ALL INDIA TEST SERIES

IIT - JAM - 2018

Full Length Test – 01

12-01-2018

PHYSICS (PH)

TIME: 3 HOURS **MAXIMUM MARKS: 100**

Section A: This section contains a total of 30 Multiple Choice Questions **(MCQ)** carrying one or two marks each. Each MCQ type question has four choices out of which only one choice is the correct answer.

There will be negative marking @ $\frac{1}{3}^{rd}$ for one marks MCQ and $\frac{2}{3}^{rd}$ negative marks for two marks MCQ for each wrong answer.

Section B: This section contains a total of 10 Multiple Select Questions **(MSQ)** carrying two marks each. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers.

Section C: This section contains a total of 20 Numerical Answer Type **(NAT)** questions carrying one or two marks each. For these NAT type questions, the answer is a signed real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these types of questions.

Note: There will be no negative marking for Section B and Section C.

SECTION A

Multiple Choice Questions (MCQ)

Q1 – Q10 Carry One Mark each. (1/3 negative marks for each wrong answer)

Q1. An object is $30\,cm$ from a spherical mirror, along the central axis. The absolute value of lateral magnification is $1/2$. The image produced is inverted. The total length of the mirror is

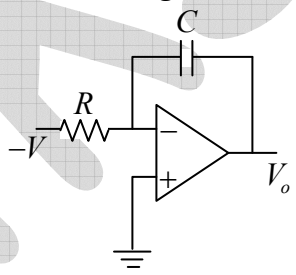
(a) $-10\,cm$ (b) $+10\,cm$ (c) $-15\,cm$ (d) $+30\,cm$

Q2. If the number density of a free electron gas in three dimension is increased 27 times. Its Fermi energy will

(a) increase by 3 times (b) decrease by 3 times

(c) increase by 9 times (d) decrease by 9 times

Q3. If a constant voltage $-V$ is applied to the input of the following OPAMP circuit for a time t, then the output voltage V_0 will approach



(a) $+V$ exponentially

(b) $-V$ exponentially

(c) $+V$ linearly

(d) $-V$ linearly

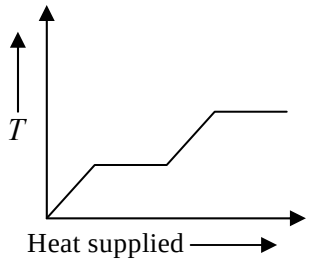
Q4. A block of density $\rho = 800\,kg/m^3$ floats with its face down in a fluid of density $\rho_f = 1200\ kg/m^3$. The block has height $H = 6.0\ cm$. By what depth h is the block submerged?

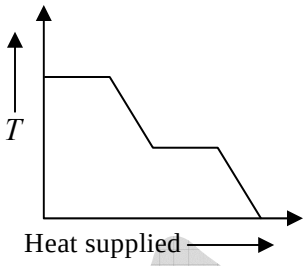
(a) $2.0\,cm$ (b) $3.0\,cm$ (c) $4.0\,cm$ (d) $5.0\,cm$

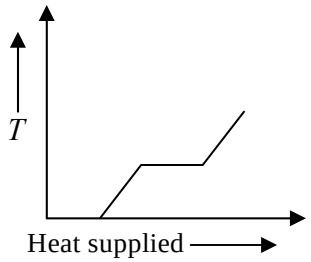
Q5. If $z_1 = e^{\frac{2\pi i}{5}} + e^{\frac{4\pi i}{5}} + e^{\frac{6\pi i}{5}} + e^{\frac{8\pi i}{5}}$ and $z_2 = 1 + \frac{i}{2} + \frac{i^2}{4} + \frac{i^3}{8} + \ldots\ldots$, then the value of $\frac{z_1}{z_2}$ is

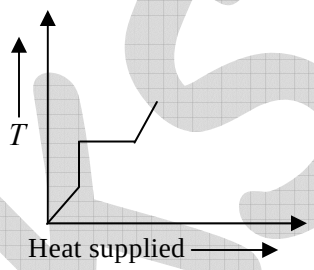
(a) $1+i$ (b) $-1-i$ (c) $1-\frac{i}{2}$ (d) $-1+\frac{i}{2}$

Q6. A block of ice at $-10^0 C$ is slowly heated and converted to steam at $100^0 C$. Which of the following curves represents the phenomenon qualitatively?

