NET-DEC-2012

Part A

Q1. A granite block of 2m × 5m × 3m size is cut into 5 cm thick slabs of 2m × 5m size. These slabs are laid over a 2m wide pavement. What is the length of the pavement that can be covered with these slabs?

(a) 100 m (b) 200 m (c) 300 m (d) 500 m

Q2. Which is the least among the following?

$0.33^{0..33}$, $0.44^{0.44}$, $\pi^{-1/\pi}$, $e^{-1/e}$

(a) $0.33^{0.33}$ (b) $0.44^{0.44}$ (c) $\pi^{-1/\pi}$ (d) $e^{-1/e}$

Q3. What is the next number in this "see and tell" sequence?

1 11 21 1211 111221 _______

(a) 312211 (b) 1112221 (c) 1112222 (d) 1112131

Q4. A vertical pole of length a stands at the centre of a horizontal regular hexagonal ground of side a. A rope that is fixed taut in between a vertex on the ground and the tip of the pole has a length

(a) a (b) $\sqrt{2}a$ (c) $\sqrt{3}a$ (d) $\sqrt{6}a$

Q5. A peacock perched on the top of a 12 m high tree spots a snake moving towards its hole at the base of the tree from a distance equal to thrice the height of the tree. The peacock flies towards the snake in a straight line and they both move at the same speed. At what distance from the base of the tree will the peacock catch the snake?

(a) 16 m (b) 18 m (c) 14 m (d) 12 m

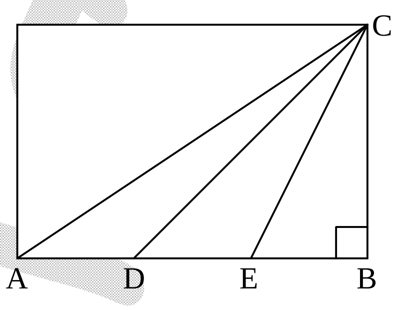
Q6. The cities of a country are connected by intercity roads. If a city is directly connected to an odd number of other cities, it is called an odd city. If a city is directly connected to an even number of other cities, it is called an even city. Then which of the following is impossible?

(a) There are an even number of odd cities (b) There are an odd number of odd cities

(c) There are an even number of even cities (d) There are an odd number of even cities

Q7. In the figure $\angle ABC = \pi/2$, $AD = DE = EB$

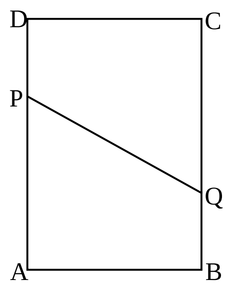
What is the ratio of the area of triangle ADC to that of triangle CDB?



(a) 1:1 (b) 1:2

(c) 1:3 (d) 1:4

Q8. A rectangular sheet ABCD is folded in such a way that vertex A meets vertex C, thereby forming a line PQ. Assuming AB = 3 and BC = 4, find PQ. Note that AP = PC and AQ = QC.



(a) 13/4 (b) 15/4

(c) 17/4 (d) 19/4

Q9. A string of diameter 1mm is kept on a table in the shape of a close flat spiral i.e. a spiral with no gap between the turns. The area of the table occupied by the spiral is $1 m^2$. Then the length of the string is

(a) 10 m (b) 10^2 m (c) 10^3 m (d) 10^4 m

Q10. 25% of 25% of a quantity is x% of the quantity where x is

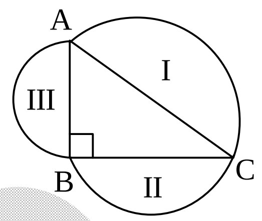
(a) 6.25 % (b) 12.5 % (c) 25 % (d) 50 %

Q11. In sequence $\{a_n\}$every term is equal to the sum of all previous terms. If $a_0 = 3$, then

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} \text{ is}$$

(a) 3 (b) 2 (c) 1 (d) e

Q12. In the figure given, angle ABC = $\pi/2$. I, II, III are the areas of semicircles on the sides opposite angles B, A and C, respectively. Which of the following is always true?



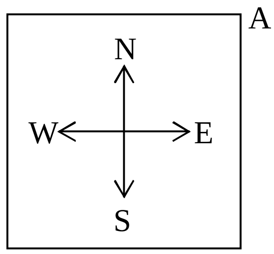
(a) $II^2 + III^2 = I^2$ (b) II + III = I

(c) $II^2 + III^2 > I^2$ (d) II + III < I

Q13. What is the minimum number of days between one Friday the 13[th] and the next Friday the 13[th]? (Assume that the year is a leap year).

