Gate Full Length Test – 01

(Solution)

22-01-2018

Ans.1: (c)

Solution: $L = I\omega + mva \Rightarrow \frac{2}{3}ma^2\frac{v}{a} + mva = \frac{5mva}{3}$.

Ans. 2: (b)

Solution: $I_N = \frac{4}{6}\times 12 = 8A$ and $R_N = \frac{6\times 3}{9} = 2\Omega$.

Ans. 3: 4

Solution: $V_{eff} = \frac{l^2}{2mr^2} + kr$, where l is angular momentum.

Condition for circular orbit $\frac{\partial V_{eff}}{\partial r} = 0 \Rightarrow -\frac{l^2}{mr^3} + k = 0 \Rightarrow l^2 \propto r^3 \Rightarrow l \propto r^{3/2}$.

Thus $\frac{l_1}{l_2} = \left(\frac{r_1}{r_2}\right)^{3/2} \Rightarrow \frac{r_1}{r_2} = \left(\frac{l_1}{l_2}\right)^{2/3} \Rightarrow \frac{r_1}{r_2} = (8)^{2/3} = 4$.

Ans. 4: (b)

Solution: $\left|\vec{E}\right|\times 2\pi r = -\frac{\partial B}{\partial t}\times \pi r^2 \Rightarrow \left|\vec{E}\right| = \frac{r}{2}\frac{\partial B}{\partial t} = \frac{1\times 10^{-2}}{2}0.4 = 2\ mV/m$

Ans. 5: (b)

Solution: $\frac{\partial Q}{\partial q}.\frac{\partial P}{\partial p} - \frac{\partial P}{\partial q}.\frac{\partial Q}{\partial p} = 1 \Rightarrow 0 - bq^{b-1}q^a = 1$

$a + b - 1 = 0$ and $b = -1$ so $a = 2$

Ans. 6: (b)

Solution: $p = \int r'\rho(r')d\tau' = \int r'\sigma(r')da' = \int x\sigma(r')da'$

$$p = \iint (r\cos\theta)\times\sigma_0\cos\theta da' = \int_0^a\int_0^{2\pi}\left(\sigma_0 r\cos^2\theta\right)rdr\,d\theta$$

$$p = \sigma_0\int_0^a r^2dr\int_0^{2\pi}\cos^2\theta d\theta = \frac{\sigma_0 a^3}{3}\times\pi = \frac{\sigma_0\pi a^3}{3} \Rightarrow \vec{p} = \frac{\sigma_0\pi a^3}{3}\hat{x}$$

Ans. 7: 1.41

Solution: $\frac{v_0}{v_{in}}=\frac{R}{R+X_C}=\frac{1}{1+\frac{X_C}{R}}=\frac{1}{1+1/j\omega CR}=\frac{1}{1-j\left(\frac{\omega_C}{\omega}\right)}$

At $\omega=\omega_C=\frac{1}{RC}\Rightarrow\frac{v_0}{v_{in}}==\frac{1}{1-j}=\frac{1}{\sqrt{2}e^{-j45^0}}=\frac{1}{\sqrt{2}}e^{j45^0}\Rightarrow\left|\frac{v_0}{v_{in}}\right|=\frac{1}{\sqrt{2}}\Rightarrow\alpha=1.41$

Ans. 8: (c)

Solution: Nuclei of a radioactive element A are being produced at a constant rate α .

Decay constant of element $=\lambda$

At $t=0$, nuclei of element present N_0

(a) Number N of nuclei of A at time t:

Net rate of formation of nuclei of element $A=\frac{dN}{dt}$

$$\therefore\frac{dN}{dt}=\alpha-\lambda N \qquad \text{or} \qquad \frac{dN}{\alpha-\lambda N}=dt$$

$$\text{or} \int_{N_0}^{N}\frac{dN}{\alpha-\lambda N}=\int_0^t dt \qquad \text{or} \ -\frac{1}{\lambda}\left[\ln\left(\alpha-\lambda N\right)\right]_{N_0}^{N}=t$$

$$\ln\left(\frac{\alpha-\lambda N}{\alpha-\lambda N_0}\right)=-\lambda t \qquad \text{or} \ \frac{\alpha-\lambda N}{\alpha-N_0\lambda}=e^{-\lambda t}$$

$$\text{or} \ \alpha-\lambda N=e^{-\lambda t}\left(\alpha-\lambda N_0\right) \qquad \text{or} \ N=\frac{1}{\lambda}\left[\alpha-\left(\alpha-N_0\lambda\right)e^{-\lambda t}\right]$$

Ans. 9: 2

Solution: $W=\int_{\infty}^{d}\vec{F}.d\vec{l}=\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{d}\frac{q^2}{4z^2}dz=\frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{4z}\right)\Bigg|_{\infty}^{d}=-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d}=-\frac{1}{16\pi\varepsilon_0}\frac{q^2}{d}=-2\times\frac{q^2}{32\pi\varepsilon_0 d}$

Ans. 10: (c)

Solution: Energy is conserved.

