

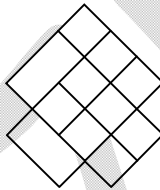
NET-JUNE-2012(Physical Science)

PART-A

- Q1. In still air, fragrance of a burning incense stick will be smelt by an observer quickest when the experiment is carried out at
- (a) low altitude and high air temperature
 - (b) high altitude and low air temperature
 - (c) low altitude and low air temperature
 - (d) high altitude and high air temperature

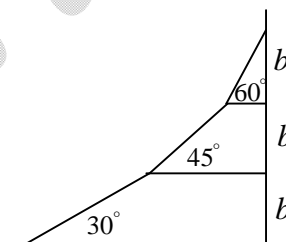
- Q2. How Many squares are there in this figure?

- (a) 9
- (b) 14
- (c) 15
- (d) 17

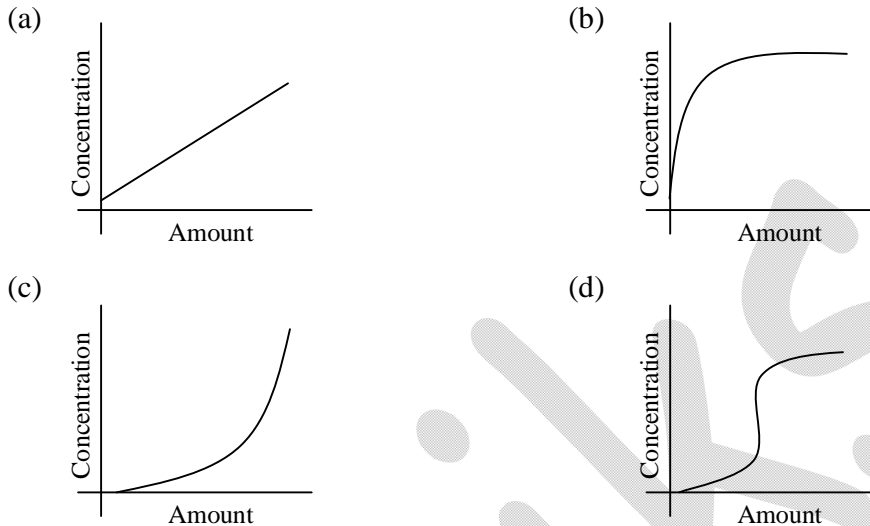


- Q3. A mountain load has 3 sections of different slopes as shown. What is the average slope m of the entire climb?

- (a) 1
- (b) $(1/3) < m < (1/2)$
- (c) $1 < m < \sqrt{3}$
- (d) $(1/\sqrt{3}) < m < 1$



Q4. Which of the following graphs shows the concentration of a sugar solution as a function of the cumulative amount of sugar added to the process of preparing a saturated solution (the temperature remaining constant)?

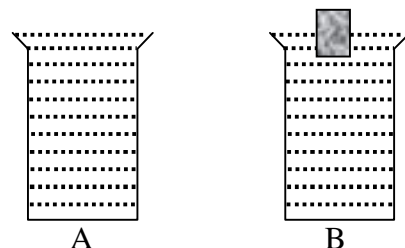


Q5. There are sand-piles which are geometrically similar but of different heights. The ratio of the mass of the sand comprising two randomly chosen piles will be equal to the ratio of the

- (a) pile heights
- (b) squares of the pile heights
- (c) cubes of the pile heights
- (d) cube-roots of the pile heights

Q6. There are two identical vessels of volume V each, one empty and the other containing a block of wood of weight w . The vessels are then filled with water up to the brim. The two arrangements are shown as A and B in the figure. If the density of water is ρ and g is the acceleration due to gravity, then

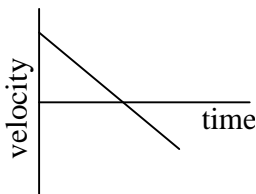
- (a) A and B have equal weights
- (b) A is heavier than B by an amount w
- (c) A is heavier than B by an amount $V\rho g - w$
- (d) B is heavier than A by an amount $V\rho g - w$