(a) 28 (b) 56 (c) 91 (d) 84

Q14. Suppose a person A is at the North-East corner of a square (see the figure below). From that point he moves along the diagonal and after covering 1/3[rd] portion of the diagonal, he goes to his left and after sometime he stops, rotates 90° clockwise and moves straight. After a few minutes he stops, rotates 180° anticlockwise. Towards which direction he is facing now?



(a) North-East (b) North-West (c) South-East (d) South-West

Q15. Cucumber contains 99% water. Ramesh buys 100 kg of cucumbers. After 30 days of storing the cucumbers lose some water. They now contain 98% water. What is the total weight of cucumbers now?

(a) 99 kg (b) 50 kg (c) 75 kg (d) 2 kg

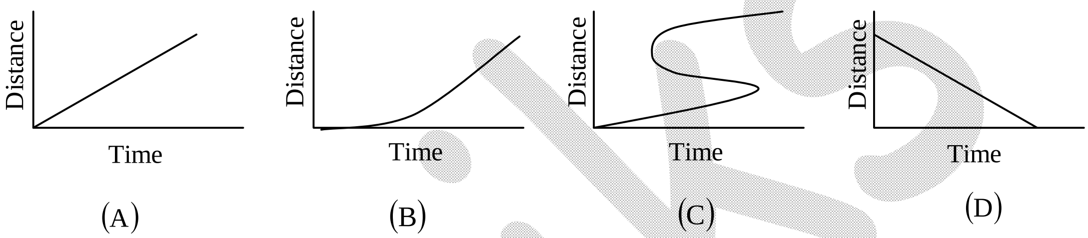
Q16. In a museum there were old coins with their respective years engraved on them, as follows:

(i) 1837 AD (ii) 1907 AD (iii) 1947 AD (iv) 200 BC

identify the fake coin(s)

(a) coin (i) (b) coin (iv) (c) coins (i) and (ii) (d) coin (iii)

Q17. A student observes the movement of four snails and plots the graphs of distance moved as a function of time as given in figures (A), (B), (C) and (D).



Which of the following is **not** correct?

(a) Graph (A) (b) Graph (B) (c) Graph (C) (d) Graph (D)

Q18. Find the missing letter:

A	E G K	C
? U Q		P R V
B	O J F	D

(a) H (b) L (c) Z (d) Y

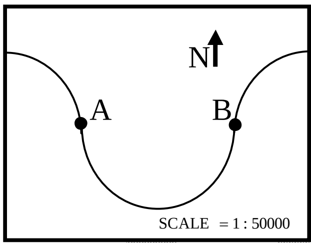
Q19. Consider the following equation

$$x^2 + 4y^2 + 9z^2 = 14x + 28y + 42z + 147$$

where x, y and z are real numbers. Then the value of x + 2y + 3z is

(a) 7 (b) 14 (c) 21 (d) not unique

Q20. The map given below shows a meandering river following a semi-circular path, along which two villages are located at A and B. The distance between A and B along the east-west direction in the map is 7 cm. What is the length of the river between A and B in the ground?



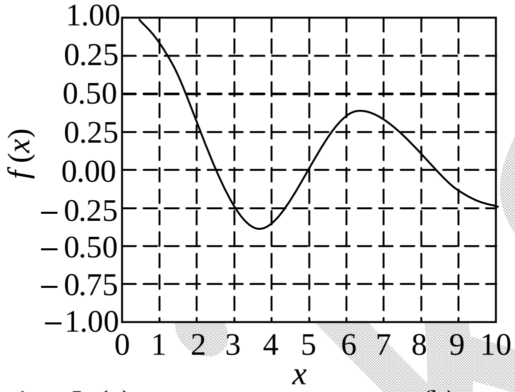
(a) 1.1 km (b) 3.5 km (c) 5.5 km (d) 11.0 km

Part B

Q21. A 2×2 matrix A has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of n such that $A^n = I$ is

(a) 20 (b) 30 (c) 60 (d) 120

Q22. The graph of the function $f(x)$ shown below is best described by



(a) The Bessel function $J_0(x)$ (b) $\cos x$

(c) $e^{-x}\cos x$ (d) $\frac{1}{x}\cos x$

Q23. In a series of five Cricket matches, one of the captains calls "Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is