Loss in kinetic energy = Gain in potential energy

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_{\min}}=5\times\left(1.6\times10^{-13}\right)J\Rightarrow r_{\min}=\frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{5\times1.6\times10^{-13}}$$

$$\text{or} \ r_{\min}=\frac{\left(9\times10^9\right)(2)(92)\left(1.6\times10^{-19}\right)^2}{5\times1.6\times10^{-13}}\Rightarrow r_{\min}=5.3\times10^{-14}m\Rightarrow r_{\min}=5.3\times10^{-12}cm$$

The distance of closest approach is of the order of $10^{-12}\,cm$

Ans. 11: 2

Solution: $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{B_1^{\parallel}/B_1^{\perp}}{B_2^{\parallel}/B_2^{\perp}} = \dfrac{B_1^{\parallel}}{B_2^{\parallel}} = \dfrac{\mu_1}{\mu_2} = \dfrac{2\mu_0}{4\mu_0} = \dfrac{1}{2}$ $\qquad \because B_1^{\perp} = B_2^{\perp},\ B_1^{\parallel}/\mu_1 = B_1^{\parallel}/\mu_2$

$$\Rightarrow \frac{\cot\theta_1}{\cot\theta_2} = 2$$

Ans. 12: 1.44

Solution: Q-value of nuclear reaction is

$$Q = \left[(4.0039 + 14.0075) - (17.0045 + 1.0081)\right] \times 931.5 = -1.12\ MeV$$

$$\mathrm{E}_{\text{threshold}} = -Q\left(1 + \frac{m_{He}}{m_N}\right) = 1.12\left(1 + \frac{4}{14}\right) = 1.44\ MeV\ .$$

Ans. 13: (b)

Solution: $F(s) = \ln\left(s^2 + w^2\right) - 2\ln s$

$$F'(s) = \frac{2s}{\left(s^2 + w^2\right)} - \frac{2}{s} \qquad \text{(I)}$$

Now Laplace transform of $tf(t)$ is $L\{tf(t)\} = -F'(s)$

$$\Rightarrow L^{-1}\{F'(s)\} = -tf(t) \qquad \text{(II)}$$

From equations (i) and (ii) we obtain $L^{-1}\left\{\dfrac{2s}{\left(s^2 + w^2\right)} - \dfrac{2}{5}\right\} = -tf(t)$

$$\Rightarrow 2\cos wt - 2 = -tf(t)$$

$$\Rightarrow f(t) = \frac{2}{t}(1 - \cos wt)$$

Ans. 14: 1.33

Solution: From the parseval's theorem $|c_n|^2 =$ square of average value of $f(x)$

$$\Rightarrow 1 + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2} = \frac{1}{2}\int_0^2 x^2 dx$$

$$\Rightarrow 1+\sum_{n=1}^{\infty}\frac{2}{n^2\pi^2}=\frac{4}{3}=1.33$$

Ans. 15: (c)

Solution: Spin $s=\frac{1}{2}$ means particles are fermions and it will obey Pauli Exclusion Principle.

Degeneracy $g=2s+1 \Rightarrow g=2$ means in every state maximum 2 identical particle can be adjusted. If we have three fermions, then in ground state two fermions will adjust and in next higher level one fermion will adjusted. Thus the energy of the lowest energy state of the system is $2\times\frac{\pi^2\hbar^2}{8ma^2}+\frac{4\pi^2\hbar^2}{8ma^2}=\frac{6\pi^2\hbar^2}{8ma^2}=\frac{3\pi^2\hbar^2}{4ma^2}$

Ans. 16: 8

Solution: Dimension of Hilbert space $(2s_1+1)\otimes(2s_2+1)=2\times 4=8$

Ans. 17: (c)

Solution: $U=kT=\frac{1}{\beta}$

Ans. 18: 4

Solution: $$\frac{I_{\max}}{I_{\min}}=\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}=\frac{\left(\sqrt{9I_0}+\sqrt{I_0}\right)^2}{\left(\sqrt{9I_0}-\sqrt{I_0}\right)^2}=\frac{\left(3\sqrt{I_0}+\sqrt{I_0}\right)^2}{\left(3\sqrt{I_0}-\sqrt{I_0}\right)^2}=\frac{16I_0}{4I_0}=4$$

Ans. 19: 2

Solution: For A, number of microstate after mixing is 2

A	B

For A, number of microstate before mixing is 1

$\Rightarrow \Delta S_A=R\ln 2-R\ln 1=R\ln 2$

Similarly, for B $\Rightarrow \Delta S_B=R\ln 2 \Rightarrow \Delta S=\Delta S_A+\Delta S_B=2R\ln 2$

Ans. 20: (d)

Solution: $\frac{dE}{dk}=\frac{\hbar^2}{2m}(6-2k)=\frac{\hbar^2}{m}(3k)$

And, $\frac{d^2E}{dk^2}=\frac{\hbar^2}{m}\cdot 3$

$\therefore m^*=\frac{\hbar^2}{d^2E/dk^2}=\frac{\hbar^2}{3\hbar^2/m}=\frac{m}{3}$

$$\Rightarrow \frac{m^*}{m} = \frac{1}{3}.$$

Ans. 21: (a)

