $\frac{c}{\alpha}$, then the value of α is ….

Q31. The state diagram corresponding to the following circuit is



(a)


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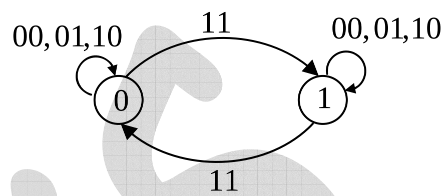
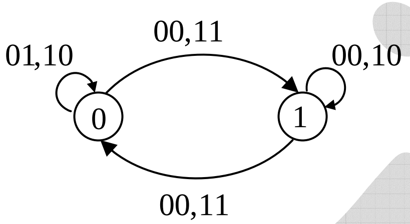
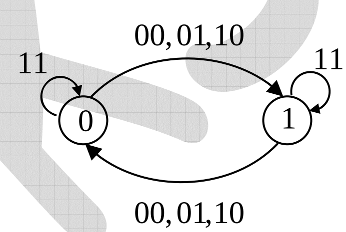


(c)


(d)


Q32. A cylindrical rod of length L has a mass density distribution given by $\rho(x)=\rho_0\left(1+\frac{x^2}{L}\right)$, where x is measured from one end of the rod and ρ_0 is a constant of appropriate dimensions. The centre of mass of the rod is αL, at a distance of from that end then value of α is …………..

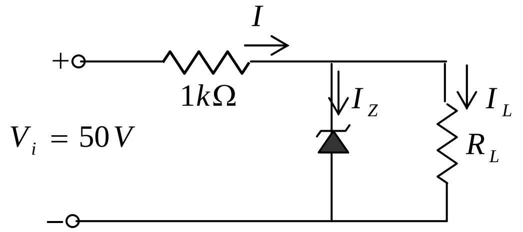
Q33. The binding energy of a light nucleus (Z, A) in MeV is given by the approximate formula

$$B(A,Z)\approx 16A-20A^{2/3}-\frac{3}{4}Z^2A^{-1/3}+30\frac{(N-Z)^2}{A}$$

where $N=A-Z$ is the neutron number. The approximate value of Z of the most stable isobar for a given $A=27$ is

(a) 12 (b) 14 (c) 16 (d) 18

Q34. For the given zener diode network, the maximum value of load resistance R_L that will maintain output voltage to $15V$ is………..Ω ($V_Z = 15\,V$, $I_{ZM} = 32mA$)



Q35. The value of the magnetic field required to maintain non-relativistic protons of energy $1MeV$ in a circular orbit of radius 100 mm is………..$Tesla$

(Given: $m_p = 1.67\times10^{-27}\,kg$, $e = 1.6\times10^{-19}\,C$)

Q36. Total binding energy of ${}^5_2\text{He}$ is approximately equal to……….. MeV

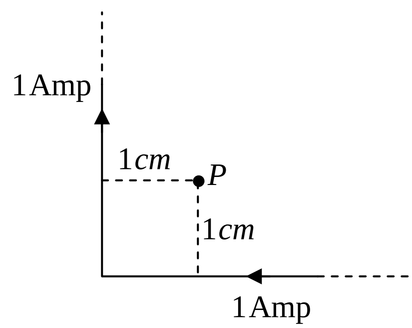
where $\left(m\left({}^5_2\text{He}\right) = 5.01220u,\ m\left({}^1\text{H}\right) = 1.007825u,\ m_n = 1.008665\,u\right)$

Q37. Consider interface between two dielectric medium having refractive index n_1 and n_2 where $n_1 > n_2$. If electromagnetic wave strikes normal to the interface such that reflection and transmission coefficients are equal then the ratio $\dfrac{n_1}{n_2}$ is………

Q38. If $\vec{A} = \hat{i}yz + \hat{j}xz + \hat{k}xy$, then the integral $\oint_C \vec{A}\cdot d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by $x = 0, x = a$ and $y = 0,\ y = b$) is

(a) $\frac{1}{2}\left(a^3 + b^3\right)$ (b) $\pi\left(ab^2 + a^2b\right)$ (c) $\pi\left(a^3 + b^3\right)$ (d) 0

Q39. Consider a L-shaped conductor carrying a current of 1 Ampere, as shown in the figure. Assume that both the arms extend to infinity. The strength of the magnetic field at the point P, which is at a distance of $1cm$ from both the arms, is approximately equal to………$\times10^{-5}\,Tesla$



Q40. Let $A=\begin{bmatrix}1&0&1\\0&3&2\\0&0&2\end{bmatrix}$ and $B=\begin{bmatrix}1&0&0\\0&3&0\\0&0&2\end{bmatrix}$. Similarity transformation of A to B can be performed by the matrix