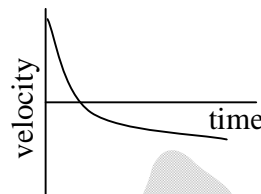
- Q7. If the father has blood group O and the mother has blood group AB, what are the possible blood groups of their children?
- (a) O, AB, A (b) A, B
(c) A, O (d) B, AB
- Q8. Nuclei of ^{32}P and ^{32}S , accelerated through the same potential difference enter a uniform, transverse magnetic field ($Z = 15$ for P and $Z = 16$ for S). As they emerge from the magnetic field
- (a) both nuclei emerge undeflected
(b) ^{32}P is deflected less than ^{32}S
(c) ^{32}P is deflected more than ^{32}S
(d) both are equally deflected
- Q9. A person chewing a bubble gum did not experience ear pain in a jet plane while landing whereas another person not chewing a gum had ear pain. The reason could be
- (a) chewing gum is a pain killer
(b) chewing equilibrates pressure on both sides of the ear drum
(c) chewing gum closes the ear drum
(d) chewing distracts the person
- Q10. The reason why a lunar eclipse does not occur at every full moon is
- (a) the position of the sun is not favourable at all full moons
(b) the orbital planes of the moon and that of the earth are inclined to each other by a small angle
(c) the shape of the earth is not a perfect sphere
(d) the moon reflects only from one hemisphere

Q11. A boy throws a stone vertically upwards with a certain initial velocity. Which of the following graphs depicts the velocity as a function of time, if the acceleration due to gravity is assumed to be uniform and constant?

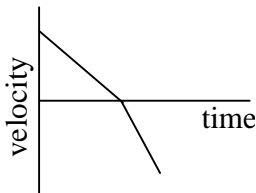
(a)



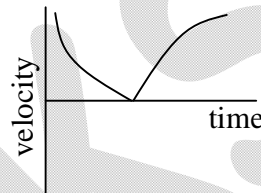
(b)



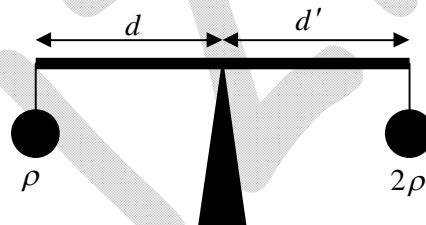
(c)



(d)



Q12. A rigid uniform bar of a certain mass has two bobs of the same size, but with different densities ρ and 2ρ suspended identically from its ends.



When the bar is level on a fulcrum as shown in the figure, d and d' are related by

(a) $2d = d'$

(b) $d > 2d'$

(c) $d = 2d'$

(d) $d < 2d'$

Q13. There are two points A and A' on the equator at longitudes 0° and 90° E, and two other points B and B' on the same longitudes respectively, but at latitude 60° S. The distances (along the latitudes) between the points A, A' and B, B' are related by

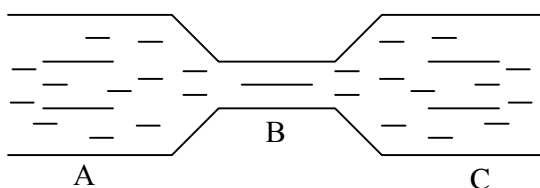
(a) $AA' = BB'$

(b) $AA' = 2BB'$

(c) $AA' = (\sqrt{3})BB'$

(d) $AA' = (\sqrt{2})BB'$

Q14.



Water is flowing through a tube as shown. The cross-sectional areas at A and C are equal and greater than the cross-sectional area at B. If the flow is steady, then the pressure on the walls at B is

- (a) less than that at A and that at C
- (b) more than that at A and that at C
- (c) same as that at A and that at C
- (d) more than that at A but less than that at C

Q15. Match the two lists

<i>Raw Material</i>	<i>Product</i>
A. Limestone	a. Porcelain
B. Gypsum	b. Glass
C. Silica sand	c. Plaster of Paris
D. Clay	d. Cement

- | | A | B | C | D |
|-----|----------|----------|----------|----------|
| (a) | a | b | c | d |
| (b) | d | c | b | a |
| (c) | a | c | d | b |
| (d) | d | a | c | b |

Q16. The ^{14}C dating method is not usually used for dating organic substances older than 60,000 years, because

- (a) such objects rarely contain carbon
- (b) such objects accumulated ^{14}C after their formation
- (c) in those times there was no production of ^{14}C
- (d) most of the ^{14}C in the sample would have decayed

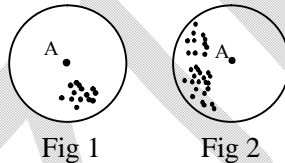
Q17. A seismograph receives a S -wave 60 s after it receives the P -wave. If the velocities of P - and S -waves are 7 km/s and 6 km/s respectively, then the distance of the seismic focus from the seismograph is

- (a) 2520 km (b) 42 km (c) 7070 km (d) 72 km

Q18. The decay of a radioactive isotope P produces a stable daughter isotope D . The ratio of the number of atoms of D to the number of atoms of P after 2 half lives would be

- (a) $1/4$ (b) $3/4$ (c) 3 (d) 2

Q19. The scatter plots represent the values measured by two similar instruments. Point A in the figures represents the true value. Which of the following is a correct description of the quality of these measurements?



