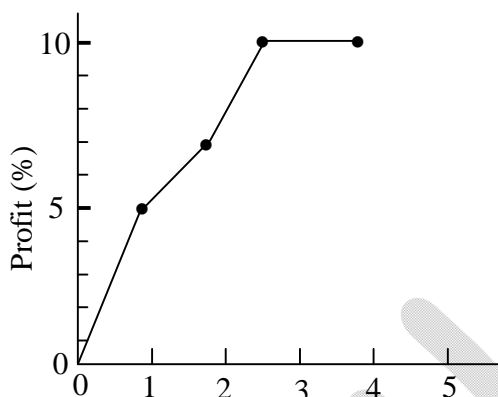


NET JUNE- 2011(Physical Sciences)PART A

1.



The cumulative profits of a company since its inception are shown in the diagram. If the net worth of the company at the end of 4th year is 99 crores, the principal it had started with was

- (a) 9.9 crores (b) 91 crores (c) 90 crores (d) 9.0 crores

2. Popular use of which of the following fertilizers increases the acidity of soil?

- (a) Potassium Nitrate (b) Ammonium sulphate
(c) Urea (d) Superphosphate of lime

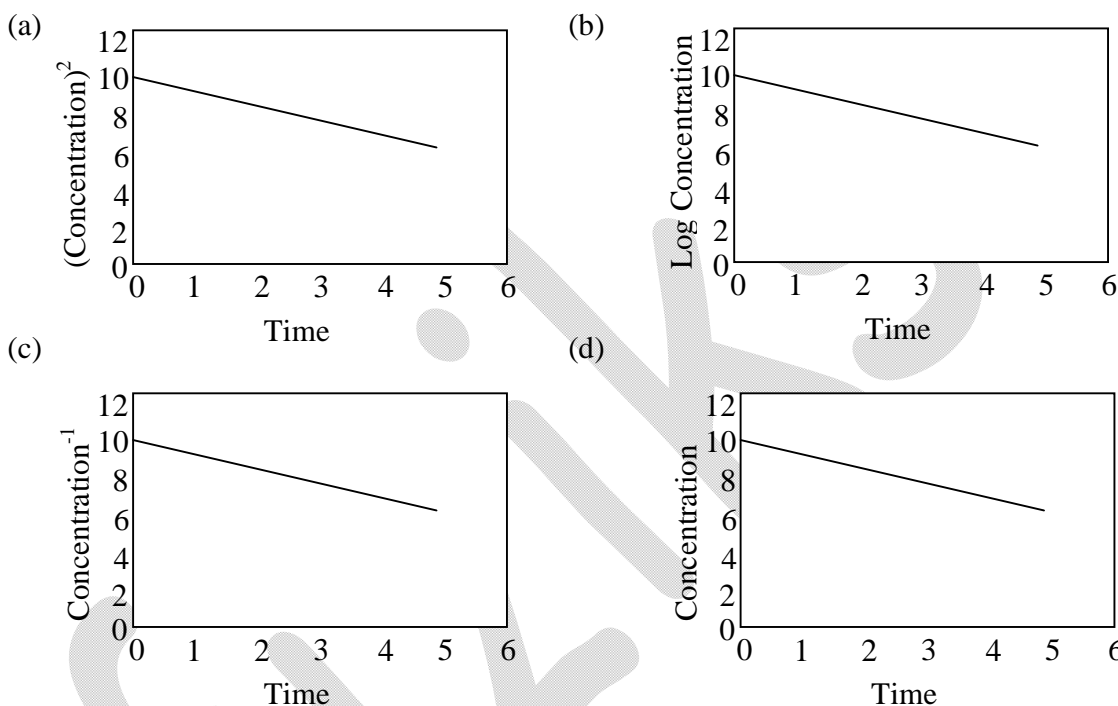
3. Exposing an organism to a certain chemical can change nucleotide bases in a gene, causing mutation. In one such mutated organism if a protein had only 70% of the primary amino acid sequence, which of the following is likely?

- (a) Mutation broke the protein
(b) The organism could not make amino acids
(c) Mutation created a terminator codon
(d) The gene was not transcribed

4. A reference material is required to be prepared with 4 ppm calcium. The amount of CaCO_3 (molecular weight = 100) required to prepare 1000 g of such a reference material is