Solution: $2B = \frac{1}{4.4} cm^{-1} \Rightarrow B = \frac{1}{8.8} cm^{-1} = 0.114\, cm^{-1} = 11.4\, m^{-1}$

where $B = \frac{h}{8\pi^2 \mu r^2} = \frac{\hbar}{4\pi B \mu C}$

$$\mu = \frac{200 \times 35}{200 + 35} \times 1.67 \times 10^{-27} kg \cong 50 \times 10^{-27} kg$$

$$\therefore r^2 = \frac{1.05 \times 10^{-34}}{4\pi \times 11.4 \times 50 \times 10^{-27} \times 3 \times 10^8} \cong 5 \times 10^{-20} m^2$$

$$r = 2.2 \overset{0}{\text{A}}$$

Ans. 22: 0.31

Solution: Hall resistance $e = BR_H = \frac{B}{ne}$

Where Fermi energy is $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$

$n \;\alpha\; E_F^{3/2}$

and, $e \,\alpha\, \frac{1}{E_F^{\;3/2}}$

$$\therefore \frac{e_A}{e_B} = \left(\frac{E_{FB}}{E_{FA}}\right)^{3/2} = \left(\frac{3\cdot 2}{7\cdot 05}\right)^{3/2} = 0\cdot 31$$

Ans. 23: (d)

Solution: (a) Forbidden as it violates the condition: $\Delta S = 0$

(b) Forbidden as the total angular momentum J changes from 0 to 0 which is forbidden for electric-dipole transition

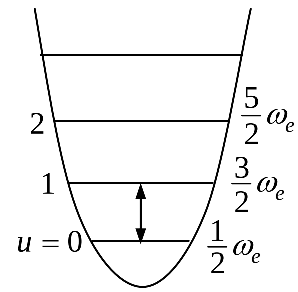
(c) Forbidden as it violates the condition $\Delta J = 0, \pm 1$

(d) Allowed electric dipole transition

Ans. 24: (a)

Solution: The energy gap between $v = 0$ and $v = 1$ is

$\Delta E = hc \times \omega_e$



Given, $\Delta E = 0.063 eV$ $\quad \therefore \omega_e = \frac{\Delta E}{hc}$

The force constant is

$$K = 4\pi^2 c^2 \mu \omega_e^2 = 4\pi^2 c^2 \mu \times \frac{(\Delta E)^2}{(hc)^2}$$

$$\therefore K = \mu \times \frac{(\Delta E)^2}{\hbar^2}$$

Now, $\mu = \frac{22 \times 35}{22 + 35} = 13.5\, amu = 13.5 \times 1.67 \times 10^{-27}\, kg$

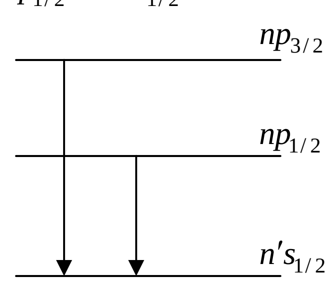
$$\mu = 2.3 \times 10^{-26} kg \cong 2 \times 10^{-26} kg$$

and $\frac{\Delta E}{\hbar} = \frac{0.063 eV}{6.58 \times 10^{-16} eV \cdot s} = \frac{6.3 \times 10^{-2} eV}{6.6 \times 10^{-16} eV \cdot s} \cong 10^{14} s^{-1}$

$$\therefore K = \left(2 \times 10^{-26} kg\right)\left(10^{14} s^{-1}\right)^2 = 2 \times 10^2 kg/s^2 = 200\, N/m$$

Ans. 25: (c)

Solution: Let probability for $np_{3/2} \to n's_{1/2}$ is denoted by p_1 and for $np_{1/2} \to n's_{1/2}$ is denoted by p_2



$$\therefore \frac{P_1}{P_2} = \frac{\text{degeneracy of state } p_{3/2}}{\text{degeneracy of state } p_{1/2}}$$

$= \frac{2J_1 + 1}{2J_2 + 1} = \frac{2 \times \frac{3}{2} + 1}{2 \times \frac{1}{2} + 1} = \frac{4}{2} = \frac{2}{1}$. Thus correct option is (c)

Ans. 26: (a)

Solution: $L = \frac{1}{2} m\dot{x}^2 - m\dot{x}\dot{y} \Rightarrow \frac{\partial L}{\partial \dot{x}} = m\dot{x} - m\dot{y} = p_x$ (i)

$\Rightarrow \quad \frac{\partial L}{\partial \dot{y}} = -m\dot{x} = p_y$ or $\dot{x} = -\frac{p_y}{m}$ (ii)

put $\dot{x} = -\frac{p_y}{m}$ in equation (i) $\Rightarrow -p_y + m\dot{y} = p_x \Rightarrow \dot{y} = \frac{p_x + p_y}{m}$

$$H = p_x\dot{x} + p_y\dot{y} - L = p_x\dot{x} + p_y\dot{y} - \frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{y}$$

put value of $\dot{x}$ and $\dot{y} \Rightarrow H = -\frac{p_x p_y}{m} - \frac{p_y^2}{2m}$

Ans. 27: $\alpha = 1$

Solution: $V(x) = \frac{1}{4}(x^2 - 2)^2 \Rightarrow \frac{\partial V}{\partial x} = \frac{2}{4}(x^2 - 2) \times 2x = 0 \Rightarrow x = 0, x = \pm\sqrt{2}$.