- (a) Fig.1: good accuracy, good precision (b) Fig.1: poor accuracy, poor precision
 Fig. 2: good accuracy, good precision Fig. 2: good accuracy, poor precision
 (c) Fig.1: poor accuracy, good precision (d) Fig.1: poor accuracy, poor precision
 Fig. 2: poor accuracy, poor precision Fig. 2: poor accuracy, good precision

Q20. Even though the concentration of CO_2 is the same at sea level and at high altitude, the photosynthetic rate is higher in a plant grown at sea level than in a plant (of the same species) grown at high altitude. The reason for this is

- (a) light intensity is more at sea level
 (b) temperature is lower at higher altitude
 (c) atmospheric pressure is higher at sea level
 (d) relative humidity is higher at sea level

PART-B

Q21. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is

- (a) $\hat{i} + \hat{j}$ (b) $\hat{j} + \hat{k}$ (c) $\hat{i} + \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{j} + 5\hat{k}$

Q22. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ are

- (a) (1, 4, 9) (b) (0, 7, 7) (c) (0, 1, 13) (d) (0, 0, 14)

Q23. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \leq |z| \leq 2$, and around $z = 1$ is

(a) $\frac{1}{2} \left[1 + z + z^2 + \dots \right] \left[1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \dots \right]$

(b) $\frac{1}{1-z} + z - (1-z)^2 + (1-z)^3 + \dots$

(c) $\frac{1}{z^2} \left[1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right] \left[1 + \frac{2}{z} + \frac{4}{z^2} + \dots \right]$

(d) $2(z-1) + 5(z-1)^2 + 7(z-1)^3 + \dots$

Q24. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second. What should be the minimum duration of counting (in seconds) so that the relative error is less than 1%?

- (a) $100 / \lambda$ (b) $10^4 / \lambda^2$ (c) $10^4 / \lambda$ (d) $1 / \lambda$

Q25. Let $u(x, y) = x + \frac{1}{2}(x^2 - y^2)$ be the real part of an analytic function $f(z)$ of the complex variable $z = x + iy$. The imaginary part of $f(z)$ is

- (a) $y + xy$ (b) xy (c) y (d) $y^2 - x^2$

Q26. Let $y(x)$ be a continuous real function in the range 0 and 2π , satisfying the inhomogeneous differential equation:

$$\sin x \frac{d^2 y}{dx^2} + \cos x \frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right).$$

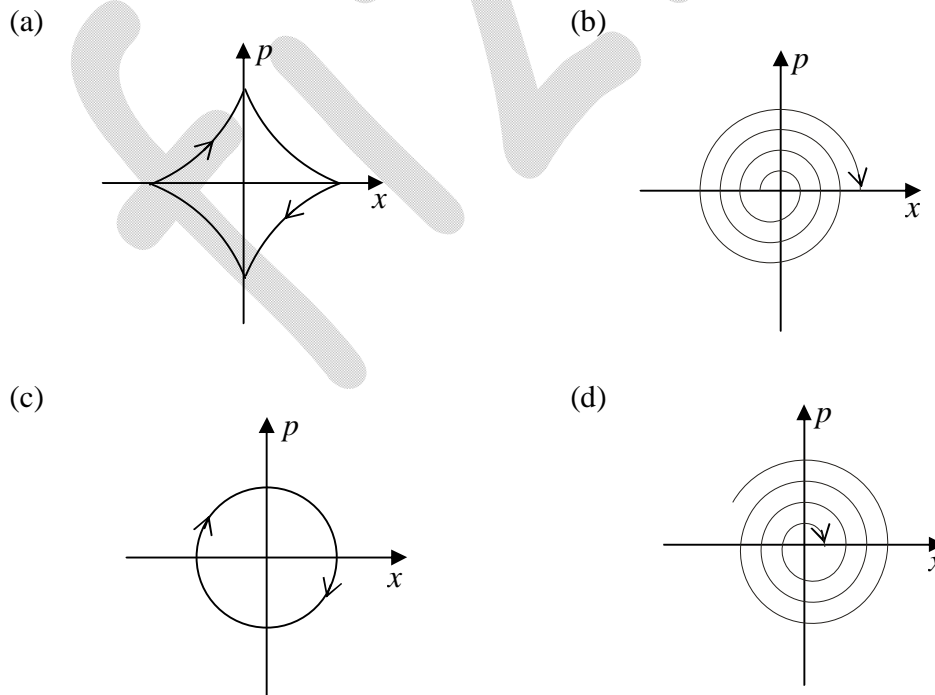
The value of dy/dx at the point $x = \pi/2$

- (a) is continuous (b) has a discontinuity of 3
(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

Q27. A ball is picked at random from one of two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white?

