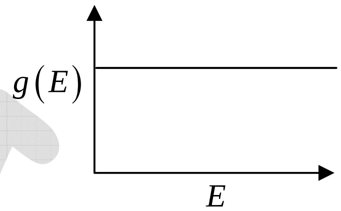
(c) Density of State in Two Dimension

$$g(E)\,dE = \frac{1}{h^2}\int\int dx.dp_x \int\int dy.dp_y \Rightarrow g(E)\,dE = \frac{A}{h^2}\iint dp_x dp_y$$

where A is volume of container $\frac{p_x^2}{2m} + \frac{p_y^2}{2m} = E \Rightarrow p_x^2 + p_y^2 = \left(\sqrt{2mE}\right)^2$

which is equation of circle with coordinate p_x, p_y



So, $\iint dp_x dp_y$ is area of circle with radius $\sqrt{2mE}$

$g(E)\,dE = \frac{A}{h^2}\iint dp_x dp_y$ is equivalent to $\frac{A}{h^2}\times\phi(E)$

where $\phi(E)$ area of circular shell between radius

$\sqrt{2m(E+dE)}$ and $\sqrt{2mE}$

$$\phi(E) = \pi\left(\left(2m(E+dE)\right)^{2/2} - (2mE)^{2/2}\right) = \pi(2mE)^{2/2}\left(\left(1+\frac{dE}{E}\right)^{2/2} - 1\right)$$

Using Taylor expansion, then $\pi(2mE)^{2/2}\left(\left(1+\frac{2}{2}\frac{dE}{E}\right)-1\right) \Rightarrow 2\pi(2m)^{2/2} E^0 dE$

$g(E)\,dE = \frac{A}{h^2}\iint dp_x dp_y$ is equivalent to $\frac{A}{h^2}\times\pi(2m)^{2/2}\,dE$

$g(E)dE$ in two dimension $g(E)dE = \pi A\left(\frac{2m}{h^2}\right)dE$, where A is area of the two dimensional space.