- (a) 10 μg (b) 4 μg (c) 4 mg (d) 10mg

5. Identify the figure which depicts a first order reaction.

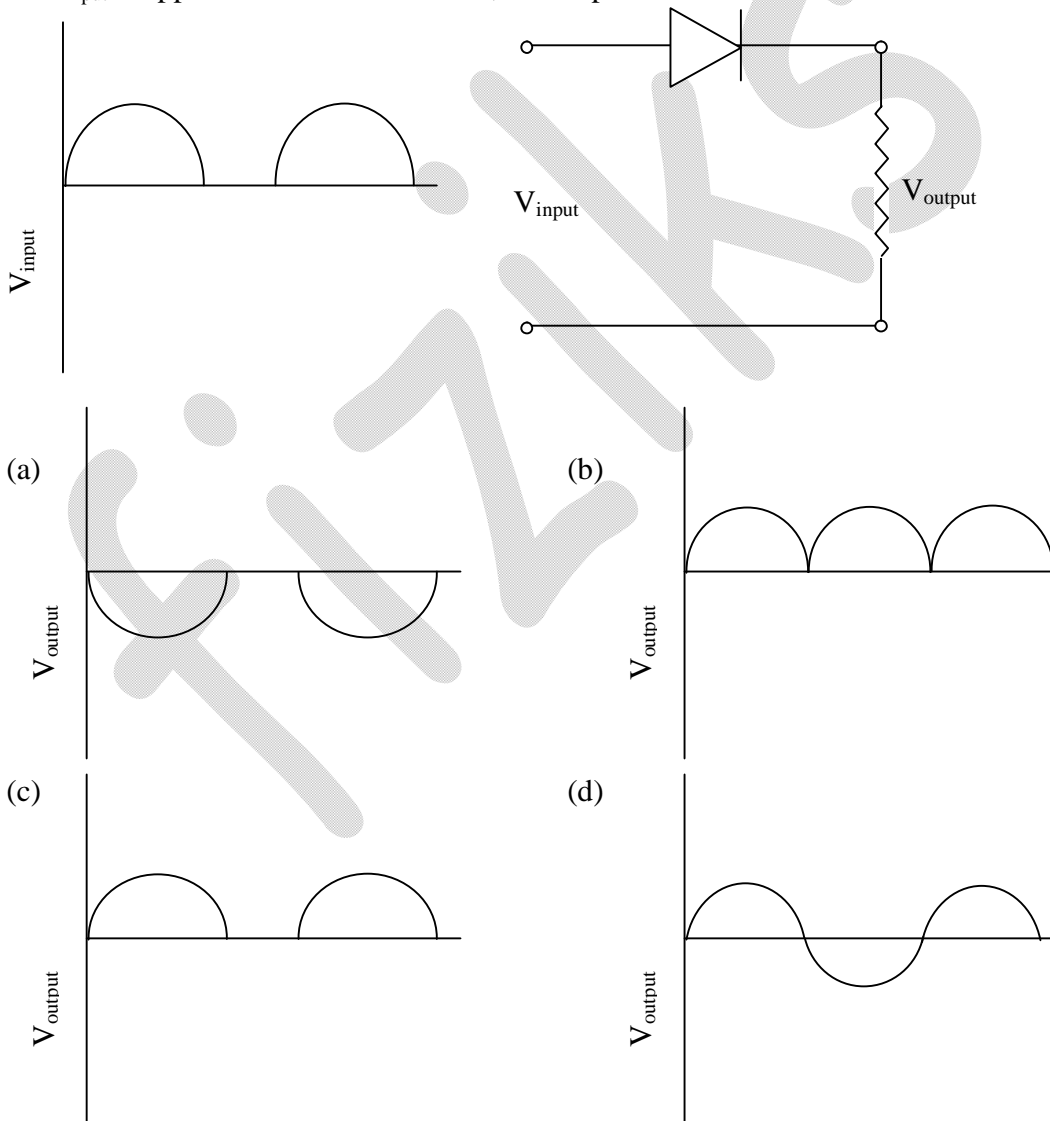


6. The speed of a car increases every minute as shown in the following Table. The speed at the end of the 19th minute would be

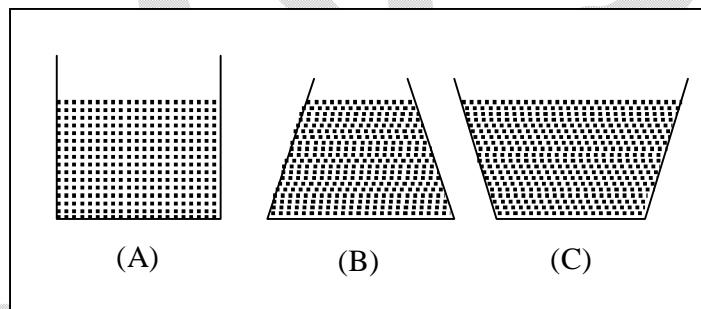
Time (minutes)	Speed (m/sec)
1	1.5
2	3.0
3	4.5
-	-
-	-
24	36.0
25	37.5

- (a) 26.5 (b) 28.0 (c) 27.0 (d) 28.5

7. If the atmospheric concentration of carbon dioxide is doubled and there are favourable conditions of water, nutrients, light and temperature, what would happen to water requirement of plants?
- It decreases initially for a short time and then returns to the original value
 - It increases
 - It decreases
 - It increases initially for a short time and then returns to the original value
8. If V_{input} is applied to the circuit shown, the output would be