- (a) 34/70 (b) 41/70 (c) 36/70 (d) 29/70

Q28. The bob of a simple pendulum, which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum?



- Q29. Two events separated by a (spatial) distance 9×10^9 m, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed $0.8c$ (where the speed of light $c = 3 \times 10^8$ m/s) is
- (a) 60 s (b) 40 s (c) 20 s (d) 0 s

- Q30. If the Lagrangian of a particle moving in one dimensions is given by $L = \frac{\dot{x}^2}{2x} - V(x)$ the Hamiltonian is

(a) $\frac{1}{2}xp^2 + V(x)$ (b) $\frac{\dot{x}^2}{2x} + V(x)$ (c) $\frac{1}{2}\dot{x}^2 + V(x)$ (d) $\frac{p^2}{2x} + V(x)$

- Q31. A horizontal circular platform rotates with a constant angular velocity Ω directed vertically upwards. A person seated at the centre shoots a bullet of mass m horizontally with speed v . The acceleration of the bullet, in the reference frame of the shooter, is
- (a) $2v\Omega$ to his right (b) $2v\Omega$ to his left
(c) $v\Omega$ to his right (d) $v\Omega$ to his left

- Q32. The magnetic field corresponding to the vector potential

$$\vec{A} = \frac{1}{2} \vec{F} \times \vec{r} + \frac{10}{r^3} \vec{r}$$

where \vec{F} is a constant vector, is

(a) \vec{F} (b) $-\vec{F}$ (c) $\vec{F} + \frac{30}{r^4} \vec{r}$ (d) $\vec{F} - \frac{30}{r^4} \vec{r}$

- Q33. An electromagnetic wave is incident on a water-air interface. The phase of the perpendicular component of the electric field, E_{\perp} , of the reflected wave into the water is found to remain the same for all angles of incidence. The phase of the magnetic field H
- (a) does not change (b) changes by $3\pi/2$
(c) changes by $\pi/2$ (d) changes by π

- Q34. The magnetic field at a distance R from a long straight wire carrying a steady current I is proportional to
- (a) IR (b) I/R^2 (c) I^2/R^2 (d) I/R

Q35. The component along an arbitrary direction \hat{n} , with direction cosines (n_x, n_y, n_z) , of the spin of a spin $-\frac{1}{2}$ particle is measured. The result is

- (a) 0 (b) $\pm \frac{\hbar}{2} n_z$ (c) $\pm \frac{\hbar}{2} (n_x + n_y + n_z)$ (d) $\pm \frac{\hbar}{2}$

Q36. A particle of mass m is in a cubic box of size a . The potential inside the box ($0 \leq x < a, 0 \leq y < a, 0 \leq z < a$) is zero and infinite outside. If the particle is in an eigenstate of energy $E = \frac{14\pi\hbar^2}{2ma^2}$, its wavefunction is

- (a) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin \frac{3\pi x}{a} \sin \frac{5\pi y}{a} \sin \frac{6\pi z}{a}$ (b) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin \frac{7\pi x}{a} \sin \frac{4\pi y}{a} \sin \frac{3\pi z}{a}$
 (c) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin \frac{4\pi x}{a} \sin \frac{8\pi y}{a} \sin \frac{2\pi z}{a}$ (d) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{a} \sin \frac{3\pi z}{a}$

Q37. Let ψ_{nlm_l} denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The wavefunction $\psi = \frac{1}{4} [\psi_{210} + \sqrt{5}\psi_{21-1} + \sqrt{10}\psi_{211}]$ is an eigenfunction only of

- (a) H, L^2 and L_z (b) H and L_z
 (c) H and L^2 (d) L^2 and L_z

Q38. The commutator $[x^2, p^2]$ is

- (a) $2i\hbar xp$ (b) $2i\hbar(xp + px)$ (c) $2i\hbar px$ (d) $2i\hbar(xp - px)$

Q39. Consider a system of non-interacting particles in d dimensional obeying the dispersion relation $\varepsilon = Ak^s$, where ε is the energy k is the wavevector, s is an integer and A constant. The density of states, $N(\varepsilon)$, is proportional to