(a) 1/8 (b) 5/8 (c) 3/16 (d) 5/16

Q24. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, is

(a) $\frac{bc\hat{i} + ca\hat{j} + ab\hat{k}}{\sqrt{a^2c^2 + b^2c^2 + a^2b^2}}$ (b) $\frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$

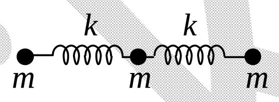
(c) $\frac{b\hat{i} + c\hat{j} + a\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (d) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

Q25. A solid cylinder of height H, radius R and density ρ, floats vertically on the surface of a liquid of density ρ_0. The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is

(a) $\dfrac{\rho g}{\rho_0 H}$ (b) $\dfrac{\rho}{\rho_0}\sqrt{\dfrac{g}{H}}$ (c) $\sqrt{\dfrac{\rho g}{\rho_0 H}}$ (d) $\sqrt{\dfrac{\rho_0 g}{\rho H}}$

Q26. Three particles of equal mass m are connected by two identical massless springs of stiffness constant k as shown in the figure:

If x_1, x_2 and x_3 denote the horizontal displacements of the masses from their respective equilibrium positions, the potential energy of the system is



(a) $\frac{1}{2}k\left[x_1^2+x_2^2+x_3^2\right]$ (b) $\frac{1}{2}k\left[x_1^2+x_2^2+x_3^2-x_2\left(x_1+x_3\right)\right]$

(c) $\frac{1}{2}k\left[x_1^2+2x_2^2+x_3^2+2x_2\left(x_1+x_3\right)\right]$ (d) $\frac{1}{2}k\left[x_1^2+2x_2^2+x_3^2-2x_2\left(x_1+x_3\right)\right]$

Q27. Let v, p and E denote the speed, the magnitude of the momentum, and the energy of a free particle of rest mass m. Then

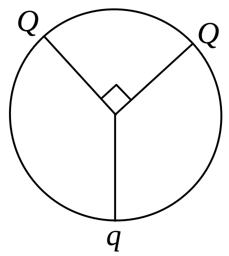
(a) $dE/dp =$ constant (b) $p = mv$

(c) $v = cp/\sqrt{p^2+m^2c^2}$ (d) $E = mc^2$

Q28. A binary star system consists of two stars S_1 and S_2, with masses m and $2m$ respectively separated by a distance r. If both S_1 and S_2 individually follow circular orbits around the centre of mass with instantaneous speeds v_1 and v_2 respectively, the speeds ratio v_1/v_2 is

(a) $\sqrt{2}$ (b) 1 (c) 1/2 (d) 2

Q29. Three charges are located on the circumference of a circle of radius R as shown in the figure below. The two charges Q subtend an angle $90°$ at the centre of the circle. The charge q is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?



(a) $q/\sqrt{2}$ (b) $\sqrt{2}q$ (c) $2q$ (d) $4q$

Q30. Consider a hollow charged shell of inner radius a and outer radius b. The volume charge density is $\rho(r)=\frac{k}{r^2}$ (k is constant) in the region $a < r < b$. The magnitude of the electric field produced at distance $r > a$ is

(a) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for all $r > a$

(b) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{kb}{\varepsilon_0 r^2}$ for $r > b$

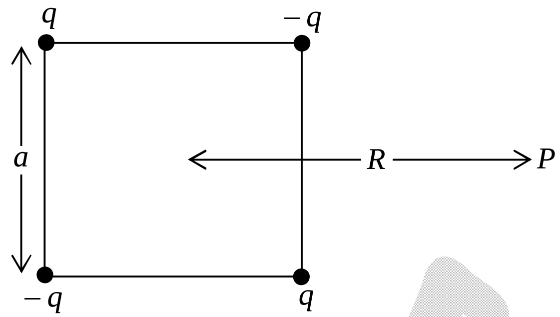
(c) $\frac{k(r-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > b$

(d) $\frac{k(r-a)}{\varepsilon_0 a^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > b$

Q31. Consider the interference of two coherent electromagnetic waves whose electric field vectors are given by $\vec{E}_1 = \hat{i}E_0\cos\omega t$ and $\vec{E}_2 = \hat{j}E_0\cos(\omega t+\varphi)$ where φ is the phase difference. The intensity of the resulting wave is given by $\frac{\varepsilon_0}{2}\langle E^2\rangle$, where $\langle E^2\rangle$ is the time average of E^2. The total intensity is

(a) 0 (b) $\varepsilon_0 E_0^2$ (c) $\varepsilon_0 E_0^2\sin^2\varphi$ (d) $\varepsilon_0 E_0^2\cos^2\varphi$

Q32. Four charges (two + q and two $-q$) are kept fixed at the four vertices of a square of side a as shown



At the point P which is at a distance R from the centre (R >> a), the potential is proportional to

(a) $1/R$ (b) $1/R^2$ (c) $1/R^3$ (d) $1/R^4$

Q33. A point charges q of mass m is kept at a distance d below a grounded infinite conducting sheet which lies in the xy - plane. For what value of d will the charge remains stationary?