9. A physiological disorder X always leads to the disorder Y. However, disorder Y may occur by itself. A population shows 4% incidence of disorder Y. Which of the following inferences is valid?
- (a) 4% of the population suffers from both X & Y
(b) Less than 4% of the population suffers from X
(c) At least 4% of the population suffers from X
(d) There is no incidence of X in the given population
10. Water is dripping out of a tiny hole at the bottom of three flasks whose base diameter is the same, and is initially filled to the same height, as shown

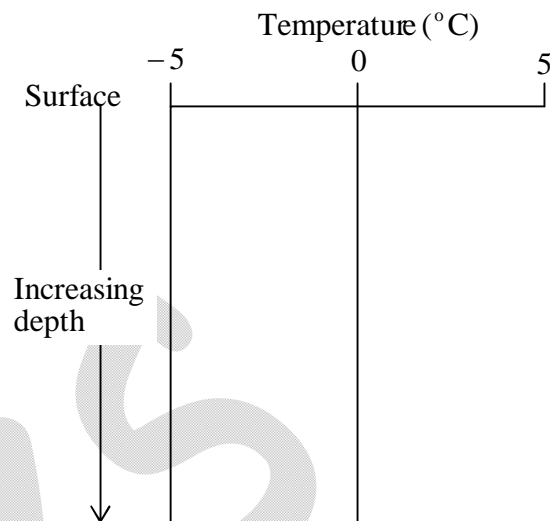


Which is the correct comparison of the rate of fall of the volume of water in the three flasks?

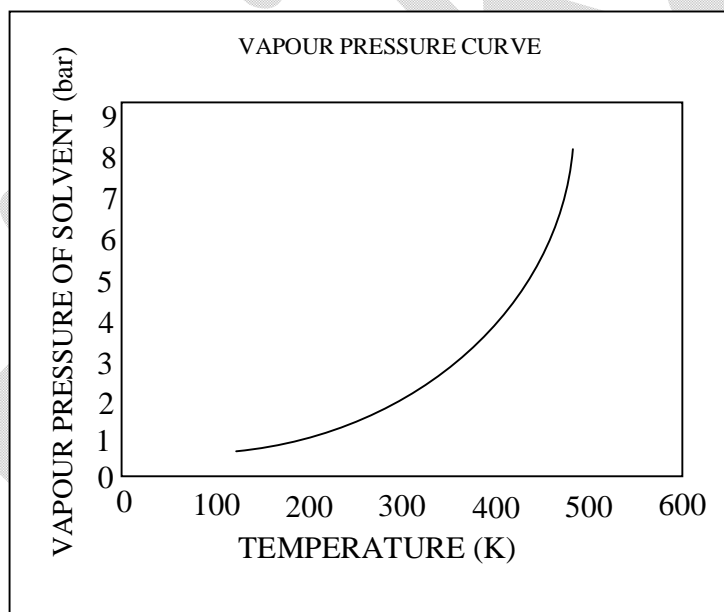
- (a) A fastest, B slowest
(b) B fastest, A slowest
(c) B fastest, C slowest
(d) C fastest, B slowest
11. How many σ bonds are present in the following molecule?
 $\text{HC} \equiv \text{CCH} = \text{CHCH}_3$
- (a) 4 (b) 10 (c) 6 (d) 13

12. The graph represents the depth profile of temperature in the open ocean; in which region this is likely to be prevalent?

- (a) Tropical region
- (b) Equatorial region
- (c) Polar region
- (d) Sub-tropical region



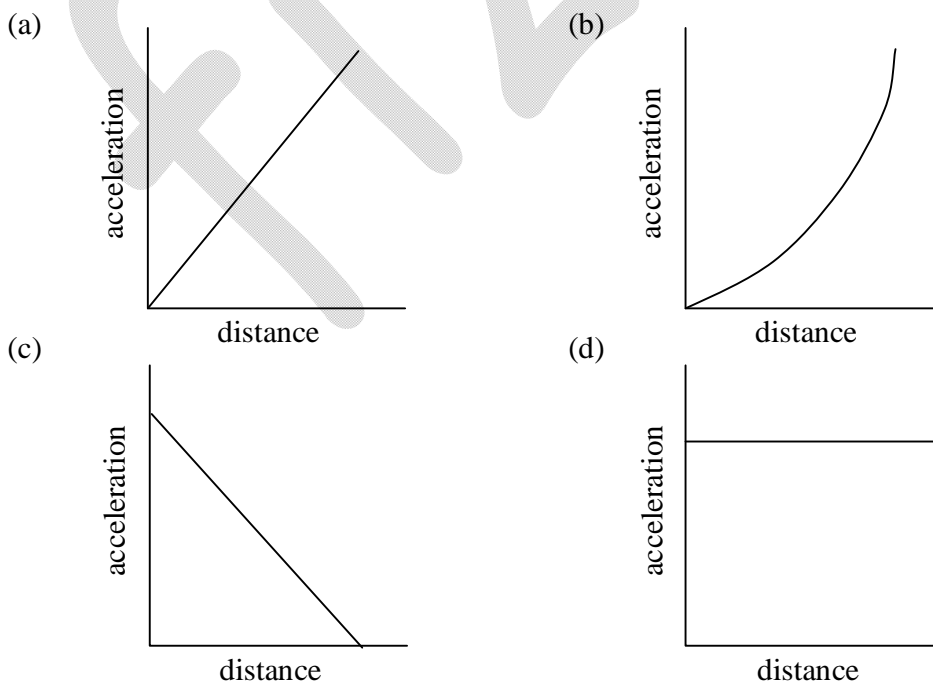
13.