$\frac{\partial^2 V}{\partial x^2} = 3x^2 - 2$. At $x = 0$, $\frac{\partial^2 V}{\partial x^2} < 0$ so V is maximum. Thus it is unstable point

$\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=\pm\sqrt{2}} = 4$ and it is stable equilibrium point with $\omega = \sqrt{\frac{\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=x_0}}{\mu}} = 2$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi.$$

Ans. 28: (a)

Solution: Let $\vec{E}(z,t) = E_0 e^{-\kappa z}\cos(kz - \omega t + \delta_E)\hat{x}$

$$\because \left|\sigma\vec{E}(z,t)\right| = \left|\varepsilon\frac{d\vec{E}(z,t)}{dt}\right| \Rightarrow \sigma = \varepsilon\omega = \varepsilon_0\varepsilon_r \times 2\pi f \Rightarrow f = \frac{\sigma}{2\pi\varepsilon_0\varepsilon_r} = \frac{2\sigma}{4\pi\varepsilon_0\varepsilon_r}$$

$$\Rightarrow f = 2 \times \frac{10^{-2}}{4} \times 9 \times 10^9 = 45\ MHz \qquad \because \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$$

Ans. 29: (d)

Solution: R_E is "shorted out" by C_E for the ac analysis. Therefore

$$Z_i = R_B \parallel \beta r_e = 470k\Omega \parallel (120)6\Omega \approx 717\Omega, \quad Z_o = R_C = 2.2\,k\Omega$$

$$A_v = -\frac{R_C}{r_e} = -\frac{2.2\,k\Omega}{5.99\,\Omega} \approx -367$$

$$A_i = -A_v\frac{Z_i}{R_L} = -(-367)\frac{717\,\Omega}{2.2\,k\Omega} \approx 120$$

Ans. 30: 1.414

Solution: $\dfrac{2000\times3600\times v}{\sqrt{1-\dfrac{v^2}{c^2}}}=2.16\times10^{12}\times1000$

$$\frac{10^5\times72\times v}{\sqrt{1-\dfrac{v^2}{c^2}}}=2160\times10^{12}\Rightarrow\frac{v}{\sqrt{1-\dfrac{v^2}{c^2}}}=c\Rightarrow v=\frac{c}{\sqrt{2}}$$

Ans. 31: (b)

Solution: $D_A=(xy)\oplus A$

Input x y	Present State A	Flip-Flop Input D_A	Next State A
0 0	0	0	0
0 0	1	1	1
0 1	0	0	0
0 1	1	1	1
1 0	0	0	0
1 0	1	1	1
1 1	0	1	1
1 1	1	0	0

Ans. 32: 0.56

Solution: $x_{cm}=\dfrac{\int_0^L xdm}{\int_0^L dm}=\dfrac{\int_0^L x\rho dx}{\int_0^L \rho dx}=\dfrac{\int_0^L x\rho_0\left(1+\dfrac{x^2}{L^2}\right)dx}{\int_0^L \rho_0\left(1+\dfrac{x^2}{L^2}\right)dx}=\dfrac{\left.\dfrac{x^2}{2}+\dfrac{x^4}{4L^2}\right|_0^L}{\int_0^L\left(1+\dfrac{x^2}{L^2}\right)dx}=\dfrac{\dfrac{L^2}{2}+\dfrac{L^4}{4L^2}}{L+\dfrac{L^3}{3L^2}}=\dfrac{L^2\left(\dfrac{1}{2}+\dfrac{1}{4}\right)}{L\left(1+\dfrac{1}{3}\right)}$

$$\Rightarrow x_{cm}=\frac{\frac{3}{4}L}{\frac{4}{3}}=\frac{9}{16}L$$

Ans. 33: (b)

Solution: $B(A,Z)\approx16A-20A^{2/3}-\dfrac{3}{4}Z^2A^{-1/3}+30\dfrac{(A-2Z)^2}{A}$

$$\frac{dB}{dZ}=0\Rightarrow -\frac{3}{2}ZA^{-1/3}-120\frac{(A-2Z)Z}{A}=0$$

$$\Rightarrow 120\frac{(A-2Z)}{A}=-\frac{3}{2}ZA^{-1/3}\Rightarrow \frac{240Z}{A}-\frac{3}{2}ZA^{-1/3}=120$$

$$\Rightarrow \frac{240Z}{A}\left(1-\frac{3}{480}\frac{1}{A^{-2/3}}\right)=120\Rightarrow Z\left(1-\frac{1}{160A^{-2/3}}\right)=\frac{A}{2}$$