(a) $q/4\sqrt{mg\pi\varepsilon_0}$ (b) $q/\sqrt{mg\pi\varepsilon_0}$

(c) There is no finite value of d (d) $\sqrt{mg\pi\varepsilon_0}/q$

Q34. The wave function of a state of the hydrogen atom is given by

$$\Psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\psi_{21-1}$$

where ψ_{nlm} is the normalized eigen function of the state with quantum numbers n, l and m in the usual notation. The expectation value of L_z in the state Ψ is

(a) $15\hbar/16$ (b) $11\hbar/16$ (c) $3\hbar/8$ (d) $\hbar/8$

Q35. The energy eigenvalues of a particle in the potential $V(x) = \frac{1}{2}m\omega^2 x^2 - ax$ are

(a) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{a^2}{2m\omega^2}$ (b) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega + \frac{a^2}{2m\omega^2}$

(c) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{a^2}{m\omega^2}$ (d) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$

Q36. If a particle is represented by the normalized wave function

$$\psi(x)=\begin{cases}\dfrac{\sqrt{15}\left(a^2-x^2\right)}{4a^{5/2}} & \text{for } -a<x<a\\ 0 & \text{otherwise}\end{cases}$$

the uncertainty Δp in its momentum is

(a) $2\hbar/5a$ (b) $5\hbar/2a$ (c) $\sqrt{10}\hbar/a$ (d) $\sqrt{5}\hbar/\sqrt{2}a$

Q37. Given the usual canonical commutation relations, the commutator $[A,B]$ of $A=i\left(xp_y-yp_x\right)$ and $B=\left(yp_z+zp_y\right)$ is

(a) $\hbar\left(xp_z-p_xz\right)$ (b) $-\hbar\left(xp_z-p_xz\right)$

(c) $\hbar\left(xp_z+p_xz\right)$ (d) $-\hbar\left(xp_z+p_xz\right)$

Q38. The entropy of a system, S, is related to the accessible phase space volume Γ by $S=k_B\ln\Gamma(E,N,V)$ where E, N and V are the energy, number of particles and volume respectively. From this one can conclude that Γ

(a) does not change during evolution to equilibrium

(b) oscillates during evolution to equilibrium

(c) is a maximum at equilibrium

(d) is a minimum at equilibrium

Q39. Let ΔW be the work done in a quasistatic reversible thermodynamic process. Which of the following statements about ΔW is correct?

(a) ΔW is a perfect differential if the process is isothermal

(b) ΔW is a perfect differential if the process is adiabatic

(c) ΔW is always a perfect differential

(d) ΔW cannot be a perfect differential

Q40. Consider a system of three spins S_1, S_2 and S_3 each of which can take values +1 and -1. The energy of the system is given by $E = -J\left[S_1S_2 + S_2S_3 + S_3S_1\right]$ where J is a positive constant. The minimum energy and the corresponding number of spin configuration are, respectively,

(a) J and 1 (b) -3J and 1 (c)-3J and 2 (d) -6J and 2

Q41. The minimum energy of a collection of 6 non-interacting electrons of spin - $\frac{1}{2}$ and mass m placed in a one dimensional infinite square well potential of width L is

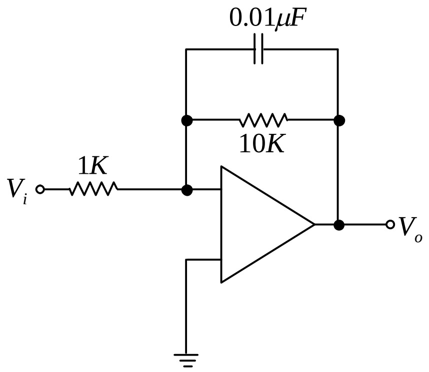
(a) $14\pi^2\hbar^2/mL^2$ (b) $91\pi^2\hbar^2/mL^2$ (c) $7\pi^2\hbar^2/mL^2$ (d) $3\pi^2\hbar^2/mL^2$