- (a) $\varepsilon^{\frac{s}{d}-1}$ (b) $\varepsilon^{\frac{d}{s}}$ (c) $\varepsilon^{\frac{d}{s}+1}$ (d) $\varepsilon^{\frac{s}{d}+1}$

Q40. The number of ways in which N identical bosons can be distributed in two energy levels, is

- (a) $N + 1$ (b) $N(N - 1) / 2$ (c) $N(N + 1) / 2$ (d) N

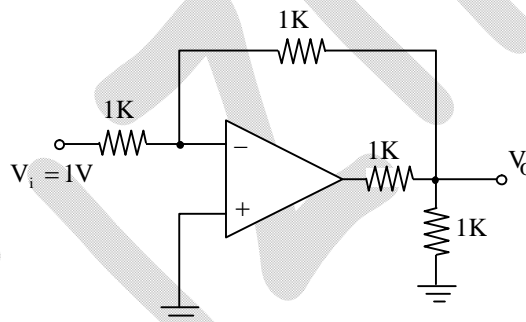
Q41. The free energy of gas of N particles in a volume V and at a temperature T is

$$F = Nk_B T \ln [a_0 V (k_B T)^{3/2} / N],$$

where a_0 is a constant and k_B denotes the Boltzmann constant. The internal energy of the gas is

- (a) $\frac{3}{2} Nk_B T$ (b) $\frac{5}{2} Nk_B T$
 (c) $Nk_B T \ln [a_0 V (k_B T)^{5/2} / N] - \frac{3}{2} Nk_B T$ (d) $Nk_B T \ln [a_0 V / (k_B T)^{5/2}]$

Q42. In the op-amp circuit shown in the figure below, the input voltage V_i is 1V. The value of the output V_o is



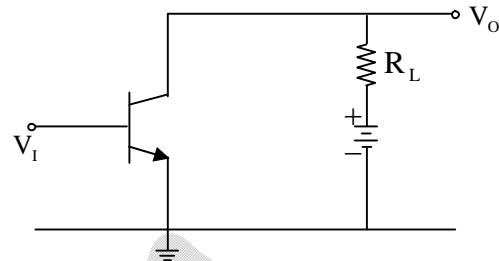
- (a) -0.33 V (b) -0.50 V (c) -1.00 V (d) -0.25 V

Q43. An LED operates at 1.5 V and 5 mA in forward bias. Assuming an 80% external efficiency of the LED, how many photons are emitted per second?

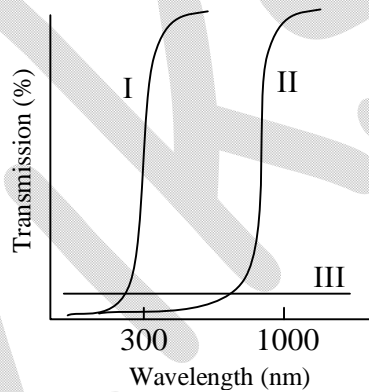
- (a) 5.0×10^{16} (b) 1.5×10^{16} (c) 0.8×10^{16} (d) 2.5×10^{16}

Q44. The transistor in the given circuit has $h_{fe} = 35\Omega$ and $h_{ie} = 1000\Omega$. If the load resistance $R_L = 1000\Omega$, the voltage and current gain are, respectively.

- (a) -35 and + 35
- (b) 35 and - 35
- (c) 35 and - 0.97
- (d) 0.98 and - 35



Q45. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond, respectively, to



- (a) insulator, semiconductor and metal
- (b) semiconductor, metal and insulator
- (c) metal, semiconductor and insulator
- (d) insulator, metal and semiconductor

PART-C

Q46. The eigenvalues of the antisymmetric matrix, $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where

n_1, n_2 and n_3 are the components of a unit vector, are

- (a) $0, i, -i$ (b) $0, 1, -1$
 (c) $0, 1 + i, -1 - i$ (d) $0, 0, 0$

Q47. Which of the following limits exists?