The normal boiling point of a solvent (whose vapour pressure curve is shown in the figure) on a planet whose normal atmospheric pressure is 3 bar, is about

- (a) 400 K
- (b) 273 K
- (c) 100 K
- (d) 500 K

14. Diabetic patients are advised a low glycaemic index diet. The reason for this is
- They require less carbohydrate than healthy individuals
 - They cannot assimilate ordinary carbohydrates
 - They need to have slow, but sustained release of glucose in their blood stream
 - They can tolerate lower, but not higher than normal blood sugar levels
15. Glucose molecules diffuse across a cell of diameter d in time τ . If the cell diameter is tripled, the diffusion time would
- increase to 9τ
 - decrease to $\tau/3$
 - increase to 3τ
 - decrease to $\tau/9$
16. The reason for the hardness of diamond is
- extended covalent bonding
 - layered structure
 - formation of cage structures
 - formation of tubular structures
17. Which of the following particles has the largest range in a given medium if their initial energies are the same?
- alpha
 - gamma
 - positron
 - electron
18. A ball is dropped from a height h above the surface of the earth. Ignoring air drag, the curve that best represents its variation of acceleration is



19. Standing on a polished stone floor one feels colder than on a rough floor of the same stone. This is because
- (a) Thermal conductivity of the stone depends on the surface smoothness
 - (b) Specific heat of the stone changes by polishing it
 - (c) The temperature of the polished floor is lower than that of the rough floor
 - (d) There is greater heat loss from the soles of the feet when in contact with the polished floor than with the rough floor
20. The acidity of normal rain water is due to
- (a) SO_2
 - (b) CO_2
 - (c) NO_2
 - (d) NO

PART B

21. A particle of unit mass moves in a potential $V(x) = ax^2 + \frac{b}{x^2}$, where a and b are positive constants. The angular frequency of small oscillations about the minimum of the potential is
- (a) $\sqrt{8b}$
 - (b) $\sqrt{8a}$
 - (c) $\sqrt{8a/b}$
 - (d) $\sqrt{8b/a}$
22. A signal of frequency 10 kHz is being digitized by an A/D converter. A possible sampling time which can be used is
- (a) 100 μs
 - (b) 40 μs
 - (c) 60 μs
 - (d) 200 μs
23. The electrostatic potential $V(x, y)$ in free space in a region where the charge density ρ is zero is given by
- $$V(x, y) = 4e^{2x} + f(x) - 3y^2.$$
- Given that the x -component of the electric field E_x , and V are zero at the origin, $f(x)$ is
- (a) $3x^2 - 4e^{2x} + 8x$
 - (b) $3x^2 - 4e^{2x} + 16x$
 - (c) $4e^{2x} - 8$
 - (d) $3x^2 - 4e^{2x}$

28. Let \vec{a} and \vec{b} be two distinct three dimensional vectors. Then the component of \vec{b} that is perpendicular to \vec{a} is given by

- (a) $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{a^2}$ (b) $\frac{\vec{b} \times (\vec{a} \times \vec{b})}{b^2}$ (c) $\frac{(\vec{a} \cdot \vec{b})\vec{b}}{b^2}$ (d) $\frac{(\vec{b} \cdot \vec{a})\vec{a}}{a^2}$

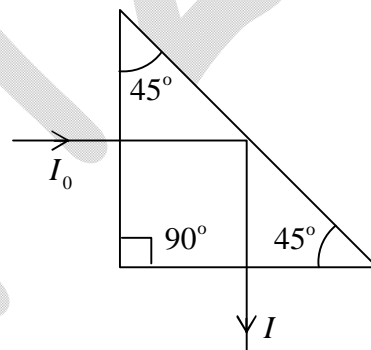
29. The wavefunction of a particle is given by $\psi = \left(\frac{1}{\sqrt{2}}\phi_0 + i\phi_1 \right)$, where ϕ_0 and ϕ_1 are the normalized eigenfunctions with energies E_0 and E_1 corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state ψ is

- (a) $\frac{E_0}{2} + E_1$ (b) $\frac{E_0}{2} - E_1$ (c) $\frac{E_0 - 2E_1}{3}$ (d) $\frac{E_0 + 2E_1}{3}$

30. A particle is confined to the region $x \geq 0$ by a potential which increases linearly as $u(x) = u_0x$. The mean position of the particle at temperature T is

- (a) $\frac{k_B T}{u_0}$ (b) $(k_B T)^2 / u_0$ (c) $\sqrt{\frac{k_B T}{u_0}}$ (d) $u_0 k_B T$