Q42. A live music broadcast consists of a radio-wave of frequency 7 MHz, amplitude-modulated by a microphone output consisting of signals with a maximum frequency of 10 kHz. The spectrum of modulated output twill be zero outside the frequency band

(a) 7.00 MHz to 7.01 MHz (b) 6.99 MHz to 7.01 MHz

(c) 6.99 MHz to 7.00 MHz (d) 6.995 MHz to 7.005 MHz

Q43. In the op-amp circuit shown in the figure, V_i is a sinusoidal input signal of frequency 10 Hz and V_0 is the output signal. The magnitude of the gain and the phase shift, respectively, close to the values



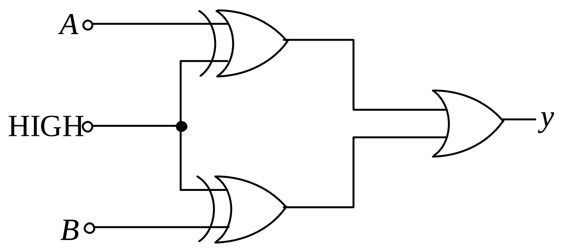
(a) $5\sqrt{2}$ and $\pi/2$

(b) $5\sqrt{2}$ and $-\pi/2$

(c) 10 and zero

(d) 10 and π

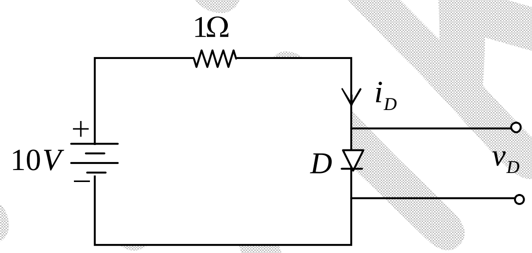
Q44. The logic circuit shown in the figure below Implements the Boolean expression



(a) $y = \overline{A \cdot B}$ (b) $y = \overline{A} \cdot \overline{B}$ (c) $y = A \cdot B$ (d) $y = A + B$

Q45. A diode D as shown in the circuit has an i-v relation that can be approximated by

$$i_D = \begin{cases} v_D^2 + 2v_D, & \text{for } v_D > 0 \\ 0, & \text{for } v_D \leq 0 \end{cases}$$



The value of v_D in the circuit is

(a) $\left(-1+\sqrt{11}\right)V$ (b) 8 V (c) 5 V (d) 2 V

Q46. The Taylor expansion of the function ln(cosh x), where x is real, about the point x = 0 starts with the following terms:

(a) $-\frac{1}{2}x^2 + \frac{1}{12}x^4 + ...$ (b) $\frac{1}{2}x^2 - \frac{1}{12}x^4 + ...$

(c) $-\frac{1}{2}x^2 + \frac{1}{6}x^4 + ...$ (d) $\frac{1}{2}x^2 + \frac{1}{6}x^4 + ...$

Q47. Given a 2×2 unitary matrix U satisfying $U^{\dagger}U = UU^{\dagger} = 1$ with $\det U = e^{i\varphi}$, one can construct a unitary matrix $V\left(V^{\dagger}V = VV^{\dagger} = 1\right)$ with det V = 1 from it by

(a) multiplying U by $e^{-i\varphi/2}$

(b) multiplying any single element of U by $e^{-i\varphi}$

(c) multiplying any row or column of U by $e^{-i\varphi/2}$

(d) multiplying U by $e^{-i\varphi}$

Q48. The value of the integral $\int_C \frac{z^3 dz}{z^2 - 5z + 6}$, where C is a closed contour defined by the equation $2|z| - 5 = 0$, traversed in the anti-clockwise direction, is

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

Q49. The function $f(x)$ obeys the differential equation $\frac{d^2 f}{dx^2} - (3 - 2i)f = 0$ and satisfies the conditions $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$. The value of $f(\pi)$ is

(a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi i}$

Q50. A planet of mass m moves in the gravitational field of the Sun (mass M). If the semi-major and semi-minor axes of the orbit are a and b respectively, the angular momentum of the planet is