- (a) $\lim_{N \rightarrow \infty} \left(\sum_{m=1}^N \frac{1}{m} + \ln N \right)$ (b) $\lim_{N \rightarrow \infty} \left(\sum_{m=1}^N \frac{1}{m} - \ln N \right)$
 (c) $\lim_{N \rightarrow \infty} \left(\sum_{m=1}^N \frac{1}{\sqrt{m}} - \ln N \right)$ (d) $\lim_{N \rightarrow \infty} \sum_{m=1}^N \frac{1}{m}$

Q48. A bag contains many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value the number will tend to

- (a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$ (c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$

Q49. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{t^2 - R^2} \cos\left(\frac{rt}{2R}\right) dt$

- (a) $-\frac{2\pi}{R}$ (b) $-\frac{\pi}{R}$ (c) $\frac{\pi}{R}$ (d) $\frac{2\pi}{R}$

Q50. The Poisson bracket $\left\{ \vec{r}, \vec{p} \right\}$ has the value

- (a) $\left| \vec{r} \right| \left| \vec{p} \right|$ (b) $\vec{r} \cdot \vec{p}$ (c) 3 (d) 1

Q51. Consider the motion of a classical particle in a one dimensional double-well potential $V(x) = \frac{1}{4}(x^2 - 2)^2$. If the particle is displaced infinitesimally from the minimum

on the x -axis (and friction is neglected), then

- (a) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = \sqrt{2}$
- (b) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = 2$
- (c) the particle will switch between the right and left wells
- (d) the particle will approach the bottom of the right well and settle there

Q52. What is proper time interval between the occurrence of two events if in one inertial frame events are separated by 7.5×10^8 m and occur 6.5 s apart?

- (a) 6.50 s
- (b) 6.00 s
- (c) 5.75 s
- (d) 5.00 s

Q53. A free particle described by a plane wave and moving in the positive z -direction undergoes scattering by a potential

$$V(r) = \begin{cases} V_0 & \text{if } r \leq R \\ 0 & \text{if } r > R \end{cases}$$

If V_0 is changed to $2V_0$, keeping R fixed, then the differential scattering cross-section, in the Born approximation.

- (a) increases to four times the original value
- (b) increases to twice the original value
- (c) decreases to half the original value
- (d) decreases to one fourth the original value

Q54. A variational calculation is done with the normalized trial

wavefunction $\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}(a^2 - x^2)$ for the one-dimensional potential well

$$V(x) = \begin{cases} 0 & \text{if } |x| \leq a \\ \infty & \text{if } |x| > a \end{cases}$$

The ground state energy is estimated to be

- (a) $\frac{5\hbar^2}{3ma^2}$ (b) $\frac{3\hbar^2}{2ma^2}$ (c) $\frac{3\hbar^2}{5ma^2}$ (d) $\frac{5\hbar^2}{4ma^2}$

Q55. A particle in one-dimension is in the potential

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \\ -V_0 & \text{if } 0 \leq x \leq l \\ 0 & \text{if } x > l \end{cases}$$

If there is at least one bound state, the minimum depth of potential is

- (a) $\frac{\hbar^2 \pi^2}{8ml^2}$ (b) $\frac{\hbar^2 \pi^2}{2ml^2}$ (c) $\frac{2\hbar^2 \pi^2}{ml^2}$ (d) $\frac{\hbar^2 \pi^2}{ml^2}$

Q56. Which of the following is a self-adjoint operator in the spherical polar coordinate system (r, θ, ϕ) ?

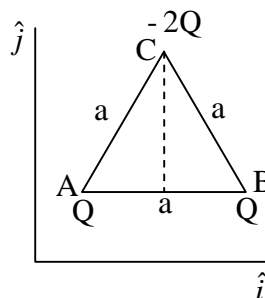
- (a) $-\frac{i\hbar}{\sin^2 \theta} \frac{\partial}{\partial \theta}$ (b) $-i\hbar \frac{\partial}{\partial \theta}$ (c) $-\frac{i\hbar}{\sin \theta} \frac{\partial}{\partial \theta}$ (d) $-i\hbar \sin \theta \frac{\partial}{\partial \theta}$

Q57. Which of the following questions is Lorentz invariant?

- (a) $|\vec{E} \times \vec{B}|^2$ (b) $|\vec{E}|^2 - |\vec{B}|^2$ (c) $|\vec{E}|^2 + |\vec{B}|^2$ (d) $|\vec{E}|^2 |\vec{B}|^2$

Q58. Charges Q , Q and $-2Q$ are placed on the vertices of an equilateral triangle ABC of sides of length a , as shown in the figure. The dipole moment of this configuration of charges, irrespective of the choice of origin, is

- (a) $+2aQ\hat{i}$ (b) $+\sqrt{3}aQ\hat{j}$
 (c) $-\sqrt{3}aQ\hat{j}$ (d) 0



Q59. The vector potential \vec{A} due to a magnetic moment \vec{m} at a point \vec{r} is given by $\vec{A} = \frac{\vec{m} \times \vec{r}}{r^3}$.