For most stable nuclei $\left.\frac{dB}{dZ}\right|_{Z=Z'}=0\Rightarrow Z'=\frac{A}{2}\left(1-\frac{A^{2/3}}{160}\right)^{-1}=\frac{27}{2}\left(1-\frac{27^{2/3}}{160}\right)^{-1}$

$$\Rightarrow Z'=13.5\left(1-\frac{9}{160}\right)^{-1}=13.5\times\frac{160}{151}=14.3\approx 14$$

Ans. 34: 5000

Solution: $R_{L_{max}}=\frac{15}{I_{L_{min}}}$ where $I_{L_{min}}=I_R-I_{ZM}=\frac{50-15}{1}-32=3\ mA$

$$\Rightarrow R_{L_{max}}=\frac{15}{3}=5k\Omega=5000\ \Omega.$$

Ans. 35: 1.44

Solution: $E=\frac{q^2B^2R^2}{2m_p}\Rightarrow 1.6\times10^{-13}=\frac{\left(1.6\times10^{-19}\right)^2B^2(0.1)^2}{2\left(1.67\times10^{-27}\right)}\Rightarrow B^2=\frac{1.6\times10^{-13}\times2\left(1.67\times10^{-27}\right)}{\left(1.6\times10^{-19}\right)^2(0.1)^2}$

$$\Rightarrow B^2=\frac{10^{-13}\times2\left(1.67\times10^{-27}\right)}{\left(1.6\times10^{-38}\right)(0.01)}=\frac{3.34\times10^{-40}}{1.6\times10^{-40}}=2.08\Rightarrow B=\sqrt{2.08}\,Tesla=1.44\,Tesla$$

Ans. 36: 27.4

Solution: $2\,^1\mathrm{H}+3\mathrm{n}\rightarrow\,^5_2\,\mathrm{He}$

$$B.E.=\left[2m\left(^1\mathrm{H}\right)+3m(n)-m\left(^5_2\mathrm{He}\right)\right]\times931.5\ MeV$$

$$\Rightarrow B.E.=\left[2\times1.007825+3\times1.008665-5.01220\right]\times931.5\ MeV$$

$$\Rightarrow B.E.=\left[0.02944\right]\times931.5\ MeV=27.4\ MeV$$

Ans. 37: 5.82

Solution: $R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} = \frac{\left(\frac{n_1}{n_2} - 1\right)^2}{\left(\frac{n_1}{n_2} + 1\right)^2} = \frac{(x-1)^2}{(x+1)^2}$ where $x = \frac{n_1}{n_2}$.

and $T = 1 - R = 1 - \frac{(x-1)^2}{(x+1)^2} = \frac{4x}{(x+1)^2}$

$\because R = T \Rightarrow \frac{(x-1)^2}{(x+1)^2} = \frac{4x}{(x+1)^2} \Rightarrow (x-1)^2 = 4x \Rightarrow x^2 - 6x + 1 = 0$

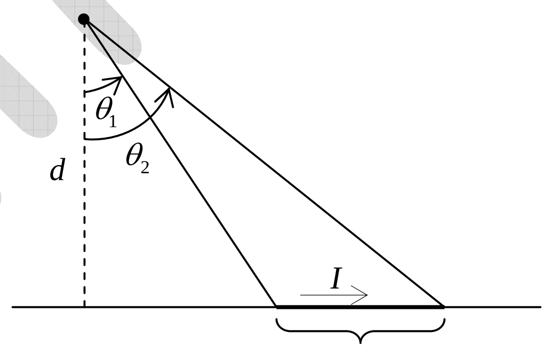
$\Rightarrow x = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 + 2\sqrt{2} = 5.82$

Ans. 38: (d)

Solution: $\oint_C \vec{A} \cdot d\vec{l} = \int_S \left(\vec{\nabla} \times \vec{A}\right) d\vec{a} = 0$ since $\vec{\nabla} \times \vec{A} = 0$.

Ans. 39: 3.4

Solution: $\because B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)$



Due to each wire field is pointing inwards.

$B_1 = B_2 = \frac{4\pi \times 10^{-7} \times 1}{4\pi \times 10^{-2}}\left(\sin 90 - \sin(-45)\right)$

$\Rightarrow B_1 = B_2 = \frac{4\pi \times 10^{-7} \times 1}{4\pi \times 10^{-2}}\left(1 + 1/\sqrt{2}\right) = 1.7 \times 10^{-5} T$

$\Rightarrow B = 2B_1 = 3.4 \times 10^{-5} T$

Ans. 40: (c)

Solution: since A is a triangular matrix, hence its eigenvalues are $1, 3$ and 2. For $\lambda = 1$, we obtain, eigenvalues of the form $X_1 = \begin{bmatrix} K \\ 0 \\ 0 \end{bmatrix}$. Taking $K = 1$, we obtain $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

For $\lambda = 2$, we obtain eigenvalues of the form $\begin{bmatrix} K \\ -2K \\ K \end{bmatrix}$. Taking $K = 1$, we obtain $X_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$. For $\lambda = 3$, we obtain eigenvectors of the form $X_3 = \begin{bmatrix} 0 \\ K \\ 0 \end{bmatrix}$. Taking $K = 1$, we obtain $X_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Thus the similarity transform of A to B can be performed by the matrix $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