(a) $\sqrt{2GMm^2(a+b)}$ (b) $\sqrt{2GMm^2(a-b)}$

(c) $\sqrt{\frac{2GMm^2 ab}{a-b}}$ (d) $\sqrt{\frac{2GMm^2 ab}{a+b}}$

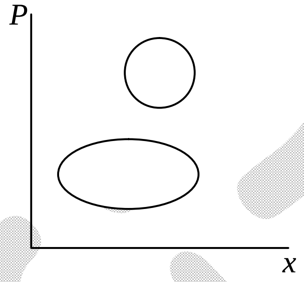
Q51. The Hamiltonian of a simple pendulum consisting of a mass m attached to a massless string of length l is $H = \frac{p_\theta^2}{2ml^2} + mgl(1-\cos\theta)$. If L denotes the Lagrangian, the value of $\frac{dL}{dt}$ is:

(a) $-\frac{2g}{l} p_\theta \sin\theta$ (b) $-\frac{g}{l} p_\theta \sin 2\theta$

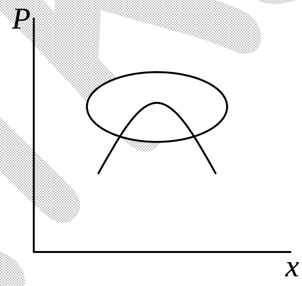
(c) $\frac{g}{l} p_\theta \cos\theta$ (d) $l p_\theta^2 \cos\theta$

Q52. Which of the following set of phase-space trajectories is not possible for a particle obeying Hamilton's equations of motion?

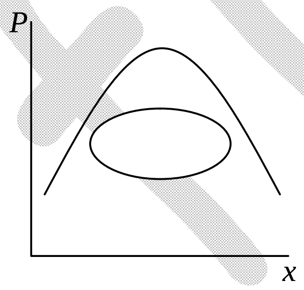
(a)



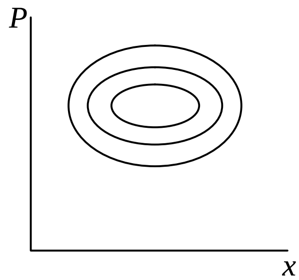
(b)



(c)



(d)

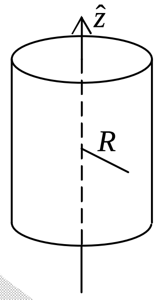


Q53. Two bodies of equal mass m are connected by a massless rigid rod of length l lying in the xy-plane with the centre of the rod at the origin. If this system is rotating about the z-axis with a frequency ω, its angular momentum is

(a) $ml^2\omega/4$ (b) $ml^2\omega/2$ (c) $ml^2\omega$ (d) $2ml^2\omega$

Q54. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current I.

The vector potential $\vec{A}$ at a distance R from the axis



(a) is constant inside and varies as R outside the solenoid

(b) varies as R inside and is constant outside the solenoid

(c) varies as $\frac{1}{R}$ inside and as R outside the solenoid

(d) varies as R inside and as $\frac{1}{R}$ outside the solenoid

Q55. Consider an infinite conducting sheet in the xy-plane with a time dependent current density $Kt\hat{i}$, where K is a constant. The vector potential at (x, y, z) is given by $\dot{A} = \frac{\mu_0 K}{4c}(ct - z)^2 \hat{i}$. The magnetic field $\vec{B}$ is

(a) $\frac{\mu_0 K t}{2}\hat{j}$ (b) $-\frac{\mu_0 K z}{2c}\hat{j}$

(c) $-\frac{\mu_0 K}{2c}(ct - z)\hat{i}$ (d) $-\frac{\mu_0 K}{2c}(ct - z)\hat{j}$

Q56. When a charged particle emits electromagnetic radiation, the electric field $\vec{E}$ and the Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ at a larger distance r from emitter vary as $\frac{1}{r^n}$ and $\frac{1}{r^m}$ respectively. Which of the following choices for n and m are correct?

(a) $n = 1$ and $m = 1$ (b) $n = 2$ and $m = 2$

(c) $n = 1$ and $m = 2$ (d) $n = 2$ and $m = 4$

Q57. The energies in the ground state and first excited state of a particle of mass $m = \frac{1}{2}$ in a potential $V(x)$ are -4 and -1, respectively, (in units in which ħ = 1). If the corresponding wavefunctions are related by $\psi_1(x) = \psi_0(x)\sinh x$, then the ground state eigenfunction is