(a) $\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$ (b) $\begin{bmatrix}1&0&0\\0&1&1\\0&2&0\end{bmatrix}$

(c) $\begin{bmatrix}1&0&1\\0&1&-2\\0&0&1\end{bmatrix}$ (d) $\begin{bmatrix}1&0&1\\0&1&1\\0&0&1\end{bmatrix}$

Q41. The Coulomb potential $V(r)=-e^2/r$ of a hydrogen atom is perturbed by adding $H'=\frac{bx^2}{2}$ (where b is a constant) to the Hamiltonian. The first order correction to the ground state energy is

(The ground state wavefunction is $\psi_0=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$)

(a) 0 (b) ba_0^2 (c) $\frac{3ba_0^2}{2}$ (d) $\sqrt{2}ba_0^2$

Q42. Consider the differential equation $\frac{d^2y}{dx^2}+9y=4\sinh x\cosh x+e^{-2x}$ subjected to the initial condition $y(0)=0$ and $y'(0)=1$. If the value of $y(\pi)$ is $\frac{2}{\alpha}$ then the value of α is…..

Q43. If $\vec{s}_1$ and $\vec{s}_2$ are the spin operators of the two electrons of a He atom, the value of $\langle\vec{s}_1.\vec{s}_2\rangle$ for the first excited state to ground state is

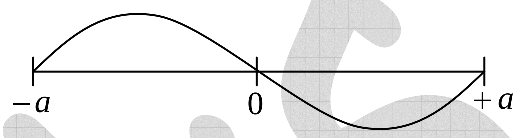
(a) $\frac{3}{2}\hbar^2,-\frac{3}{2}\hbar^2$ (b) $\frac{3}{4}\hbar^2,-\frac{3}{4}\hbar^2$ (c) $-\frac{3}{4}\hbar^2,\frac{1}{4}\hbar^2$ (d) $\frac{1}{4}\hbar^2,\frac{3}{4}\hbar^2$

Q44. In the radiation emitted by a black body, the ratio of the spectral densities at frequencies 4ν and 2ν will very with ν as:

(a) $\left[e^{2h\nu/k_BT}-1\right]^{-1}$ (b) $\left[e^{2h\nu/k_BT}+1\right]^{-1}$ (c) $\left[e^{2h\nu/k_BT}-1\right]$ (d) $\left[e^{2h\nu/k_BT}+1\right]$

Q45. The value of integral $\int_{-\infty}^{+\infty}\frac{x+5}{x^3-x}dx$ is …….

Q46. A particle is confined in a one dimensional box with impenetrable walls at $x = \pm a$. Its energy eigenvalue is $4\,eV$ and the corresponding eigenfunction is as shown below.



Then energy of second excited state is ……..eV

Q47. If T_N is the Neel temperature for anti – ferromagnet, then its susceptibility is given by

(a) $\chi=\frac{C}{T-T_N}$ (b) $\chi=\frac{C}{T+2T_N}$ (c) $\chi=\frac{C}{T+T_N}$ (d) $\chi=\frac{C}{T_N-T}$

Q48. The recoil energy E_R with which an hydrogen atom recoil when emitting a photon in a transition from $n=4$ level to $n=1$ level is $\left[m_H=1.008\,amu\right]$

(a) $1.73\times10^{-4}eV$ (b) $1.73\times10^{-5}eV$ (c) $1.73\times10^{-6}eV$ (d) $1.73\times10^{-7}eV$

Q49. Consider an elastic scattering of particles in $l=0$ states. If the corresponding phase shift δ_0 is 90^0 and the magnitude of the incident wave vector is equal to $2\sqrt{\pi}\;\; fm^{-1}$ then the total scattering cross section in units of fm^2 is _______.

Q50. A solid melts into a liquid via first order phase transition. The relationship between the pressure P and the temperature T of the phase transition is $P=-2T+P_0$, where P_0 is a constant. The entropy change associated with the phase transition is 1.0 $J\,mole^{-1}\,K^{-1}$. Here $\Delta v=v_{liquid}-v_{solid}$ is the magnitude of change in molar volume at the phase transition, then the value of Δv is …..