(a)


(b)


(c)


(d)


Q7. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy $6eV$ fall on it is $4eV$. The stopping potential in volt is

(a) 2 (b) 4 (c) 6 (d) 10

Q8. In the expansion of Fourier sine series for function $f(x) = e^x,\ 0 < x < \pi$, the value of b_3 is

(a) $\frac{(1-e^{\pi})}{\pi}$ (b) $\frac{3(1-e^{\pi})}{5\pi}$ (c) $\frac{(1+e^{\pi})}{\pi}$ (d) $\frac{3(1+e^{\pi})}{5\pi}$

Q9. A simple pendulum has a time period T_1 when taken on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2 / T_1 is

(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

Q10. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having nonzero velocities. The ratio of the de Broglie wavelengths of the particles, λ_1 / λ_2, is

(a) m_1 / m_2 (b) m_2 / m_1 (c) 1.0 (d) $\sqrt{m_2} / \sqrt{m_1}$

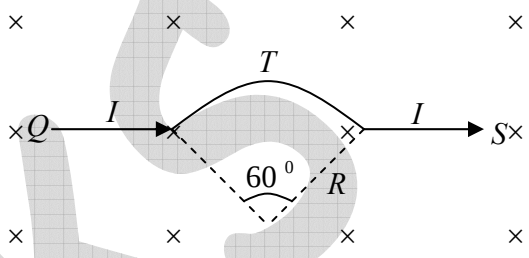
Q11 – Q30 Carry Two Marks each (2/3 negative marks for each wrong answer)

Q11. The quality factor of a sonometer wire of frequency $400\,Hz$ is 7000. The time in which its energy decays to $\frac{1}{e}$ of its initial value is

(a) $1.6\,\text{sec}$ (b) $2.8\,\text{sec}$ (c) $1.4\,\text{sec}$ (d) $4.2\,\text{sec}$

Q12. A circular arc QTS is kept in an external magnetic field $\vec{B}_0$ as shown in the figure. The arc carries a current I. The magnetic field is directed normal and into the page. The force acting on the arc is

($\hat{k}$ is a unit vector out of page)



(a) $2IB_0R\hat{k}$ (b) $IB_0R\hat{k}$

(c) $-2IB_0R\hat{k}$ (d) $-IB_0R\hat{k}$

Q13. For a radioactive material, its activity A and rate of change of its activity R are defined as $A=-\frac{dN}{dt}$ and $R=\frac{dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t=0$. Their rates of change of activities at $t=2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q}=\frac{n}{e}$, then the value of n is

(a) 2 (b) 3 (c) 4 (d) 5

Q14. Electron beam accelerated from rest through a potential difference of $80V$ will be diffracted from the (111) planes of an FCC crystal of lattice parameter $3.5\,\text{Å}$. The Bragg angle is

(a) $\sin^{-1}(0.34)$ (b) $\sin^{-1}(0.52)$ (c) $\sin^{-1}(0.61)$ (d) $\sin^{-1}(0.70)$

Q15. Let $y=y(t)$ be the solution to the differential equation $y''-8y'+16y=32t$ subjected to the initial conditions $y(0)=1$ and $y'(0)=2$, then the value of $y\left(\frac{1}{2}\right)$ is

(a) 0 (b) 2 (c) 5 (d) 12

Q16. A left circularly polarized light $\left(\lambda = 5893\,\overset{0}{\text{A}}\right)$ is incident normally on a calcite crystal (with its optic axis cut parallel to the surface) of thickness $0.00514\,mm$. What is the state of polarization of the emergent beam? ($\mu_0 = 1.65836$ and $\mu_e = 1.48641$)

(a) Right elliptically polarized

(b) Left elliptically polarized

(c) Right circularly polarized

(d) Left circularly polarized

Q17. A conducting sphere of radius R is placed in a uniform electric field $\vec{E}_0$ directed along $+z$ axis. The electric potential for outside points is given as $V_{out} = -E_0\left(1-\frac{R^3}{r^3}\right)r\cos\theta$, where r is the distance from the centre and θ is the polar angle. The charge density on the surface of the sphere is