Ans. 41: (b)

Solution: $H' = bx^2$ put $x = r\sin\theta\cos\phi$

$$E_1^1 = \langle \psi_1 | H' | \psi_1 \rangle, |\psi_1\rangle = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} = \int \psi_1^* H' \psi_1 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\frac{b}{\pi a_0^3} \int_0^\infty r^2 e^{-\frac{2r}{a_0}} r^2 dr \int_0^\pi \sin^3\theta d\theta \int_0^{2\pi} \cos^2\phi d\phi = ba_0^2$$

Ans. 42: 13

Solution: The solution of corresponding homogenous differential equation is

$$y_h = c_1 \cos 3x + c_2 \sin 3x$$

we have $2\sinh x \cosh x = \sinh 2x$

$\Rightarrow 4\sinh x \cosh x = 2\sinh 2x$

Now $2\sinh 2x = 2\left(\frac{e^{2x} - e^{-2x}}{2}\right) = e^{2x} - e^{-2x}$

Thus the given differential equation can be written as

$$\frac{d^2y}{dx^2} + 9y = e^{2x} - e^{-2x} + e^{-2x} = e^{2x}$$

Thus, the particular solution is

$$y_p = \frac{1}{D^2+9}\left(e^{2x}\right) = \frac{1}{13}$$

Thus, the general solution of the given differential equation is

$$y = y_h + y_p = c_1\cos 3x + c_2 \sin 3x + \frac{1}{13}$$

Given, $y(0)=0$ and $y'(0)=1$

Hence $c_1 + \frac{1}{13} = 0 \Rightarrow c_1 = -\frac{1}{13}$

Thus $y = -\frac{1}{13}\cos 3x + c_2\sin 3x + \frac{1}{13} \Rightarrow y' = \frac{3}{13}\sin 3x + 3c_2\cos 3x$

Thus $y = -\frac{1}{13}\cos 3x + \frac{1}{3}\sin 3x + \frac{1}{13}$

$$\Rightarrow y(\pi) = -\frac{1}{13}(-1) + \frac{1}{13} = \frac{2}{13}$$

Ans. 43: (c)

Solution: $\vec{s} = \vec{s}_1 + \vec{s}_2$, $s_1 = \frac{1}{2}, s_1 = \frac{1}{2}, s = 0,1,$ $\langle \vec{s}_1.\vec{s}_2 \rangle = \frac{s(s+1)\hbar^2 - s_1(s_1+1)\hbar^2 - s_2(s_2+1)\hbar^2}{2}$

For $s=1, \langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{2\hbar^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2} = \frac{\hbar^2}{4}$

$$s=0, \langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{0\hbar^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2} = -\frac{3}{4}\hbar^2$$

Ans. 44: (b)

Solution: $\frac{\frac{1}{\exp\frac{4h\nu}{kT}-1}}{\frac{1}{\exp\frac{2h\nu}{kT}-1}} \Rightarrow \frac{\exp\frac{2h\nu}{kT}-1}{\exp\frac{4h\nu}{kT}-1} = \frac{\exp\frac{2h\nu}{kT}-1}{\left(\exp\frac{2h\nu}{kT}-1\right)\left(\exp\frac{2h\nu}{kT}+1\right)} = \frac{1}{\left(\exp\frac{2h\nu}{kT}+1\right)}$

$$= \left[e^{2h\nu/k_BT}+1\right]^{-1}$$

Ans. 45: 0

$$\oint_c \frac{z+5}{z^3-z}dz = \oint_c \frac{z+5}{z\left(z^2-1\right)}dz$$

$$z=0, z^2-1=0 \Rightarrow z^2=1 \Rightarrow z=\pm 1$$

$\therefore z=0,\pm 1$ lie inside contour

$$\text{Res}\left(z=0\right)=\lim_{z\to 0}\frac{\left(z-0\right)\left(z+5\right)}{z\left(z-1\right)\left(z+1\right)}$$

$$=\lim_{z\to 0}\frac{z\left(z+5\right)}{z\left(z-1\right)\left(z+1\right)}$$

$$=\frac{0+5}{\left(0-1\right)\left(0+1\right)}=\frac{5}{-1}=-5$$

$$\text{Res}\left(z=1\right)=\lim_{z\to 1}\frac{\left(z-1\right)\left(z+5\right)}{z\left(z-1\right)\left(z+1\right)}$$

$$=\frac{1+5}{1\times\left(1+1\right)}=\frac{6}{2}=3$$

$$\text{Res}\left(z=-1\right)=\lim_{z\to -1}\frac{\left[z-\left(-1\right)\right]\left(z+5\right)}{z\left(z-1\right)\left(z+1\right)}$$

$$=\lim_{2\to -1}\frac{\left(z+1\right)\left(z+5\right)}{z\left(z-1\right)\left(z+1\right)}$$

$$=\frac{-1+5}{\left(-1\right)\left(-1-1\right)}=\frac{4}{\left(-1\right)\left(-2\right)}=\frac{4}{2}=2$$

$$\therefore \oint_c \frac{z+5}{z\left(z^2-1\right)}dz = 2\pi i\left(-5+3+2\right)=0$$

$$\therefore \int_{-\infty}^{+\infty}\frac{x+5}{x^3-x}dx=0$$

Ans. 46: 9

Solution: The given state is representation of first exited state whose energy is $4eV$.

