(b) Calculation of Pressure Exerted by an Ideal Gas

Suppose there are N molecules per cubic meter each of mass m, and it is assumed that n_i number of molecule have velocity v_i in a cubical box (Figure 3).

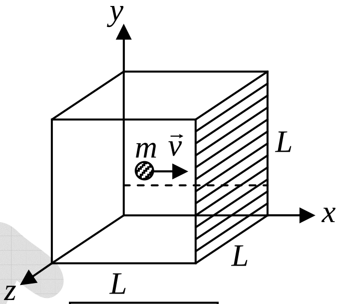

Figure 1

Mathematically, $\sum n_i = N$ and $v_i^2 = v_{ix}^2 + v_{iy}^2 + v_{iz}^2$, where v_{ix}, v_{iy} and v_{iz} are x, y and z components of velocity of gases respectively.

Figure given above, shows a typical gas molecule, of mass m and velocity $\vec{v}$, that is about to collide with the shaded wall, where the time Δt between collisions is the time the molecule take to travel to the opposite wall and back again at speed v_{ix}.

From assumptions of kinetic theory of gases, $v_{ix}^2 = v_{iy}^2 = v_{iz}^2 = \frac{v_i^2}{3}$

Suppose, molecules are kept in the cubic container of parameter L.

A molecule moving in the x- direction will have momentum mv_{ix} normal to face of the cube before collision.

Hence, the momentum Δp_{ix} delivered to the wall by the molecule during collision is

$$\Delta p_{ix} = mv_{ix} - (-mv_{ix}) = 2mv_{ix}$$

Force acting on the wall by molecule is $f_{ix} = \frac{n_i 2mv_{ix}}{\Delta t} = \frac{n_i 2mv_{ix}^2}{2L} = \frac{n_i mv_{ix}^2}{L}$

Pressure exerted on the walls of container by molecules, $P_{ix} = \frac{mn_i v_{ix}^2}{L^3}$

So, the pressure in the x- direction exerted by all molecules is,

$$P_x = \sum P_{ix} = \frac{m}{L^3}\sum n_i v_{ix}^2$$

Average value of v^2 is given by,

$$\langle v_x^2 \rangle = \frac{\sum_i n_i v_{ix}^2}{\sum n_i} = \frac{\sum_{i=1} n_i v_{ix}^2}{n}$$

For three dimensional system, $\left\langle v_x^2\right\rangle+\left\langle v_y^2\right\rangle+\left\langle v_z^2\right\rangle=\left\langle v^2\right\rangle$ and

For isotropic system, $\left\langle v_x^2\right\rangle=\left\langle v_y^2\right\rangle=\left\langle v_z^2\right\rangle=\dfrac{\left\langle v^2\right\rangle}{3}$

So, P_x can be written as

$$P_x=\frac{m}{L^3}N\left\langle v_x^2\right\rangle\ ,\ P=P_x=\frac{1}{3}\frac{m}{L^3}N\left\langle v^2\right\rangle=\frac{1}{3}\frac{mN\left\langle v^2\right\rangle}{V}$$

$$PV=\frac{1}{3}mN\left\langle v^2\right\rangle$$

where V is volume of the container and $\left\langle v^2\right\rangle$ is average value of square of velocity.