(a) $\psi_0(x) = \sqrt{\sec hx}$ (b) $\psi_0(x) = \sec hx$

(c) $\psi_0(x) = \sec h^2 x$ (d) $\psi_0(x) = \sec h^3 x$

Q58. The perturbation

$$H' = \begin{cases} b(a-x), & -a < x < a \\ 0, & \text{otherwise} \end{cases}$$

acts on a particle of mass m confined in an infinite square well potential

$$V(x) = \begin{cases} 0, & -a < x < a \\ \infty, & \text{otherwise} \end{cases}$$

The first order correction to the ground state energy of the particle is

(a) $\frac{ba}{2}$ (b) $\frac{ba}{\sqrt{2}}$ (c) $2ba$ (d) ba

Q59. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty Δx in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is

(a) $\Delta x = \sqrt{\hbar / 2m\omega}$ (b) $\Delta x = \sqrt{\hbar / m\omega}$

(c) $\Delta x = \sqrt{2\hbar / m\omega}$ (d) $\Delta x = \sqrt{4\hbar / m\omega}$

Q60. What would be the ground state energy of the Hamiltonian

$$H=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}-\alpha\delta(x)$$

if variational principle is used to estimate it with the trial wavefunction $\psi(x)=Ae^{-bx^2}$ with b as the variational parameter?

[Hint: $\int_{-\infty}^{\infty} x^{2n}e^{-2bx^2}dx=(2b)^{-n-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)$]

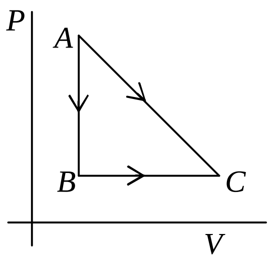
(a) $-m\alpha^2/2\hbar^2$ (b) $-2m\alpha^2/\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $m\alpha^2/\pi\hbar^2$

Q61. The free energy difference between the superconducting and the normal states of a material is given by $\Delta F=F_S-F_N=\alpha|\psi|^2+\frac{\beta}{2}|\psi|^4$, where ψ is an order parameter and α and β are constants such that $\alpha>0$ in the normal and $\alpha<0$ in the superconducting state, while $\beta>0$ always. The minimum value of ΔF is

(a) $-\alpha^2/\beta$ (b) $-\alpha^2/2\beta$ (c) $-3\alpha^2/2\beta$ (d) $-5\alpha^2/2\beta$

Q62. A given quantity of gas is taken from the state A → C reversibly, by two paths, A → C directly and A → B → C as shown in the figure.

During the process A → C the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process A → B → C the work done by the gas is 30 J, the heat absorbed is



(a) 20 J (b) 80 J

(c) 220 J (d) 280 J

Q63. Consider a one-dimensional Ising model with N spins, at very low temperatures when almost all spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy $2J$. In a configuration with r spin flips, the energy of the system is $E = -NJ + 2rJ$ and the number of configuration is ${}^{N}C_r ; r$ varies from 0 to N. The partition function is

(a) $\left(\frac{J}{k_B T}\right)^N$

(b) $e^{-NJ/k_B T}$

(c) $\left(\sinh \frac{J}{k_B T}\right)^N$

(d) $\left(\cosh \frac{J}{k_B T}\right)^N$

Q64. A magnetic field sensor based on the Hall Effect is to be fabricated by implanting As into a Si film of thickness 1 μm. The specifications require a magnetic field sensitivity of 500 mV/Tesla at an excitation current of 1 mA. The implantation dose is to be adjusted such that the average carrier density, after activation, is

(a) 1.25×10^{26} m^{-3}

(b) 1.25×10^{22} m^{-3}

(c) 4.1×10^{21} m^{-3}

(d) 4.1×10^{20} m^{-3}

Q65. Band-pass and band-reject filters can be implemented by combining a low pass and a high pass filter in series and in parallel, respectively. If the cut-off frequencies of the low pass and high pass filters are ω_0^{LP} and ω_0^{HP}, respectively, the condition required to implement the band-pass and band-reject filters are, respectively,

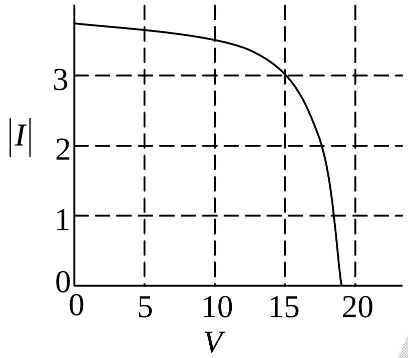
(a) $\omega_0^{HP} < \omega_0^{LP}$ and $\omega_0^{HP} < \omega_0^{LP}$

