

4(c). Volume Integrals

A volume integral is an expression of the form

$$\int_V T d\tau,$$

where T is a scalar function and $d\tau$ is an infinitesimal volume element. In Cartesian coordinates,

$$d\tau = dx dy dz.$$

For example, if T is the density of a substance (which might vary from point to point) then the volume integral would give the total mass. Occasionally we shall encounter volume integrals of vector functions:

$$\int \vec{A} d\tau = \int (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) d\tau = \hat{x} \int A_x d\tau + \hat{y} \int A_y d\tau + \hat{z} \int A_z d\tau;$$

because the unit vectors are constants, they come outside the integral.