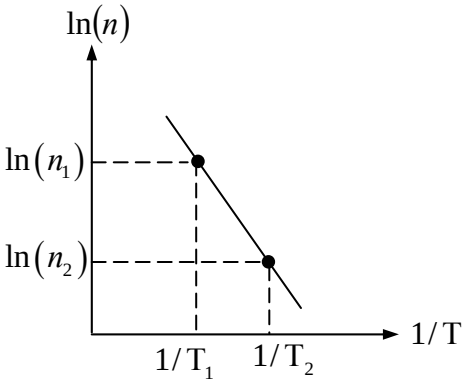
(a) $6\varepsilon_0 E_0\cos\theta$ (b) $\varepsilon_0 E_0\cos\theta$ (c) $3\varepsilon_0 E_0\cos\theta$ (d) $\frac{\varepsilon_0}{3}\mathrm{E}_0\cos\theta$

Q18. In an intrinsic semiconductor, the free carrier concentration n (in cm^{-3}) varies with temperature T (in Kelvin) as shown in the figure below.

If $n_1 = 3\times10^{14}\,m^{-3}$ and $n_2 = 1\times10^{8}\,m^{-3}$ and $1/T_1 = 2\times10^{-3}\,K^{-1}$ and $1/T_2 = 6\times10^{-3}\,K^{-1}$.

(Assume effective density of states and band gap energy to be independent of temperature).

Then the band gap energy of the semiconductor material:



(a) $1.3eV$ (b) $1.1eV$ (c) $0.65eV$ (d) $0.35eV$

Q19. If the quality factor of an under damped harmonic oscillator of frequency $512\,H$ be 8×10^4, then how many oscillations does the oscillator make in time, in which its energy is reduce to $1/e$ of its energy in absence of damping,

(a) 12840 (b) 12740 (c) 12640 (d) 12540

Q20. Let the function $\text{sgn}(x)$ is defined as:

$$\text{sgn}(x)=\begin{cases}1, & \text{if } x>0\\ -1, & \text{if } x<0\end{cases}$$

Then $\dfrac{d}{dx}\text{sgn}(x)$ is

(a) 0 (b) $\delta(x)$ (c) $2\delta(x)$ (d) $\delta'(x)$

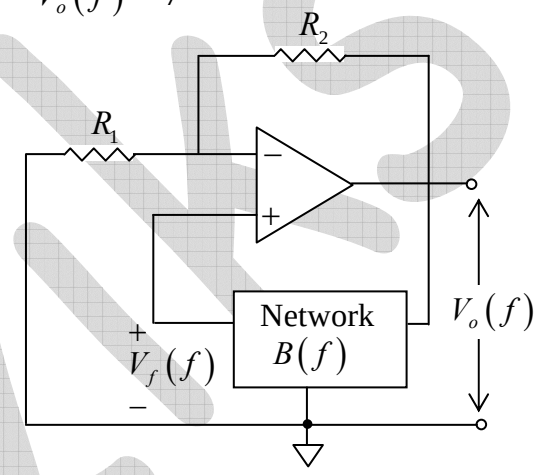
Q21. The circuit in figure employs positive feedback and is intended to generate sinusoidal oscillation. If at a frequency f_0, $B(f)=\dfrac{V_f(f)}{V_o(f)}=\dfrac{1}{7}$, then to sustain oscillation at this frequency

(a) $R_2=5R_1$

(b) $R_2=6R_1$

(c) $R_2=\dfrac{R_1}{6}$

(d) $R_2=\dfrac{R_1}{5}$



Q22. An orthorhombic crystal whose primitive translational are $a=1.21\,\text{Å}$, $b=1.84\,\text{Å}$ and $c=1.97\,\text{Å}$ respectively. If a plane with Miller indices $(23\bar{1})$ cuts an intercepts along z-axis is

(a) $-4.21\,\text{Å}$ (b) $-3.94\,\text{Å}$ (c) $-3.26\,\text{Å}$ (d) $-1.2\,\text{Å}$

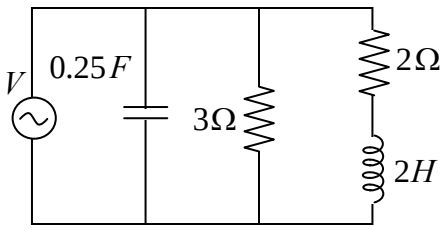
Q23. Find the resonance frequency of the circuit shown in the figure below

(a) 0.10 *Hz*

(b) 0.16 *Hz*

(c) 0.22 *Hz*

(d) 0.28 *Hz*



Q24. If Q is heat exchanged and α is volume expansivity then $\left(\frac{\partial Q}{\partial P}\right)_T$ is equivalent to