(b) $\omega_0^{HP} < \omega_0^{LP}$ and $\omega_0^{HP} > \omega_0^{LP}$

(c) $\omega_0^{HP} > \omega_0^{LP}$ and $\omega_0^{HP} < \omega_0^{LP}$

(d) $\omega_0^{HP} > \omega_0^{LP}$ and $\omega_0^{HP} > \omega_0^{LP}$

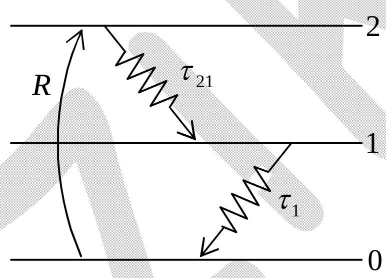
Q66. The output characteristics of a solar panel at a certain level of irradiance is shown in the figure below.



If the solar cell is to power a load of 5 Ω, the power drawn by the load is

(a) 97 W (b) 73 W (c) 50 W (d) 45 W

Q67. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate R is 10^{20} atoms cm^{-3} s^{-1} and the decay routes are as shown with $\tau_{21} = 20$ ns and $\tau_1 = 1\mu s$, the equilibrium populations of states 2 and 1 are, respectively,

(a) 10^{14} cm^{-3} and 2×10^{12} cm^{-3} (b) 2×10^{12} cm^{-3} and 10^{14} cm^{-3}.

(c) 2×10^{12} cm^{-3} and 2×10^{6} cm^{-3} (d) zero and 10^{20} cm^{-3}

Q68. Consider a hydrogen atom undergoing a $2P \rightarrow 1S$ transition. The lifetime t_{sp} of the 2P state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV. Assuming that the refractive index of the medium $n_0 = 1$, the ratio of Einstein coefficients for stimulated and spontaneous emission $B_{21}(\omega)/A_{21}(\omega)$ is given by

(a) 0.683×10^{12} $m^3J^{-1}s^{-1}$ (b) 0.146×10^{-12} Jsm^{-3}.

(c) 6.83×10^{12} $m^3J^{-1}s^{-1}$ (d) 1.463×10^{-12} Jsm^{-3}.

Q69. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0.98$. The mirrors are separated by a distance $d = 20$ cm and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes δv and the width Δv_p of each mode of the laser cavity are:

(a) $\delta v = 75kHz, \Delta v_p = 24kHz$ (b) $\delta v = 100kHz, \Delta v_p = 100kHz$

(c) $\delta v = 750MHz, \Delta v_p = 2.4MHz$ (d) $\delta v = 2.4MHz, \Delta v_p = 750MHz$

Q70. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to

(a) $\hbar\omega/2$ (b) $\hbar\omega$ (c) $3\hbar\omega/2$ (d) 0

Q71. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta(\cos k_x a + \cos k_y a + \cos k_z a),$$

where β is a constant and a is the lattice constant. The effective mass at the boundary of the first Brillouin zone is

(a) $\frac{2\hbar^2}{5\beta a^2}$ (b) $\frac{4\hbar^2}{5\beta a^2}$ (c) $\frac{\hbar^2}{2\beta a^2}$ (d) $\frac{\hbar^2}{3\beta a^2}$

Q72. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is a^3, is

(a) $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$ (b) $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$ (c) $\left(\frac{\pi^2}{a^3}\right)^{1/3}$ (d) $\frac{1}{a}$

Q73. The muon has mass 105 MeV/c^2 and mean lifetime 2.2 μs in its rest frame. The mean distance traversed by a muon of energy 315 MeV/c^2 before decaying is approximately

(a) 3×10^5 km (b) 2.2 cm (c) 6.6 μm (d) 1.98 km

Q74. Consider the following particles: the proton p, the neutron n, the neutral pion π^0 and the delta resonance Δ^+. When ordered in terms of decreasing lifetime, the correct arrangement is as follows:

(a) π^0, n, p, Δ^+ .
(b) p, n, Δ^+, π^0.
(c) p, n, π^0, Δ^+.
(d) Δ^+, n, π^0, p

Q75. The single particle energy difference between the p-orbitals (i.e. $P_{3/2}$ and $P_{1/2}$) of the nucleus $^{114}_{50}Sn$ is 3 MeV. The energy difference between the states in its $1f$ orbitals is

(a) -7 MeV
(b) 7 MeV
(c) 5 MeV
(d) -5 MeV