Q51. Two classical particles are distributed among $N(>2)$ sites on a ring. Each site can accommodate only one particle. If two particles occupy two nearest neighbour sites, then the energy of the system is increased by ε. The average Helmholtz free energy of the system at temperature T is

(a) $-kT\ln\left[N-2+\exp\left(\frac{-2\varepsilon}{kT}\right)\right]$ (b) $-kT\ln\left[N-2+\exp\frac{2\varepsilon}{kT}\right]$

(c) $-kT\ln\left[N-3+\exp\left(\frac{-2\varepsilon}{kT}\right)\right]$ (d) $-kT\ln\left[N-3+\exp\frac{2\varepsilon}{kT}\right]$

Q52. The atom He^3 has spin $\frac{1}{2}$ and is fermion. The density of liquid He^3 is $0\cdot081\,gcm^{-3}$ at absolute zero. The Fermi energy E_F is ……..$\times10^{-4}eV$ (Mass of $He^3=5\cdot03\times10^{-24}\,g$).

Q53. Let N_{MB}, N_{BE} denote the number of ways in which two particles can be distributed in two energy states according to Maxwell-Boltzmann and Bose-Einstein statistics respectively. Then ratio $N_{MB}:N_{BE}$ is….. (Answer must be upto two digits after declined).

Q54. Monoatomic X-rays of wavelength $1A^0$ are incident on a simple cubic crystal. The first order Bragg reflection from (311) plane occurs at an angle of 30^0 from the plane. The lattice parameter on the crystal in $\left(\overset{0}{\text{A}}\right)$ is

(a) $\sqrt{\frac{11}{2}}$ (b) $\sqrt{11}$ (c) $\sqrt{3}$ (d) 1

Q55. The $J=0\rightarrow J=1$ rotational absorption line occurs at $11.53\times10^{10}\,Hz$ in $^{12}C^{16}O$ and $11.02\times10^{10}\,Hz$ in $^{?}C^{16}O$. The mass number of the unknown carbon isotop is

(a) 14 (b) 13 (c) 15 (d) 12

GENERAL APPTITUDE

Q56 – Q60 carry 1 Mark each.

Q56. Which of the phrases (a), (b), (c) and (d) given below in the statements should replace the phrase printed in bold in the sentence to make it grammatically correct. ?

Airline companies pay nearly 25 billion dollars for **their right of fly** over the countries other than their parent country.

(a) their right to fly (b) their right in flying

(c) their right to flying (d) their right of flight

Q57. Find out the word which is most OPPOSITE in meaning to the word printed in bold '**Bleaker.**

(a) hopeful (b) Warn (c) Cozy (d) Sheltered

Q58. What should come next in the following letter series?

$c\,b\,a\,a\,c\,b\,a\,a\,b\,c\,b\,a\,a\,b\,c\,c\,b\,a\,a\,b\,c\,d\,c\,b\,a\,a$

(a) a (b) b (c) c (d) d

Q59. The following three words are alike in a certain manner and so from a group. Which one of the four options belong to the same group?

(a) Cuboid (b) Diameter (c) Diagonal (d) Perimeter

Q60. There are 30 people in a ground. If all shakes hands with one another, how many hand shakes are possible?

(a) 870 (b) 435 (c) 30! (d) 29!+1

Q61 – Q65 carry 2 Marks each.

Q61. Read the following information carefully and answer the question given below it

$P \div Q$ means P is the father of Q

$P + Q$ means P is the mother of Q

$P - Q$ means P is the brother of Q

$P \times Q$ means P is the sister of Q

Which of the following shows that A is the aunt E?

(a) $A - B + C \div D \times E$ (b) $A \times B \div C \times D - E$

(c) $A \div B \times C + D - E$ (d) $A + B - C \times D \div E$

Q62. In a six story building (consisting of floors numbered $1, 2, 3, 4, 5$ and 6) the ground floor is numbered 1, the floor above it is numbered 2 and so on. The third floor is unoccupied. The building houses five persons viz P, Q, R, S and T each living on a different floor. On which of the floor does T lives if S lives between the floors on which R and T live and there are two floor between T's floor and Q's floor.

(a) 1^{st} floor (b) 2^{nd} floor (c) 3^{rd} floor (d) 4^{th} floor

Q63. If $\log_x y = 100$ and $\log_3 x = 10$, then the value of y is

(a) 3^{10} (b) 3^{100} (c) 3^{1000} (d) 3^{10000}

Q64. Angles of elevation of a pole are 60^0 and 45^0 from points at distance m and n on ground respectively. Here m, when measured from base of pole is less than n. What is the height of the pole?

(a) $\sqrt{mn\sqrt{3}}$ units (b) $\sqrt{mn\sqrt[4]{3}}$ units (c) $\sqrt{3mn}$ units (d) $\sqrt{mn}$ units

Q65. A sum of money gets doubled in 9 years when the interest is compounded annually. In how many years will the amount become four times?

(a) 12 years (b) 27 years (c) 18 years (d) 25 years