If \vec{m} is directed along the positive z -axis, the x -component of the magnetic field, at the point \vec{r} , is

- (a) $\frac{3myz}{r^5}$ (b) $-\frac{3mxy}{r^5}$
 (c) $\frac{3mxz}{r^5}$ (d) $\frac{3m(z^2 - xy)}{r^5}$

Q60. A system has two normal modes of vibration, with frequencies ω_1 and $\omega_2 = 2\omega_1$. What is the probability that at temperature T , the system has an energy less than $4\hbar\omega_1$?

[In the following $x = e^{\beta\hbar\omega_1}$ and Z is the partition function of the system.]

- (a) $x^{3/2}(x + 2x^2)/Z$ (b) $x^{3/2}(1 + x + x^2)/Z$
 (c) $x^{3/2}(1 + 2x^2)/Z$ (d) $x^{3/2}(1 + x + 2x^2)/Z$

Q61. The magnetization M of a ferromagnet, as a function of the temperature T and the magnetic field H , is described by the equation $M = \tanh\left(\frac{T_c}{T}M + \frac{H}{T}\right)$. In these units, the

zero-field magnetic susceptibility in terms of $M(0) = M(H = 0)$ is given by

- (a) $\frac{1 - M^2(0)}{T - T_c(1 - M^2(0))}$ (b) $\frac{1 - M^2(0)}{T - T_c}$
 (c) $\frac{1 - M^2(0)}{T + T_c}$ (d) $\frac{1 - M^2(0)}{T}$

Q62. Bose condensation occurs in liquid He^4 kept at ambient pressure at 2.17 K. At which temperature will Bose condensation occur in He^4 in gaseous state, the density of which is 1000 times smaller than that of liquid He^4 ? (Assume that it is a perfect Bose gas.)

- (a) 2.17 mK (b) 21.7 mK (c) 21.7 μK (d) 2.17 μK

Q63. Consider black body radiation contained in a cavity whose walls are at temperature T . The radiation is in equilibrium with the walls of the cavity. If the temperature of the walls is increased to $2T$ and the radiation is allowed to come to equilibrium at the new temperature, the entropy of the radiation increases by a factor of

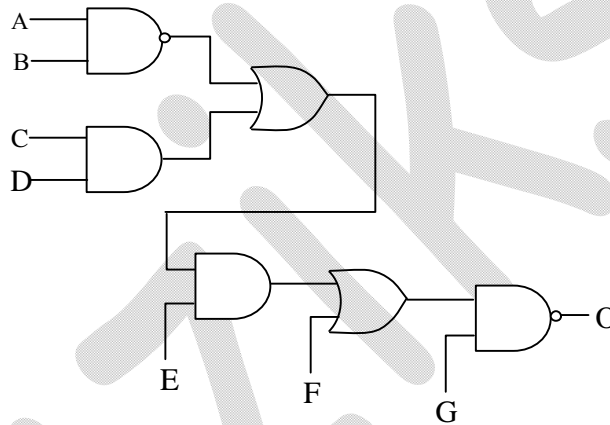
- (a) 2 (b) 4 (c) 8 (d) 16

Q64. The output, O , of the given circuit in cases I and II, where

Case I: $A, B = 1; C, D = 0; E, F = 1$ and $G = 0$

Case II: $A, B = 0; C, D = 0; E, F = 0$ and $G = 1$

are respectively



- (a) 1, 0 (b) 0, 1 (c) 0, 0 (d) 1, 1

Q65. A resistance strain gauge is fastened to a steel fixture and subjected to a Stress of 1000 kg/m^2 . If the gauge factor is 3 and the modulus of elasticity of steel is $2 \times 10^{10} \text{ kg/m}^2$ then the fractional change in resistance of the strain gauge due to the applied stress is

(**Note:** The gauge factor is defined as the ratio of the fractional change in resistance to the fractional change in length.)