31. Circularly polarized light with intensity I_0 is incident normally on a glass prism as shown in the figure. The index of refraction of glass is 1.5. The intensity I of light emerging from the prism is



- (a) I_0 (b) $0.96 I_0$ (c) $0.92 I_0$ (d) $0.88 I_0$

32. The acceleration due to gravity (g) on the surface of Earth is approximately 2.6 times that on the surface of Mars. Given that the radius of Mars is about one half the radius of Earth, the ratio of the escape velocity on Earth to that on Mars is approximately

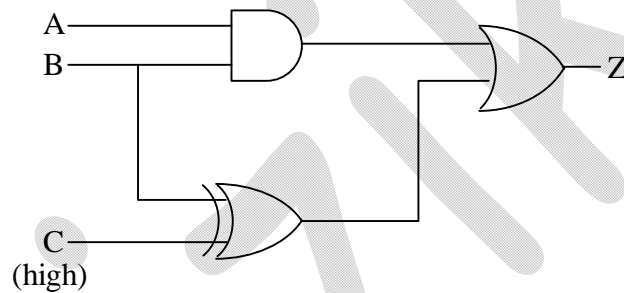
- (a) 1.1 (b) 1.3 (c) 2.3 (d) 5.2

33. A plane electromagnetic wave is propagating in a lossless dielectric. The electric field is given by

$$\vec{E}(x, y, z, t) = E_0(\hat{x} + A\hat{z})\exp\left[ik_0\left\{-ct + (x + \sqrt{3}z)\right\}\right]$$

where c is the speed of light in vacuum, E_0 , A and k_0 are constant and \hat{x} and \hat{z} are unit vectors along the x - and z -axes. The relative dielectric constant of the medium ϵ_r and the constant A are

- (a) $\epsilon_r = 4$ and $A = -\frac{1}{\sqrt{3}}$ (b) $\epsilon_r = 4$ and $A = +\frac{1}{\sqrt{3}}$
 (c) $\epsilon_r = 4$ and $A = \sqrt{3}$ (d) $\epsilon_r = 4$ and $A = -\sqrt{3}$
34. Consider the digital circuit shown below in which the input C is always high (1).



The truth table for the circuit can be written as

A	B	Z
0	0	
0	1	
1	0	
1	1	

The entries in the Z column (vertically) are

- (a) 1010 (b) 0100 (c) 1111 (d) 1011

35. The energy levels of the non-relativistic electron in a hydrogen atom (i.e. in a Coulomb potential $V(r) \propto -1/r$) are given by $E_{nlm} \propto -1/n^2$, where n is the principal quantum number, and the corresponding wave functions are given by ψ_{nlm} , where l is the orbital angular momentum quantum number and m is the magnetic quantum number. The spin of the electron is not considered. Which of the following is a correct statement?
- (a) There are exactly $(2l + 1)$ different wave functions ψ_{nlm} , for each E_{nlm} .
 (b) There are $l(l + 1)$ different wave functions ψ_{nlm} , for each E_{nlm} .
 (c) E_{nlm} does not depend on l and m for the Coulomb potential.
 (d) There is a unique wave function ψ_{nlm} for each E_{nlm} .
36. The Hamiltonian of an electron in a constant magnetic field \vec{B} is given by $H = \mu \vec{\sigma} \cdot \vec{B}$, where μ is a positive constant and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ denotes the Pauli matrices. Let $\omega = \mu B / \hbar$ and I be the 2×2 unit matrix. Then the operator $e^{iHt/\hbar}$ simplifies to
- (a) $I \cos \frac{\omega t}{2} + \frac{i \vec{\sigma} \cdot \vec{B}}{B} \sin \frac{\omega t}{2}$ (b) $I \cos \omega t + \frac{i \vec{\sigma} \cdot \vec{B}}{B} \sin \omega t$
 (c) $I \sin \omega t + \frac{i \vec{\sigma} \cdot \vec{B}}{B} \cos \omega t$ (d) $I \sin 2\omega t + \frac{i \vec{\sigma} \cdot \vec{B}}{B} \cos 2\omega t$
37. The Hamiltonian of a system with n degrees of freedom is given by $H(q_1, \dots, q_n; p_1, \dots, p_n; t)$, with an explicit dependence on the time t . Which of the following is correct?
- (a) Different phase trajectories cannot intersect each other.
 (b) H always represents the total energy of the system and is a constant of the motion.
 (c) The equations $\dot{q}_i = \partial H / \partial p_i$, $\dot{p}_i = -\partial H / \partial q_i$ are not valid since H has explicit time dependence.
 (d) Any initial volume element in phase space remains unchanged in magnitude under time evolution.

