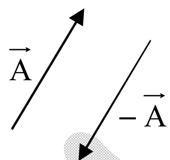
1. Vector Algebra

Vector quantities have both *direction* as well as *magnitude* such as velocity, acceleration, force and momentum etc. We will use $\vec{A}$ for any general vector and its magnitude by $|\vec{A}|$.

In diagrams vectors are denoted by arrows: the length of the arrow is proportional to the magnitude of the vector, and the arrowhead indicates its direction. Minus $\vec{A}\,(-\vec{A})$ is a vector with the same magnitude as $\vec{A}$ but of opposite direction.



1(a). Vector Operations

We define four vector operations: addition and three kinds of multiplication.

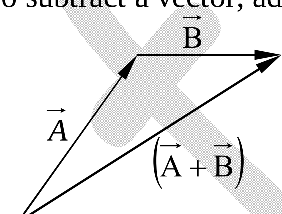
(i) Addition of two vectors

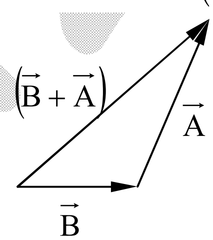
Place the tail of $\vec{B}$ at the head of $\vec{A}$; the sum, $\vec{A}+\vec{B}$, is the vector from the tail of $\vec{A}$ to the head of $\vec{B}$.

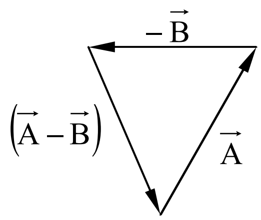
Addition is *commutative*: $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

Addition is associative: $\left(\vec{A}+\vec{B}\right)+\vec{C}=\vec{A}+\left(\vec{B}+\vec{C}\right)$

To subtract a vector, add its opposite: $\vec{A}-\vec{B}=\vec{A}+\left(-\vec{B}\right)$



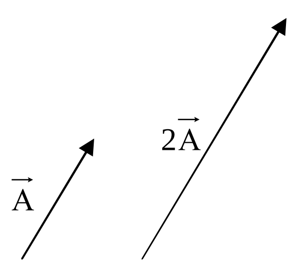




(ii) Multiplication by scalar

Multiplication of a vector by a positive scalar *a*, multiplies the *magnitude* but leaves the direction unchanged. (If *a* is negative, the direction is reversed.) Scalar multiplication is distributive:

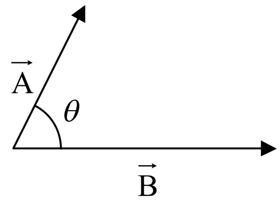
$$a\left(\vec{A}+\vec{B}\right)=a\vec{A}+a\vec{B}$$



(iii) Dot product of two vectors

The dot product of two vectors is define by

$$\vec{A}.\vec{B} = AB\cos\theta$$



where θ is the angle they form when placed tail to tail. Note that $\vec{A}.\vec{B}$ is itself a scalar. The dot product is commutative,

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

and distributive, $\vec{A}.\left(\vec{B}+\vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

Geometrically $\vec{A}.\vec{B}$ is the product of A times the projection of $\vec{B}$ along $\vec{A}$ (or the product of B times the projection of $\vec{A}$ along $\vec{B}$).

If the two vectors are parallel, $\vec{A}.\vec{B} = AB$

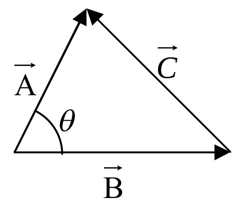
If two vectors are perpendicular, then $\vec{A}.\vec{B} = 0$

Law of cosines

Let $\vec{C} = \vec{A} - \vec{B}$ and then calculate dot product of $\vec{C}$ with itself.

$$\vec{C}.\vec{C} = \left(\vec{A}-\vec{B}\right).\left(\vec{A}-\vec{B}\right) = \vec{A}.\vec{A} - \vec{A}.\vec{B} - \vec{B}.\vec{A} + \vec{B}.\vec{B}$$

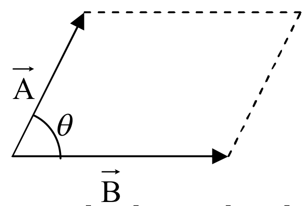
$$C^2 = A^2 + B^2 - 2AB\cos\theta$$



(iv) Cross product of two vectors

The cross product of two vectors is define by

$$\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$$



where $\hat{n}$ is a unit vector(vector of length 1) pointing perpendicular to the plane of $\vec{A}$ and $\vec{B}$ .Of course there are two directions perpendicular to any plane "in" and "out."

The ambiguity is resolved by the **right-hand rule:**

let your fingers point in the direction of first vector and curl around (via the smaller angle) toward the second; then your thumb indicates the direction of $\hat{n}$. (In figure $\vec{A}\times\vec{B}$ points into the page; $\vec{B}\times\vec{A}$ points out of the page)

The cross product is distributive,

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}\times\vec{B}\right)+\left(\vec{A}\times\vec{C}\right)$$

but not commutative.

In fact, $\left(\vec{B}\times\vec{A}\right)=-\left(\vec{A}\times\vec{B}\right)$.

Geometrically, $\left|\vec{A}\times\vec{B}\right|$ is the area of the parallelogram generated by $\vec{A}$ and $\vec{B}$. If two vectors are parallel, their cross product is zero.

In particular $\vec{A}\times\vec{A}=0$ for any vector $\vec{A}$