- (a) 1.5×10^{-7} (b) 3.0×10^{-7} (c) 0.16×10^{-10} (d) 0.5×10^{-7}

- Q66. Consider a sinusoidal waveform of amplitude 1 V and frequency f_0 . Starting from an arbitrary initial time, the waveform is sampled at intervals of $\frac{1}{2f_0}$. If the corresponding Fourier spectrum peaks at a frequency \bar{f} and an amplitude \bar{A} , then
- (a) $\bar{f} = 2f_0$ and $\bar{A} = 1V$ (b) $\bar{f} = 2f_0$ and $0 \leq \bar{A} \leq 1V$
 (c) $\bar{f} = 0$ and $\bar{A} = 1V$ (d) $\bar{f} = \frac{f_0}{2}$ and $\bar{A} = \frac{1}{\sqrt{2}}V$
- Q67. The first absorption spectrum of $^{12}\text{C}^{16}\text{O}$ is at 3.842 cm^{-1} while that of $^{13}\text{C}^{16}\text{O}$ is at 3.673 cm^{-1} . The ratio of their moments of inertia is
- (a) 1.851 (b) 1.286 (c) 1.046 (d) 1.038
- Q68. The spin-orbit interaction in an atom is given by $H = a \mathbf{L} \cdot \mathbf{S}$, where \mathbf{L} and \mathbf{S} denote the orbital and spin angular momenta, respectively, of the electron. The splitting between the levels $^2P_{3/2}$ and $^2P_{1/2}$ is
- (a) $\frac{3}{2}a\hbar^2$ (b) $\frac{1}{2}a\hbar^2$ (c) $3a\hbar^2$ (d) $\frac{5}{2}a\hbar^2$
- Q69. The spectral line corresponding to an atomic transition from $J = 1$ to $J = 0$ states splits in a magnetic field of 1 kG into three components separated by $1.6 \times 10^{-3} \text{ \AA}$. If the zero field spectral line corresponds to 1849 \AA , what is the g-factor corresponding to the $J = 1$ state? (You may use $\frac{hc}{\mu_0} \approx 2 \times 10^4 \text{ cm}$)
- (a) 2 (b) 3/2 (c) 1 (d) 1/2
- Q70. The energy required to create a lattice vacancy in a crystal is equal to 1 eV. The ratio of the number densities of vacancies $n(1200 \text{ K})/n(300 \text{ K})$ when the crystal is at equilibrium at 1200 K and 300 K, respectively, is approximately
- (a) $\exp(-30)$ (b) $\exp(-15)$ (c) $\exp(15)$ (d) $\exp(30)$

Q71. The dispersion relation of phonons in a solid is given by

$$\omega^2(k) = \omega_0^2(3 - \cos k_1 a - \cos k_2 a - \cos k_3 a)$$

The velocity of the phonons at large wavelength is

- (a) $\omega_0 a / \sqrt{3}$ (b) $\omega_0 a$ (c) $\sqrt{3} \omega_0 a$ (d) $\omega_0 a / \sqrt{2}$

Q72. Consider an electron in a box of length L with periodic boundary condition

$$\psi(x) = \psi(x + L). \text{ If the electron is in the } \psi_k(x) = \frac{1}{\sqrt{L}} e^{ik \cdot x} \text{ with energy } \epsilon_k = \frac{\hbar^2 k^2}{2m}, \text{ what is}$$

the correction to its energy, to second order of perturbation theory, when it is subjected to a weak periodic potential $V(x) = V_0 \cos gx$, where g is an integral multiple of the $2\pi/L$?

- (a) $V_0^2 \epsilon_g / \epsilon_k^2$
 (b) $-\frac{mV_0^2}{2\hbar} \left(\frac{1}{g^2 + 2kg} + \frac{1}{g^2 - 2kg} \right)$
 (c) $V_0^2 (\epsilon_k - \epsilon_g) / \epsilon_g^2$
 (d) $V_0^2 / (\epsilon_k + \epsilon_g)$

Q73. The ground state of ${}_{12}^{207}Pb$ nucleus has spin-parity $J^p = \frac{1}{2}^-$, while the first excited state

has $J^p = \frac{5}{2}^-$. The electromagnetic radiation emitted when the nucleus makes a transition

from the first excited state to ground state are

- (a) E2 and E3 (b) M2 or E3
 (c) E2 or M3 (d) M2 or M3

Q74. The dominant interactions underlying the following processes

A. $K^- + p \rightarrow \Sigma^- + \pi^+$

B. $\mu^- + \mu^+ \rightarrow K^- + K^+$

C. $\Sigma^+ \rightarrow p + \pi^0$

are

- (a) A: strong, B: electromagnetic and; C: weak
- (b) A: strong, B: weak and; C: weak
- (c) A: weak, B: electromagnetic and; C: strong
- (d) A: weak, B: electromagnetic and; C: weak

Q75. If Higgs bosons of mass m_H moving with a speed $\beta = \frac{v}{c}$ decays into a pair photons, then

the invariant mass of the photon is

[Note: The invariant mass of a system of two particles, with four momenta p_1 and p_2

is $(p_1 + p_2)^2$.]

- (a) βm_H
- (b) m_H
- (c) $m_H / \sqrt{1 - \beta^2}$
- (d) $\beta m_H / \sqrt{1 - \beta^2}$