38. If the perturbation $H' = ax$, where a is a constant, is added to the infinite square well potential

$$V(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq \pi \\ \infty & \text{otherwise.} \end{cases}$$

The correction to the ground state energy, to first order in a , is

- (a) $\frac{a\pi}{2}$ (b) $a\pi$ (c) $\frac{a\pi}{4}$ (d) $\frac{a\pi}{\sqrt{2}}$

39. Let $p_n(x)$ (where $n = 0, 1, 2, \dots$) be a polynomial of degree n with real coefficients,

defined in the interval $2 \leq n \leq 4$. If $\int_2^4 p_n(x)p_m(x)dx = \delta_{nm}$, then

(a) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(-3-x)$ (b) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}(3+x)$

(c) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$ (d) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$

40. A cavity contains blackbody radiation in equilibrium at temperature T . The specific heat per unit volume of the photon gas in the cavity is of the form $C_V = \gamma T^3$, where γ is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature T . The new internal energy per unit volume is

- (a) $4\gamma T^4$ (b) $2\gamma T^4$ (c) γT^4 (d) $\frac{\gamma T^4}{4}$

PART C (COMPULSORY)

41. Consider a system of N non-interacting spins, each of which has classical magnetic moment of magnitude μ . The Hamiltonian of this system in an external magnetic field \vec{H} is $\sum_{i=1}^N \vec{\mu}_i \cdot \vec{H}$, where $\vec{\mu}_i$ the magnetic moment of the i^{th} spin. The magnetization per spin at

temperature T is

- (a) $\frac{\mu^2 H}{k_B T}$ (b) $\mu \left[\coth\left(\frac{\mu H}{k_B T}\right) - \frac{k_B T}{\mu H} \right]$
 (c) $\mu \sinh\left(\frac{\mu H}{k_B T}\right)$ (d) $\mu \tanh\left(\frac{\mu H}{k_B T}\right)$

42. Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$?

- (a) $(3 + x - iy)^7$ (b) $(1 + x + iy)^4 (7 - x - iy)^3$
 (c) $(1 - 2x - iy)^4 (3 - x - iy)^3$ (d) $(x + iy - 1)^{1/2}$

43. A particle in one dimension moves under the influence of a potential $V(x) = ax^6$, where a is a real constant. For large n the quantized energy level E_n depends on n as:

- (a) $E_n \sim n^3$ (b) $E_n \sim n^{4/3}$ (c) $E_n \sim n^{6/5}$ (d) $E_n \sim n^{3/2}$

44. The Lagrangian of a particle of charge e and mass m in applied electric and magnetic fields is given by $L = \frac{1}{2} m \vec{v}^2 + e \vec{A} \cdot \vec{v} - e\phi$, where \vec{A} and ϕ are the vector and scalar

potentials corresponding to the magnetic and electric fields, respectively. Which of the following statements is correct?

- (a) The canonically conjugate momentum of the particle is given by $\vec{p} = m\vec{v}$
 (b) The Hamiltonian of the particle is given by $H = \frac{\vec{p}^2}{2m} + \frac{e}{m} \vec{A} \cdot \vec{p} + e\phi$
 (c) L remains unchanged under a gauge transformation of the potentials
 (d) Under a gauge transformation of the potentials, L changes by the total time derivative of a function of \vec{r} and t

45. A static, spherically symmetric charge distribution is given by $\rho(r) = \frac{A}{r} e^{-Kr}$ where A and K are positive constants. The electrostatic potential corresponding to this charge distribution varies with r as

- (a) re^{-Kr} (b) $\frac{1}{r} re^{-Kr}$ (c) $\frac{1}{r^2} re^{-Kr}$ (d) $\frac{1}{r} (1 - e^{-Kr})$

46. Consider two independently diffusing non-interacting particles in 3-dimensional space, both placed at the origin at time $t = 0$. These particles have different diffusion constants D_1 and D_2 . The quantity $\langle [\vec{R}_1(t) - \vec{R}_2(t)]^2 \rangle$ where $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are the positions of the particles at time t , behaves as:

- (a) $6t(D_1 + D_2)$ (b) $6t(D_1 - D_2)$ (c) $6t\sqrt{D_1^2 + D_2^2}$ (d) $6t\sqrt{D_1 D_2}$

47. A resistance is measured by passing a current through it and measuring the resulting voltage drop. If the voltmeter and the ammeter have uncertainties of 3% and 4%, respectively, then

A. The uncertainty in the value of the resistance is

- (a) 7.0 % (b) 3.5 % (c) 5.0 % (d) 12.0 %

B. The uncertainty in the computed value of the power dissipated in the resistance is

- (a) 7% (b) 5% (c) 11% (d) 9%

48. In the absence of an applied torque a rigid body with three distinct principal moments of inertia given by I_1 , I_2 and I_3 is rotating freely about a fixed point inside the body. The Euler equations for the components of its angular velocity ($\omega_1, \omega_2, \omega_3$) are

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1} \omega_2 \omega_3, \quad \dot{\omega}_2 = \frac{I_3 - I_1}{I_2} \omega_1 \omega_3, \quad \dot{\omega}_3 = \frac{I_1 - I_2}{I_3} \omega_1 \omega_2$$

