1(c). Thevenin's Theorem

Thevenin's theorem is a method used to change a complex circuit into a simple equivalent circuit. Thevenin's theorem states that any linear network of voltage source and resistances, if viewed from any two points in the network can be replaced by an equivalent resistance R_{TH} in series with an equivalent source V_{TH}. Figure (a) shows the original linear network with terminals a and b; figure (b) shows its connection to an external network or load; and figure (c) shows the Thevenin equivalent V_{TH} and R_{TH} that can be substituted for the linear network at the terminals a and b. The polarity of V_{Th} is such that it will produce current from a to b in the same direction as in the original network. R_{TH} is the Thevenin resistance across the network terminals a and b with each internal voltage source short-circuited. V_{TH} is the Thevenin voltage that would appear across the terminals a and b with the voltage sources in place and no load connected across a and b. For this reason, V_{TH} is also called the open-circuit voltage.

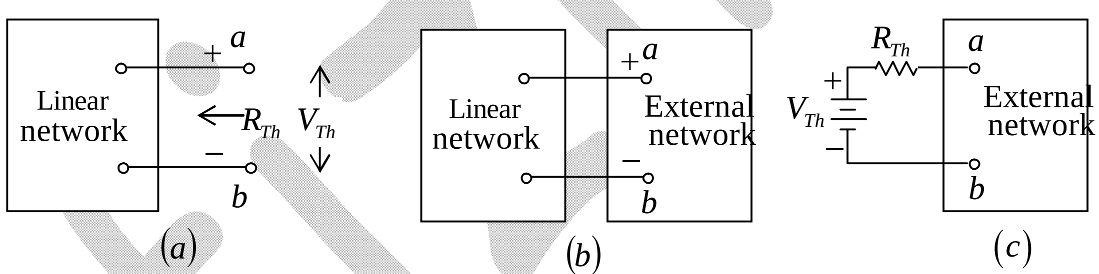
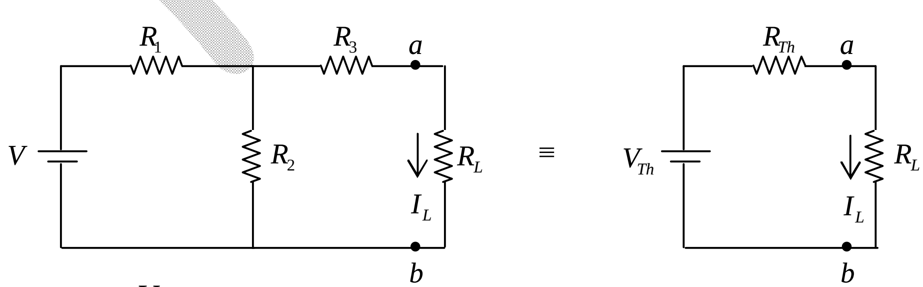


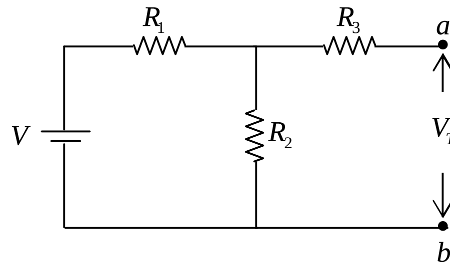
Figure: Thevenin equivalent, V_{TH} and sereies R_{TH}

Consider two loop network as shown in figure.



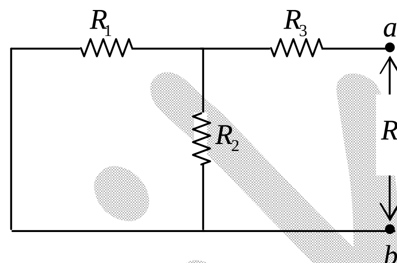
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Thevenin Voltage (V_{TH}) (Open-circuit voltage)



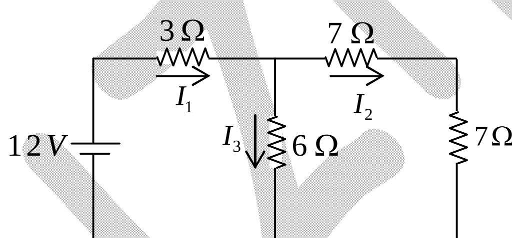
$$V_{TH} = \frac{VR_2}{R_1 + R_2}$$

Thevenin Resistance (R_{TH})



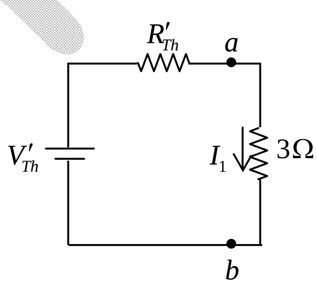
$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

Example: Draw Thevinin's equivalent circuit and find current across each element for the circuit shown in figure below.