(a) 0 (b) $TV\alpha$ (c) $-TV\alpha$ (d) $\frac{TV}{\alpha}$

Q25. The double integral of the function $f(x,y)=xy(x+y)$ over the region bounded by the curves $y^2=x$ and $y=x$ is

(Answer must be upto two digits after the decimal places)

(a) 0.05 (b) 0.04 (c) 0.20 (d) 0.50

Q26. When an ideal diatomic gas is heated at constant pressure, the fraction of heat energy supplied which increases the internal energy of the gas is

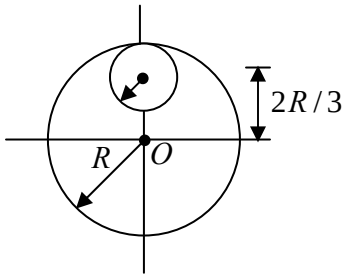
(a) $2/5$ (b) $3/5$ (c) $3/7$ (d) $5/7$

Q27. The wave function of a particle of mass m in deep square well potential of width a is $\psi(0)=\frac{1}{\sqrt{5}}(|\phi_1\rangle+2|\phi_2\rangle)$, where $|\phi_1\rangle$ and $|\phi_2\rangle$ are ground state and first excited state wave functions. What will be the wave function at time $t=\frac{2ma^2}{\pi\hbar}$?

(a) $\frac{1}{\sqrt{5}}[|\phi_1\rangle+2|\phi_2\rangle]$ (b) $\frac{1}{\sqrt{5}}[|\phi_1\rangle-2|\phi_2\rangle]$

(c) $\frac{1}{\sqrt{5}}[2|\phi_1\rangle+|\phi_2\rangle]$ (d) $\frac{1}{\sqrt{5}}[2|\phi_2\rangle-|\phi_1\rangle]$

Q28. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(a) $4MR^2$ (b) $\frac{40}{9}MR^2$ (c) $10MR^2$ (d) $\frac{37}{9}MR^2$

Q29. A body of rest mass m_0 moving at speed v collide and stick to an identical body at rest.

The rest mass of mass M of the final clump is (take $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$)

(a) $M = m_0\sqrt{2+\gamma}$ (b) $M = m_0\sqrt{2-\gamma}$

(c) $M = m_0\sqrt{2(1+\gamma)}$ (d) $M = m_0\sqrt{2(1-\gamma)}$

Q30. A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is.

(a) $\sqrt{\mu/\alpha}$ (b) $\mu/\sqrt{a}$ (c) $1/\sqrt{\mu\alpha}$ (d) infinitesimal.

SECTION B

Multiple Select Type Questions (MSQ)

Q31 – Q40 Carry Two Marks each (No negative marking for any wrong answer)

Q31. Two simple harmonic oscillations of right angle to each other with phase difference δ

$$x = a\sin(\omega t) \text{ and } y = b\sin(\omega t + \delta)$$

Superimpose, which of the following statements are correct for the resultant motion of the particle?

(a) At $\delta = \frac{\pi}{2}$ and $a = b$, the motion is right circularly polarized

(b) At $\delta = \pi$, the motion is linearly polarized.

(c) At $\delta = \frac{5\pi}{2}$, the motion is right elliptically polarized

(d) At $\delta = \frac{9\pi}{2}$, the motion is right circularly polarized.

Q32. Let $\tilde{E} = \hat{x}E_0 \exp\left[i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right]$, where $\vec{k} = \hat{z}\left(k\cos\phi + ik\sin\phi\right)$ and $\hat{x}$, $\hat{y}$ and $\hat{z}$ are Cartesian unit vectors, represent an electric field of a plane electromagnetic wave of frequency ω. Which one of the following statements is/are not true?

(a) The magnitude of the electric field is attenuated as the wave propagates

(b) The energy of the e.m. wave flows along the x - direction

(c) The magnitude of the electric field of the wave is a constant

(d) The speed of the wave is the same as c (speed of light in free space)

Q33. Which of the following statements are true?

(a) Octal equivalent of decimal number $(478)_{10}$ is $(736)_8$

(b) Octal equivalent of decimal number $(443)_{10}$ is $(673)_8$

(c) Octal equivalent of decimal number $(415)_{10}$ is $(637)_8$

(d) Octal equivalent of decimal number $(247)_{10}$ is $(366)_8$

Q34. A critical point for a function $f(x)$ is a point in the domain of $f(x)$ at which the first derivative is either zero or undefined. If $f(x) = \dfrac{|x-1|}{x^2}$, then which of the following statements is/are correct?

