1(a). Kirchhoff's Voltage Law (K.V.L.) and Kirchhoff's Current Law (K.C.L.)

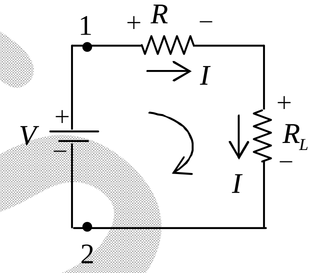
Kirchhoff's Voltage Law (K.V.L.)

Sum of the voltage in any closed loop must be zero considering same sign convention.

Sign Convention:

Lets consider that voltage is positive if we move from plus to minus then voltage is negative if we move from minus to plus. We can consider otherwise also.

Lets apply KVL in single loop network as shown in figure.



$$-V + IR + IR_L = 0 \text{ or } +V - IR - IR_L = 0$$

$$\Rightarrow I = \frac{V}{R+R_L} = \frac{V}{R_{eq,12}}$$

$\Rightarrow R_{eq,12} = R + R_L$ (Series equivalent of resistance)

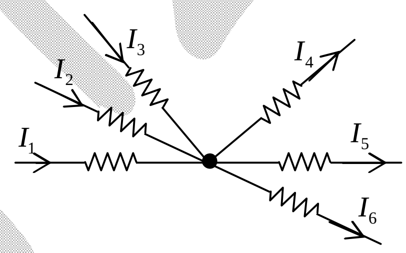
Voltage Division Rule

Voltage drop across R is $V_R = IR = \dfrac{VR}{R+R_L}$

Voltage drop across R_L is $V_{R_L} = IR_L = \dfrac{VR_L}{R+R_L}$

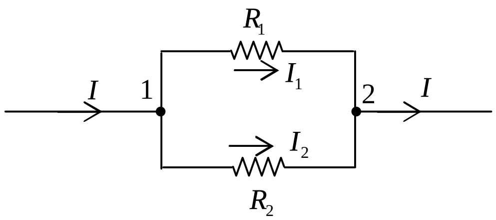
Kirchhoff's Current Law (K.C.L.)

At any node sum of incoming current is equal to sum of outgoing current.



Thus $I_1 + I_2 + I_3 = I_4 + I_5 + I_6$

Current Division Rule



According to KCL at node 1 and 2: $I = I_1 + I_2 \quad ...(i)$

Since parallel voltages are equal: $V_{12} = V = I_1R_1 = I_2R_2 \quad ...(ii)$

Solve equation (i) and (ii) for I_1 and I_2:

$$I_1 = \frac{R_2}{R_1+R_2}I \text{ and } I_2 = \frac{R_1}{R_1+R_2}I$$

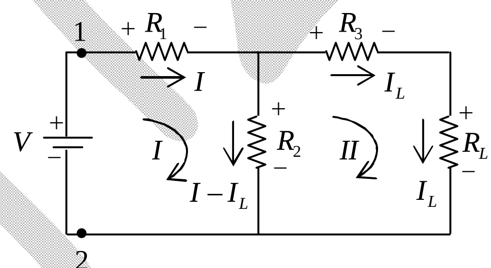
Then $V_{12} = V = I_1R_1 = I_2R_2 = I\dfrac{R_1R_2}{R_1+R_2} = IR_{eq,12}$

$\Rightarrow R_{eq,12} = \dfrac{R_1R_2}{R_1+R_2}$ (Parallel equivalent of resistance)

Two Loop Network

We can find current through each resistance using KVL and current division rule.

Lets apply KVL in two loop network as shown in figure.

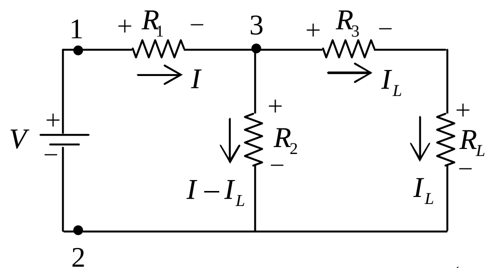


In loop (I): $-V + IR_1 + (I - I_L)R_2 = 0 \quad ...(i)$

In loop (II): $I_LR_2 + I_LR_L - (I - I_L)R_2 = 0 \quad ...(ii)$

Solve equation (i) and (ii) for I and I_L.

Lets apply current division rule in two loop network as shown in figure.



Current through R_1 is $I = \frac{V}{R_{eq,12}}$ where $R_{eq,12} = R_1 + \frac{R_2(R_3 + R_L)}{R_2 + (R_3 + R_L)}$

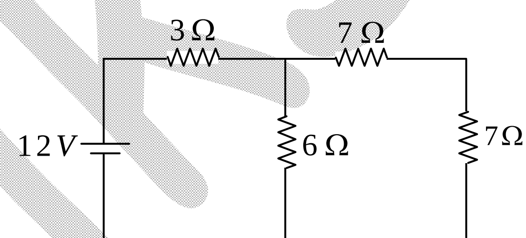
Lets apply current division rule at node 3:

$$I_L = \frac{R_2}{R_2 + (R_3 + R_L)} I \quad \text{and} \quad I - I_L = \frac{(R_3 + R_L)}{R_2 + (R_3 + R_L)} I$$

Example: Find current across each element in the circuit shown in figure below.

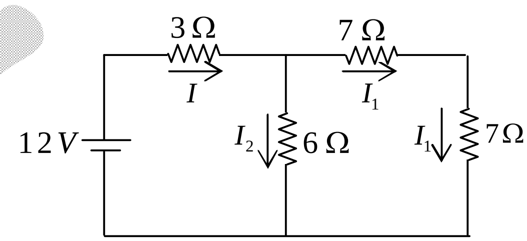
(a) Using current division rule

(b) Using KVL



Solution:

(a) Current $I = \frac{12}{3 + \frac{6 \times 14}{6 + 14}} = \frac{5}{3} A$



$$I_1 = \frac{6}{6 + (7 + 7)} \times \frac{5}{3} = \frac{1}{2} A$$

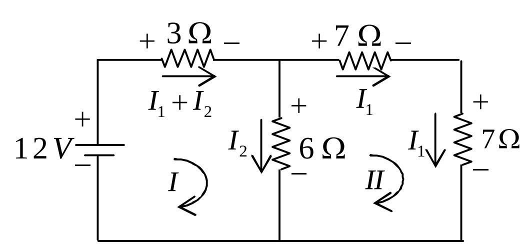
$$I_2 = \frac{(7 + 7)}{6 + (7 + 7)} \times \frac{5}{3} = \frac{7}{6} A$$

(b) Apply KVL in Loop I

$$-12 + 3(I_1 + I_2) + 6I_2 = 0 \Rightarrow 3I_1 + 9I_2 = 12$$

Apply KVL in Loop II

$$7I_1 + 7I_1 - 6I_2 = 0 \Rightarrow 7I_1 = 3I_2$$



Solving above equations, we get $I_1 = \frac{1}{2} A$ and $I_2 = \frac{7}{6} A$