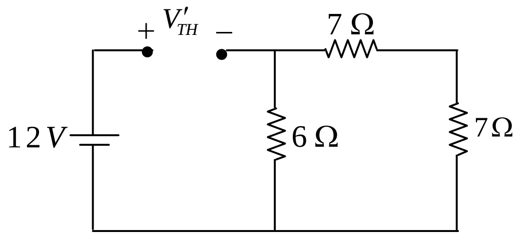


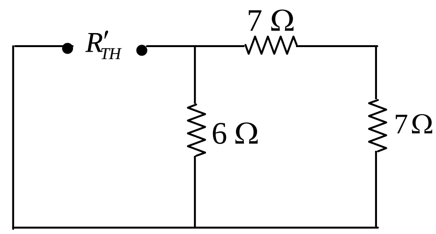
Solution:

For I_1:



$$I_1 = \frac{V'_{Th}}{R'_{Th} + 3\Omega}$$



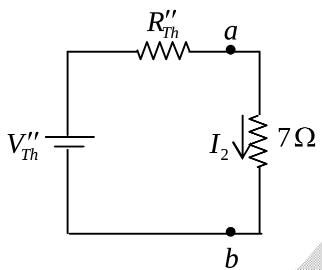


$V'_{Th} = 12\,V$

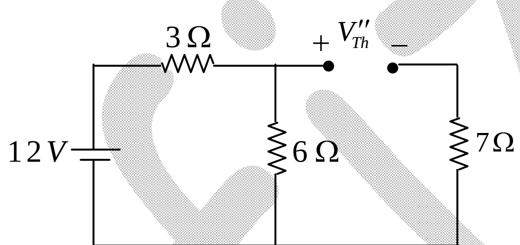
$R'_{Th} = \dfrac{6\times 14}{6+14} = \dfrac{21}{5}\Omega$

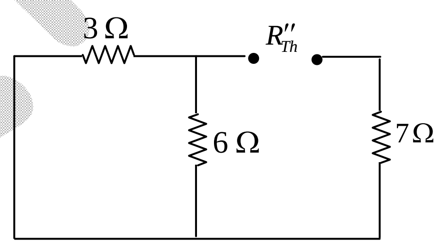
Thus $I_1 = \dfrac{V'_{Th}}{R'_{Th} + 3\Omega} = \dfrac{12}{21/5 + 3\Omega} = \dfrac{60}{36} = \dfrac{5}{3}A$

For I_2 :



$I_2 = \dfrac{V''_{Th}}{R''_{Th} + 7\Omega}$



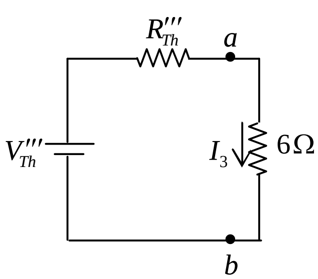


$V''_{Th} = \dfrac{6}{6+3}\times 12\,V = 8\,V$

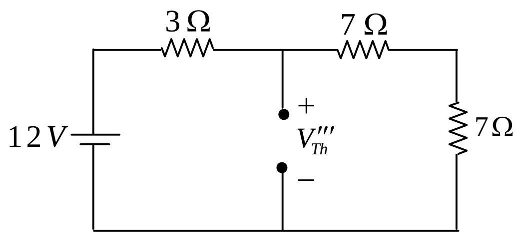
$R''_{Th} = \dfrac{6\times 3}{6+3} + 7 = 9\,\Omega$

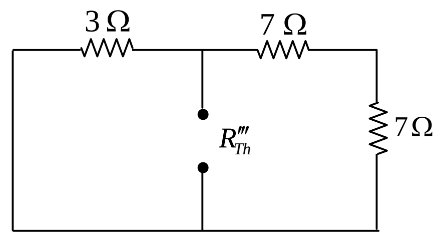
Thus $I_2 = \dfrac{V''_{Th}}{R''_{Th} + 7\Omega} = \dfrac{8V}{9\Omega + 7\Omega} = \dfrac{1}{2}A$

For I_3 :



$I_3 = \dfrac{V'''_{Th}}{R'''_{Th} + 6\Omega}$





$$V_{Th}''' = \frac{14}{14+3} \times 12V = \frac{168}{17} V \qquad R_{Th}'' = \frac{14 \times 3}{14+3} = \frac{42}{17} \Omega$$

Thus $I_3 = \dfrac{V_{Th}'''}{R_{Th}''' + 3\Omega} = \dfrac{168/17V}{42/17\Omega + 6\Omega} = \dfrac{168}{144} A = \dfrac{7}{6} A$