(c) Average Energy, RMS and Most Probable Energy

For the Maxwell-Boltzmann energy distribution law, average energy $\langle E\rangle$ of the particles is

$$\langle E\rangle = \frac{\int_0^\infty E\, f(E)\, g(E)\, dE}{\int_0^\infty dn} \Rightarrow \int_0^\infty E\phi(E)\, dE$$

$$\langle E\rangle = \frac{1}{N}\frac{2\pi N}{(\pi k_B T)^{3/2}}\int_0^\infty E^{3/2}\exp\left(-\frac{E}{k_B T}\right)dE \Rightarrow \frac{1}{N}\frac{2\pi N}{(\pi k_B T)^{3/2}}\times (k_B T)^{5/2}\left\lceil\frac{5}{2}\right. = \frac{3k_B T}{2}$$

Hence, the average of a particle is $\frac{1}{2}k_B T$ per degree of freedom, for three degree of freedom it is $\frac{3}{2}k_B T$

Mean square of energy $\langle E^2\rangle = \dfrac{\int_0^\infty E^2\, f(E)\, g(E)\, dE}{\int_0^\infty dn} \int_0^\infty E^2\phi(E)\, dE$

$$\langle E^2\rangle = \frac{1}{N}\frac{2\pi N}{(\pi k_B T)^{3/2}}\int_0^\infty E^{5/2}\exp\left(-\frac{E}{k_B T}\right)dE \Rightarrow \frac{1}{N}\frac{2\pi N}{(\pi k_B T)^{3/2}}\times (k_B T)^{7/2}\left\lceil\frac{7}{2}\right. = \frac{15(k_B T)^2}{4}$$

Root mean Energy $\sqrt{\langle E^2\rangle} = \sqrt{\frac{15}{4}}k_B T$

Most probable Energy

Probability density $\phi(E) = \dfrac{2\pi}{(\pi k_B T)^{3/2}}E^{1/2}\exp\left(-\dfrac{E}{k_B T}\right)$ then

The most probable energy is given by $\dfrac{d\phi}{dE} = 0 \Rightarrow E = \dfrac{k_B T}{2}$

Energy distribution in different dimension

- $f(E) = \dfrac{N}{V}\left(\dfrac{h^2}{2\pi m k_B T}\right)^{3/2}\exp\left(-\dfrac{E}{k_B T}\right)$ distribution function in three dimension, where V is volume .

- $f(E)=\frac{N}{A}\left(\frac{h^2}{2\pi mk_BT}\right)\exp\left(-\frac{E}{k_BT}\right)$ distribution function in two dimension, where A is area.
- $f(E)=\frac{N}{L}\left(\frac{h^2}{2\pi mk_BT}\right)^{\frac{1}{2}}\exp\left(-\frac{E}{k_BT}\right)$ distribution function in one dimension, where L is length.