If E_n is energy of n^th state and E_0 is energy of ground state then $E_n=n^2E_0$.

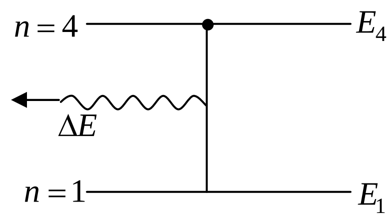
So $E_2=4E_0=4eV \Rightarrow E_0=1eV$

For second excited state $n=3$. So $E_3=9eV$

Ans. 47: (c)

Ans. 48: (d)

Solution: $E_n = -\frac{13.6}{n^2}[eV]$



For $n=1$: $E_1 = -13.6\ eV$

For $n=4$: $E_4 = -0.85\ eV$

where $\Delta E = E_1 - E_4 = 12.75\ eV$

The momentum of the emitted photon is $p = \frac{E}{C} = 12.75\ eV/C$

According to momentum conservation, the momentum of the recoil atom is same as of photon but with opposite direction. Thus kinetic energy of the recoil atom is

$$E_R = \frac{p^2}{2m_H}$$

where $m_H = 1.008\,amu = 1.008 \times 931.4941\,MeV/C^2 = 938.94\,MeV/C^2$

$$\therefore E_R = \frac{(12.75\,eV/C)^2}{938.94\times10^6\,eV/C^2} = 1.73\times10^{-7}\ eV$$

Ans. 49: 1

Solution: $\sigma = \frac{4\pi}{k^2}\sum_{l=0}(2l+1)\sin\delta_0$ for $l=0$, it is given $\delta_0 = 90^0$ and $k = 2\sqrt{\pi}\quad fm^{-1}$

$$\sigma = \frac{4\pi}{4\pi}\sin 90 = 1$$

Ans. 50: $0\cdot5$

Solution: Since $P = -2T + P_0 \Rightarrow \frac{dP}{dT} = -2$

It is given $L = T\left(\frac{dP}{dT}\right)\Delta v \Rightarrow L = -2T\Delta v \quad \Rightarrow \frac{dL}{dT} = -2\Delta v$

Since $dS = 1.0$ Jmole^{-1} K^{-1} $1.0\ J\ mole^{-1}\ K^{-1}$

$$dS = \frac{dQ}{T} = \frac{mdL}{dT} = 1 \Rightarrow 1 = -2\Delta v \Rightarrow \Delta v = -\frac{1}{2} = 0.5$$

Ans. 51: (c)

$$Z = N - 3 + \exp\left(-\frac{2\varepsilon}{kT}\right)$$

$$F = -kT\ln Z = -kT\ln\left[N - 3 + \exp\left(\frac{-2\varepsilon}{kT}\right)\right]$$

Ans. 52: 4.24

Solution: The number density

$$n = \frac{\rho}{M} = \frac{0\cdot081\,g/cm^3}{5\cdot03\times10^{-24}\,g} = 1\cdot617\times10^{22}\,cm^{-3}$$

$$= 1\cdot617\times10^{28}\,m^{-3}$$

$\therefore$ Fermi energy $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$

$$= \frac{\left(1\cdot05\times10^{-34}\,J\cdot s\right)^2}{2\times5\cdot03\times10^{-27}}\left(3\pi^2\times1\cdot62\times10^{28}\right)^{2/3}$$

$$= 6\cdot78\times10^{-23}\,J = \frac{6\cdot78\times10^{-23}}{1\cdot6\times10^{-19}}(eV)$$

$\therefore\ 4\cdot24\times10^{-4}\,eV$

Ans. 53: 1.33

Solution: $N = 2,\ g = 2,\ n = 2$

For Maxwell $W = \frac{\lfloor N}{\lfloor n} g^n = \frac{\lfloor 2}{\lfloor 2} 2^2 = 4$

For boson $N = 2\ \ g = 2\ \ n = 2$; $\quad W = \frac{\lfloor n+g-1}{\lfloor n \lfloor g-1} = \frac{\lfloor 2+2-1}{\lfloor 2 \lfloor 2-1} = 3$

$$N_{MB} : N_{BE} = \frac{4}{3}$$

Ans. 54: (b)

Solution: $2d\sin\theta = \lambda \Rightarrow a = \frac{\lambda\sqrt{\hbar^2 + k^2 + l^2}}{2\sin\theta}$

$$a = \frac{1\overset{0}{A}\times\sqrt{3^2 + 1^2 + 1^2}}{2\sin 30^0} = \sqrt{11}\,.$$

Ans. 55: (b)

Solution: For $J=0 \to J=1$ the wavenumber of emitted photon is $\bar{\nu}=2B(J+1)=2B$ where

$$B=\frac{h}{8\pi^2 4r^2 c}$$

For ${}^{12}C^{16}O$: $\quad \bar{\nu}_1 = 2B_1 \quad$ (i)