(a) $f(x)$ is not differential at $x = 2$.

(b) Function $f(x)$ has three critical points.

(c) $f(x)$ is not differential at $x = 1$.

(d) Function $f(x)$ has two critical points.

Q35. A dielectric sphere of radius R carries a polarization $\vec{P} = kr^2\hat{r}$, where k is a constant and r is the distance from the centre and k is a constant. In spherical polar coordinate system $\hat{r}$, $\hat{\theta}$ and ϕ are the unit vectors. Which one of the following statements is/are correct?

(a) The bound volume charge density inside the sphere is $-4kr$.

(b) The bound volume charge density inside the sphere is $-2kr$.

(c) The electric field inside the sphere at a distance d from the centre is $-\dfrac{kd^2}{\varepsilon_0}\hat{r}$.

(d) The electric field inside the sphere at a distance d from the centre is $-\dfrac{kR^2}{\varepsilon_0}\hat{r}$.

Q36. Let $1.00\ kg$ of liquid water at 100^0C be converted to steam at 100^0C by boiling at standard atmospheric pressure ($1\ atm$). The volume of that water changes from an initial value of $1\times10^{-3}m^3$ as a liquid to $1.671m^3$, such that pressure remains constant, then which of the following is/are correct?

(a) The work done by the system is $169kJ$

(b) There is not heat transferred to gas

(c) The change in internal energy of system is zero

(d) The change of internal energy of system is $2087kJ$.

Q37. A particle is in the ground state of an infinite square well potential is given by,

$$V(x)=\begin{cases}0, & \text{for } -a\le x\le a\\ \infty, & \text{otherwise}\end{cases}$$

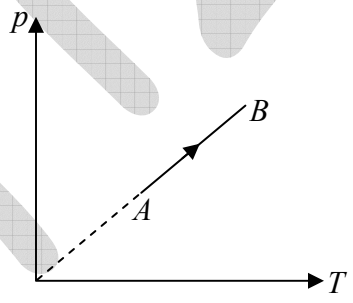
(a) The ground state wave function is $\sqrt{\frac{2}{a}}\cos\frac{\pi x}{a}$

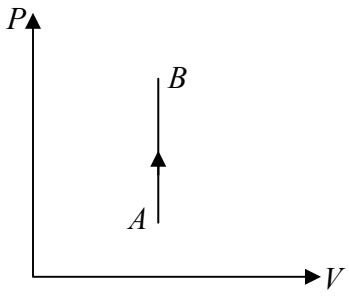
(b) The ground state wave function has $\sqrt{\frac{1}{a}}\cos\frac{\pi x}{2a}$

(c) The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is $\frac{1}{2}+\frac{1}{\pi}$

(d) The energy difference between ground state and first excited state is $\frac{3\pi^2\hbar^2}{8ma^2}$

Q38. During the process AB of an ideal gas as shown in figure below, which of the following options is/are correct?





(a) Work done on the gas is zero

(b) Density of the gas is constant

(c) Slope of line A from the T - axis is inversely proportional to the number of moles of the gas

(d) Slope of line AB from the T - axis is directly proportional to the number of moles of the gas

Q39. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A}$ and $\vec{L}$ where $\vec{A}$ is a constant vector, and $\vec{L}$ is the angular momentum of the body about that point. From this, it follows that

(a) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time

(b) the component of $\vec{L}$ in the direction A does not change with time

(c) the magnitude of $\vec{L}$ does not change with time

(d) $\vec{L}$ does not change with time

Q40. In Bohr's model of the hydrogen atom

(a) the radius of the n^{th} orbit is proportional to n^2

(b) the total energy of the electron in n^{th} orbit is inversely proportional to n.

(c) the angular momentum of electron in an n^{th} orbit is an integral multiple of $\frac{h}{2\pi}$.

(d) the magnitude of potential energy of the electron in any orbit is greater than its kinetic energy

SECTION C

Numerical Answer Type Questions (NAT)

Q41 – Q50 Carry One Mark each (No negative marking for any wrong answer).