A. The equilibrium points in $(\omega_1, \omega_2, \omega_3)$ space are

- (a) (1, -1, 0), (-1, 0, 1) and (0, -1, 1)
 (b) (1, 1, 0), (1, 0, 1) and (0, 1, 1)
 (c) (1, 0, 0), (0, 1, 0) and (0, 0, 1)
 (d) (1, 1, 1), (-1, -1, -1) and (0, 0, 0)

B. The constants of motion are

- (a) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1\omega_1 + I_2\omega_2 + I_3\omega_3$
 (b) $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$
 (c) $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ and $\omega_1 + \omega_2 + \omega_3$
 (d) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

49. In a system consisting of two spin $\frac{1}{2}$ particles labeled 1 and 2, let $\vec{S}^{(1)} = \frac{\hbar}{2}\vec{\sigma}^{(1)}$ and $\vec{S}^{(2)} = \frac{\hbar}{2}\vec{\sigma}^{(2)}$ denote the corresponding spin operators. Here $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ and $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices.

A. In the standard basis the matrices for the operators $S_x^{(1)}S_y^{(2)}$ and $S_y^{(1)}S_x^{(2)}$ are respectively,

- (a) $\frac{\hbar^2}{4} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \frac{\hbar^2}{4} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
 (b) $\frac{\hbar^2}{4} \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \frac{\hbar^2}{4} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$
 (c) $\frac{\hbar^2}{4} \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \frac{\hbar^2}{4} \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$
 (d) $\frac{\hbar^2}{4} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \frac{\hbar^2}{4} \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

B. These two operators satisfy the relation

- (a) $\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\} = S_z^{(1)}S_z^{(2)}$ (b) $\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\} = 0$
 (c) $[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}] = iS_z^{(1)}S_z^{(2)}$ (d) $[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}] = 0$

50. Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

A. The eigenvalues of M are

- (a) 0, 1, 2 (b) 0, 0, 3 (c) 1, 1, 1 (d) -1, 1, 3

B. The exponential of M simplifies to (I is the 3×3 identity matrix)

(a) $e^M = I + \left(\frac{e^3 - 1}{3}\right)M$ (b) $e^M = I + M + \frac{M^2}{2!}$

(c) $e^M = I + 3^3 M$ (d) $e^M = (e - 1)M$

Choose any 10 of the remaining questions

51. The radius of a ${}^{64}_{29}\text{Cu}$ nucleus is measured to be 4.8×10^{-13} cm.

A. The radius of a ${}^{27}_{12}\text{Mg}$ nucleus can be estimated to be

- (a) 2.86×10^{-13} cm (b) 5.2×10^{-13} cm
(c) 3.6×10^{-13} cm (d) 8.6×10^{-13} cm

B. The root-mean-square (rms) energy of a nucleon in a nucleus of atomic number A in its ground state varies as:

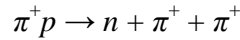
- (a) $A^{4/3}$ (b) $A^{1/3}$ (c) $A^{-1/3}$ (d) $A^{-2/3}$

52. The character table of C_{3v} the group of symmetries of an equilateral triangle is given below

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1C_1$	1	1	b
$3C_2$	1	a	c
$2C_3$	1	1	d

In the above C_1, C_2, C_3 denote the three classes of C_{3v} containing 1, 3 and 2 elements respectively, and $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters of the three irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$ of C_{3v} .

57. A beam of pions (π^+) is incident on a proton target, giving rise to the process



A. Assuming that the decay proceeds through strong interactions, the total isospin I and its third component I_3 for the decay products, are

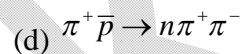
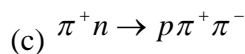
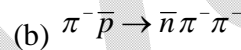
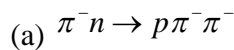
(a) $I = \frac{3}{2}, I_3 = \frac{3}{2}$

(b) $I = \frac{5}{2}, I_3 = \frac{5}{2}$

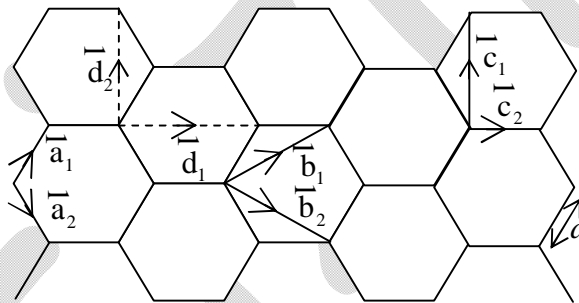
(c) $I = \frac{5}{2}, I_3 = \frac{3}{2}$

(d) $I = \frac{1}{2}, I_3 = -\frac{1}{2}$

B. Using isospin symmetry, the cross-section for the above process can be related to that of the process



58. The two dimensional lattice of graphene is an arrangement of Carbon atoms forming a honeycomb lattice of lattice spacing a , as shown below. The Carbon atoms occupy the vertices.