For ${}^{?}C^{16}O$: $\quad \bar{\nu}_2 = 2B_2 \quad$ (ii)

Taking ratio of (i) and (ii)

$$\frac{B_1}{B_2}=\frac{\bar{\nu}_1}{\bar{\nu}_2}=\frac{\nu_1/c}{\nu_2/c}=\frac{\nu_1}{\nu_2}$$

where $\frac{B_1}{B_2}=\frac{\mu_2}{\mu_1}$

$$\therefore \frac{\mu_2}{\mu_1}=\frac{\nu_1}{\nu_2} \Rightarrow \mu_2=\frac{\nu_1}{\nu_2}\times\mu_1 \quad \text{(iii)}$$

Now, $\frac{\nu_1}{\nu_2}=\frac{11.53\times10^{10}\,Hz}{11.02\times10^{10}\,Hz}=1.046$

while $\mu_1=\frac{m_{12_C}\times m_{16_O}}{m_{12_C}+m_{16_O}}=\frac{12\times16}{12+16}amu$ and $\mu_2=\frac{m_{x_C}\times m_{16_O}}{m_{x_C}+m_{16_O}}=\frac{x\times16}{x+16}amu$

$\therefore$ From equation (iii), we get

$$\frac{x\times16}{x+16}=1.046\times\frac{12\times16}{12+16}\Rightarrow\frac{x}{x+16}=1.046\times\frac{12}{28}=1.046\times\frac{3}{7}=\frac{3.138}{7}$$

$$\Rightarrow 7x=3.14x+50.2\Rightarrow 3.86x=50.2\Rightarrow x=13.$$

Ans. 56: (a)

Solution: We should use correct preposition with right and it should be their right to fly.

Ans. 57: (a)

Solution: Bleaker means not hopeful or not encouraging. So, hopeful is the most opposite in meaning to it.

Ans. 58: (b)

Solution: The letter series are as follows:

cbaa cbaab cbaabc cbaabcd cbaa (*b*)

So, the required letter is ‘*b*’

Ans. 59: (b)

Solution: Radius, circumference and diameter are related to a circle.

Ans. 60: (b)

Solution: There are 30 people. A handshakes needs 2 people. This simply means in how many ways 2 people can be selected out of 30.

So, the answer is $30C_2$

So, $30C_2 = \dfrac{30!}{2!(30-2)!} = \dfrac{30\times 29}{2}$

435, number of hand shake.

Ans. 61: (b)

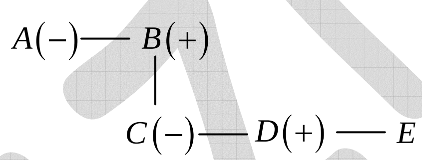
Solution: In choice, (a), generation gap is: $0+1+1+0=2$. So, A is E's grandma's generation.

In choice (b), gap is: $0+1+0+0=1$.

In choice, (c), gap is: $1+0+1+0=2$. So, A is E's grandma's generation.

So, choice, $1,3$ and 4 are eliminated.

Correct choice: (b)



In this figure, plus sign used for males and minus sign is used for females. The long bar indicates married couple.

Ans. 62: (d)

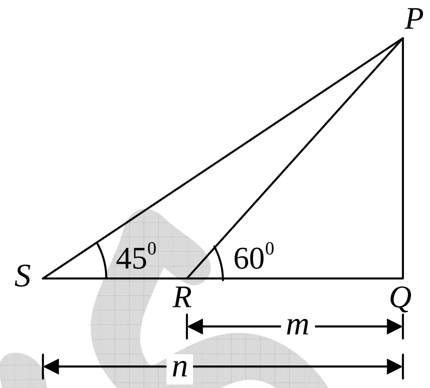
Solution: From the question:

Floor	Person
6^{th}	R
5^{th}	S
4^{th}	T
3^{rd}	Vacant
2^{nd}	P
1^{st}	Q

Ans. 63: (c)

Solution: $\log_3 x = 10$ or $x = 3^{10}$, again, $\log_x y = 100$ or $y = x^{100} = \left(3^{10}\right)^{100} = y = 3^{1000}$

Ans. 64: (a)



Solution: Height of pole $= PQ$, and $\tan 60^0 = \frac{PQ}{m}$

or, $\sqrt{3} = \frac{PQ}{m}$ and $\tan 45^0 = 1 = \frac{PQ}{n}$

Hence, $\sqrt{3} \times 1 = \frac{PQ}{m} \times \frac{PQ}{n}$

$\therefore PQ^2 = \sqrt{3}mn \quad \therefore PQ = \sqrt{mn\sqrt{3}}$ unit

Ans. 65: (c)

Solution: Let the principal be P and rate of interest be r %, then according to the question:

$$2P = P\left(1+\frac{r}{100}\right)^9 \text{ or } \left(1+\frac{r}{100}\right) = 2^{1/9}$$

Also, $4P = P\left(1+\frac{r}{100}\right)^n$ or $2^2 = 2^{n/9}$

or $\frac{n}{9} = 2 \quad \therefore \quad n = 18$ years.