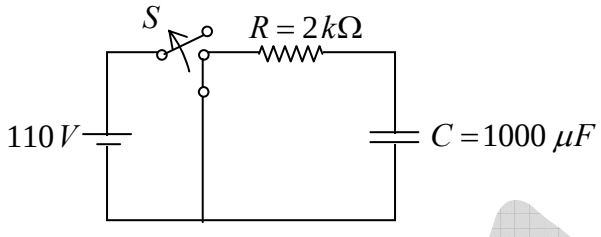
Q41. Number of slits in grating required to resolve the two closely spaced wavelengths $5800\,\overset{0}{\text{A}}$ and $5802\,\overset{0}{\text{A}}$ in the second order is……

Q42. An electromagnetic wave with $\vec{E}(z,t) = E_0 \cos(\omega t - kz)\hat{i}$ is travelling in free space and crosses a disc of radius $2m$ placed perpendicular to the z-axis. If $E_0 = 60\,V\,m^{-1}$, the average power, in Watt, crossing the disc along the z- direction is……$Watt$

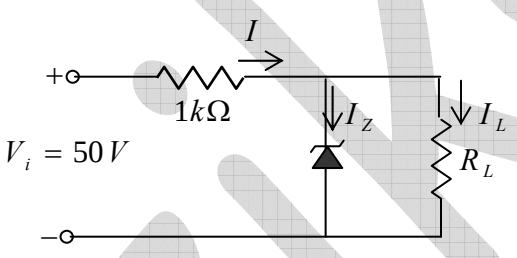
Q43. Let $w = \left(x^3 - y^3 - 2xy + 6\right)$. The value of $\frac{\partial^2 w}{\partial x^2}$, at a point where $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0$, is…..

Q44. A $2\cdot 0\,m$ long simple pendulum oscillating in an elevator which is accelerating upward at a rate of $2m/\sec^2$. The frequency of pendulum is …..Hz

Q45. The RC series circuit shown in figure below has reached steady state when the capacitor has charged to $110V$, the switch is opened. The voltage across capacitance $1\sec$ after the circuit is opened is…… Volts.



Q46. For the given zener diode network, the maximum value of load resistance R_L that will maintain output voltage to $15V$ is……Ω $(V_Z = 15\ V, I_{ZM} = 32mA)$



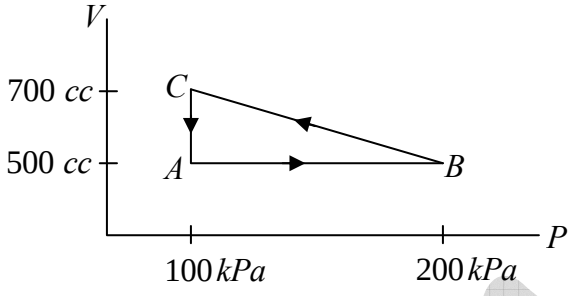
Q47. Consider two different systems each with three identical non-interacting particles. Both have single particle states with energies $\varepsilon_0, 3\varepsilon_0$ but state ε_0 is doubly degenerate. One system is populated by spin $\frac{1}{2}$ fermions and the other by bosons. If E_F and E_B are the ground state energies of the fermionic and bosonic systems, respectively, the value of $E_F - E_B = \alpha$, then the value of α is……

Q48. A particle is in a state which is a superposition of the ground state $|m\rangle$ and the first excited state $|n\rangle$ of a one-dimensional quantum harmonic oscillator with potential

$$V(x) = \begin{cases} \infty, & x < 0 \\ \frac{1}{2}m\omega^2 x^2, & x > 0 \end{cases}.$$

The state is given by $\Phi = \frac{1}{\sqrt{5}}|m\rangle + \frac{2}{\sqrt{5}}|n\rangle$. The expectation value of the energy of the particle in this state (in units of $\hbar\omega, \omega$ being the frequency of the oscillator) is …….

Q49. A gas is taken through a cyclic process $ABCA$ as shown in figure below. If $2.4\,cal$ of heat is given in the process, the value of J is…… $Joule/cal$



Q50. A solid sphere of mass M and radius R having moment of inertia I about its diameter is recast into a solid disc of radius r and thickness t. The moment of inertia of the disc about an axis passing the edge and perpendicular to the plane remains I. If $R=\alpha r$ then the value of α is ……. (Answer must be upto two digits after decimal)