A. The Wigner-Seitz cell has an area of

(a) $2a^2$

(b) $\frac{\sqrt{3}}{2} a^2$

(c) $6\sqrt{3}a^2$

(d) $\frac{3\sqrt{3}}{2} a^2$

B. The Bravais lattice for this array is a

(a) rectangular lattice with basis vectors \vec{d}_1 and \vec{d}_2

(b) rectangular lattice with basis vectors \vec{c}_1 and \vec{c}_2

(c) hexagonal lattice with basis vectors \vec{a}_1 and \vec{a}_2

(d) hexagonal lattice with basis vectors \vec{b}_1 and \vec{b}_2

59. Consider the decay process $\tau^- \rightarrow \pi^- + \nu_\tau$ in the rest frame of the τ^- . The masses of the τ^- , π^- and ν_τ are M_τ , M_π and zero respectively.

A. The energy of τ^- is

- (a) $\frac{(M_\tau^2 - M_\pi^2)c^2}{2M_\tau}$ (b) $\frac{(M_\tau^2 + M_\pi^2)c^2}{2M_\tau}$
 (c) $(M_\tau - M_\pi)c^2$ (d) $\sqrt{M_\tau M_\pi}c^2$

B. The velocity of π^- is

- (a) $\frac{(M_\tau^2 - M_\pi^2)c}{M_\tau^2 + M_\pi^2}$ (b) $\frac{(M_\tau^2 + M_\pi^2)c}{M_\tau^2 - M_\pi^2}$ (c) $\frac{M_\pi c}{M_\tau}$ (d) $\frac{M_\tau c}{M_\pi}$

60. A narrow beam of X-rays with wavelength 1.5 \AA is reflected from an ionic crystal with an *fcc* lattice structure with a density of 3.32 g cm^{-3} . The molecular weight is 108 AMU ($1 \text{ AMU} = 1.66 \times 10^{-24} \text{ g}$)

A. The lattice constant is

- (a) 6.00 \AA (b) 4.56 \AA (c) 4.00 \AA (d) 2.56 \AA

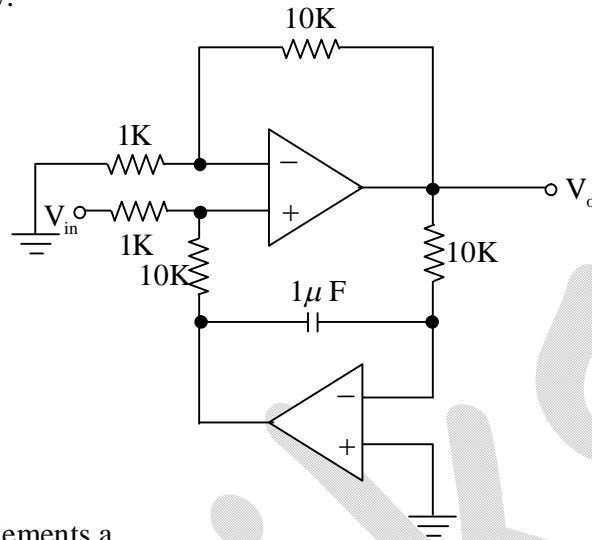
B. The sine of the angle corresponding to (111) reflection is

- (a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

61. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately

- (a) 0.68 (b) 0.48 (c) 0.28 (d) 0.91

62. A time varying signal V_{in} is fed to an op-amp circuit with output signal V_o as shown in the figure below.



The circuit implements a

- (a) high pass filter with cutoff frequency 16 Hz
 (b) high pass filter with cutoff frequency 100 Hz
 (c) low pass filter with cutoff frequency 16 Hz
 (d) low pass filter with cutoff frequency 100 Hz
63. The Hamiltonian of a particle of unit mass moving in the xy -plane is given to be:
- $$H = xp_x - yp_y - \frac{1}{2}x^2 + \frac{1}{2}y^2$$
- in suitable units. The initial values are given to be $(x(0), y(0)) = (1, 1)$ and $(p_x(0), p_y(0)) = \left(\frac{1}{2}, -\frac{1}{2}\right)$. During the motion, the curves traced out by the particles in the xy -plane and the $p_x p_y$ -plane are
- (a) both straight lines
 (b) a straight line and a hyperbola respectively
 (c) a hyperbola and an ellipse, respectively
 (d) both hyperbolas

64. Consider an ideal Bose gas in three dimensions with the energy-momentum relation $\varepsilon \propto p^s$ with $s > 0$. The range of s for which this system may undergo a Bose-Einstein condensation at a non-zero temperature is
- (a) $1 < s < 3$ (b) $0 < s < 2$ (c) $0 < s < 3$ (d) $0 < s < \infty$
65. Two gravitating bodies A and B with masses m_A and m_B , respectively, are moving in circular orbit. Assume that $m_B \gg m_A$ and let the radius of the orbit of body A be R_A . If the body A is losing mass adiabatically, its orbital radius R_A is proportional to
- (a) $\frac{1}{m_A}$ (b) $\frac{1}{m_A^2}$ (c) m_A (d) m_A^2