(e) Average, RMS and Most Probable Speed

Average Speed

$$\langle v\rangle=\int_0^\infty vf(v)dv=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int_0^\infty v\exp\left(-\frac{mv^2}{2k_BT}\right)v^2dv$$

$$=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}\left(\frac{2k_BT}{m}\right)^{\frac{3+1}{2}}\Gamma\frac{3+1}{2}=\sqrt{\frac{8k_BT}{\pi m}}$$

Mean square Speed

$$\langle v^2\rangle=\int_0^\infty v^2f(v)dv=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int_0^\infty v^2\exp\left(-\frac{mv^2}{2k_BT}\right)v^2dv$$

$$=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}\left(\frac{2k_BT}{m}\right)^{\frac{4+1}{2}}\Gamma\frac{4+1}{2}=\frac{3k_BT}{m}$$

Root Mean Square speed

$$\sqrt{\langle v^2\rangle}=\sqrt{\int_0^\infty v^2f(v)dv}=\sqrt{\frac{3k_BT}{m}}$$

Most probable speed

$$v_p=\frac{df(v)}{dv}=0\Rightarrow v_p=\sqrt{\frac{2k_BT}{m}}$$

Average velocity of $\langle v_x\rangle,\langle v_x^2\rangle$

$$\langle v_x\rangle=\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}v_x\exp\left(-\frac{m\left(v_x^2+v_y^2+v_z^2\right)}{2k_BT}\right)dv_xdv_ydv_z=0$$

$$\langle v_x\rangle=\langle v_x\rangle=\langle v_x\rangle=0$$

$$\langle v_x^2\rangle=\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}v_x^2\exp\left(-\frac{m\left(v_x^2+v_y^2+v_z^2\right)}{2k_BT}\right)dv_xdv_ydv_z=\frac{k_BT}{m}$$

Similarly, $\langle v_x^2\rangle=\langle v_y^2\rangle=\langle v_z^2\rangle=\dfrac{k_BT}{m}$