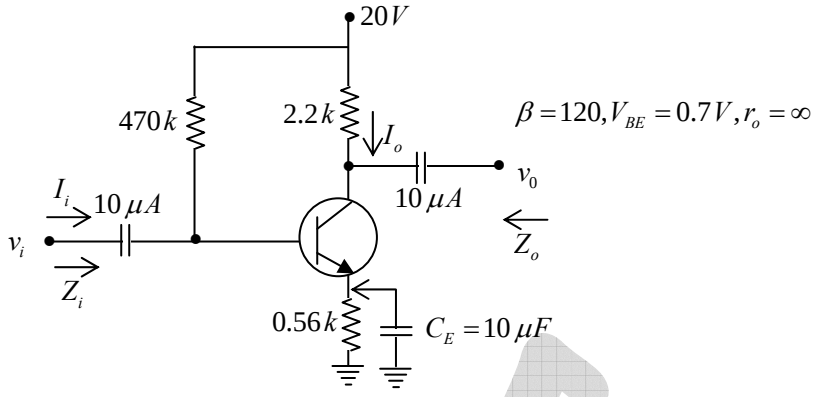
Q51 – Q60 Carry Two Marks each (No negative marking for any wrong answer).

Q51. When an open metal pipe is cut into two pieces, the lowest resonance frequency for the air column in one piece is $256\,Hz$ and that for the other is $440\,Hz$. The length of the original pipe was…… cm (velocity of sound $=343\,m/\sec$)

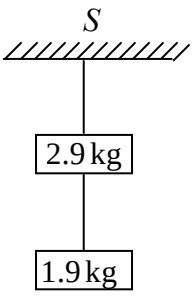
Q52. The magnetic field (in Am^{-1}) inside a long solid cylindrical conductor of radius a = 0.1 m is, $\vec{H}=\frac{10^4}{r}\left[\frac{1}{\alpha^2}\sin(\alpha r)-\frac{r}{\alpha}\cos(\alpha r)\right]\hat{\phi}$, where $\alpha=\frac{\pi}{2a}$. The total current in the conductor is ……Amperes (Answer must be rounded to the nearest integer)

Q53. The temperature of equal masses of three different liquids A, B and C is 12^0C, 18^0C and 28^0C, respectively. When A and B are mixed the temperature is $16^\circ C$. When B and C are mixed, it is $23^\circ C$. The temperature when A and C are mixed will be ……^0C.

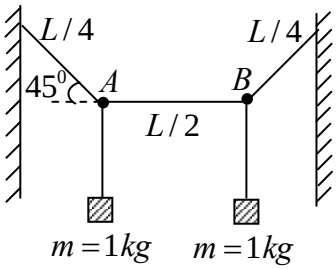
Q54. For the network shown in figure, the value of A_i is $\approx$ …….. (use $r_e = 6\Omega$)



Q55. Two block of mass $2.9\,kg$ and $1.9\,kg$ are suspended from a rigid support S by two inextensible wires each of length 1 meter, see fig. the upper wire has negligible mass and the lower wire has a uniform mass of $0.2\,kg/m$. The whole system of blocks, wires and support have an upward acceleration of $0.2\,m/s^2$. Acceleration due to gravity is $9.8\,m/s^2$. If the tension at the mid-point of the lower wire is T_1 and tension at the mid-point of the upper wire is T_2, then $\frac{T_2}{T_1}$ is ……..



Q56. A light string of mass $2g$ and length $L = 100\,cm$ has its ends tied to two walls. Two objects, each of mass $1kg$ are suspended from the string as shown in the figure. It a wave pulse is sent from point A, then total time it will take to travel to point B is …..$\times 10^{-3}$ sec.



Q57. The value of the magnetic field required to maintain non-relativistic protons of energy $1MeV$ in a circular orbit of radius 100 mm is……Tesla

(Given: $m_p = 1.67\times10^{-27}\,kg$, $e = 1.6\times10^{-19}\,C$)

(Answer must be upto two digits after decimal)

Q58. For vector function $\vec{A} = 2r\cos^2\phi\hat{r} + 3r^2\sin z\hat{\phi} + 4z\sin^2\phi\hat{z}$ the value of $\vec{\nabla}.\vec{A}$ is…….

Q59. If matrix $A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a,b,c are positive real numbers. If $abc=1$ and $A^T A = I$, then the largest value of $a^3+b^3+c^3$ is ……..

Q60. The wave function for three dimensional cubical potential of width a is given by $\left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi x}{a}\sin\frac{2\pi x}{a}\sin\frac{3\pi x}{a}$ the corresponding degeneracy for energy is given by ……
