

NET/JRF (DEC-2011)

- Q2. Given that the ground state energy of the hydrogen atom is -13.6 eV , the ground state energy of positronium (which is a bound state of an electron and a positron) is
 (a) $+6.8 \text{ eV}$ (b) -6.8 eV (c) -13.6 eV (d) -27.2 eV

Ans. : (b)

Solution: The energy expression for Positronium atom is $E_n = -\frac{13.6}{2n^2} (\text{eV})$

$$\text{For } n = 1, E_1 = \frac{-13.6}{2} (\text{eV}) = -6.8 \text{ eV}, \quad \therefore E_1 = -6.8 \text{ eV}$$

- Q3. A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at 770 cm^{-1} , the approximate positions of the Stokes and the anti-Stokes lines are
 (a) 481.5 nm and 520 nm (b) 481.5 nm and 500 nm
 (c) 500 nm and 520 nm (d) 500 nm and 600 nm

Ans. :

Solution: Given $\lambda_0 = 500 \text{ nm} = 5 \times 10^{-5} \text{ cm}$, $\bar{\nu}_{\text{stoke}} = 770 \text{ cm}^{-1} \therefore \bar{\nu}_0 = 20,000 \text{ cm}^{-1}$

$$\text{Raman shift } \Delta\bar{\nu} = \bar{\nu}_0 - \bar{\nu}_{\text{stoke}} = 19230 \text{ cm}^{-1}$$

$$\text{Wave number of anti-stokes line is } \bar{\nu}_{\text{anti-stoke}} = \Delta\bar{\nu} + \bar{\nu}_0 = 39,230 \text{ cm}^{-1}$$

$$\text{In wavelength term } \lambda_{\text{anti-stoke}} = 2.549 \times 10^{-7} = 254.9 \text{ nm} \text{ and } \lambda_{\text{stoke}} = 12987 \text{ nm}$$

- Q4. If the hyperfine interaction in an atom is given by $H = a\vec{S}_e \cdot \vec{S}_p$ where \vec{S}_e and \vec{S}_p denote the electron and proton spins, respectively, the splitting between the 3S_1 and 1S_0 state is
 (a) $a\hbar^2 / \sqrt{2}$ (b) $a\hbar^2$ (c) $a\hbar^2 / 2$ (d) $2a\hbar^2$

Ans. : (b)

Solution: Total spin is $\vec{S} = \vec{S}_e + \vec{S}_p \Rightarrow S^2 = S_e^2 + S_p^2 + 2\vec{S}_e \cdot \vec{S}_p \Rightarrow \vec{S}_e \cdot \vec{S}_p = \frac{1}{2}[S^2 - S_e^2 - S_p^2]$

$$\Rightarrow H = a\vec{S}_e \cdot \vec{S}_p = \frac{a}{2}[S^2 - S_e^2 - S_p^2] \text{ where } S_e^2 = S_p^2 = S(S+1)\hbar^2 = \frac{3}{4}\hbar^2$$

$$\Rightarrow H = \frac{a}{2}\left(S^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2\right) = \frac{a}{2}\left[S^2 - \frac{3}{2}\hbar^2\right]$$

$$\text{For } ^3S_1 : S = 1 \Rightarrow S^2 = S(S+1)\hbar^2 = 2\hbar^2, \text{ For } ^1S_0 : S = 0 \Rightarrow S^2 = S(S+1)\hbar^2 = 0\hbar^2$$

$$\therefore H_1 = \frac{a}{2}\left[2 - \frac{3}{2}\right]\hbar^2 = \frac{a}{4}\hbar^2 \text{ for } ^3S_1 \text{ and } H_2 = \frac{a}{2}\left[0 - \frac{3}{2}\right]\hbar^2 = \frac{-3}{4}a\hbar^2 \text{ for } ^1S_0$$

\therefore The splitting between 3S_1 and 1S_0 is $\Delta H = H_1 - H_2 = \left(\frac{1}{4} + \frac{3}{4}\right) a\hbar^2 = a\hbar^2$

Q5. The ratio of intensities of the D_1 and D_2 lines of sodium at high temperature is

- (a) 1:1 (b) 2:3 (c) 1:3 (d) 1:2

Ans. : (d)

Solution: The electronic transition for D_2 and D_1 line is

$$D_2 : ^2P_{3/2} \rightarrow ^2S_{1/2}, D_1 : ^2P_{1/2} \rightarrow ^2S_{1/2} \therefore \frac{I(D_2)}{I(D_1)} = \frac{2J_2 + 1}{2J_1 + 1} = \frac{2 \times \frac{3}{2} + 1}{2 \times \frac{1}{2} + 1} = \frac{4}{2} = \frac{2}{1}$$

Q6. An atom of mass M can be excited to a state of mass $(M + \Delta)$ by photon capture. The frequency of a photon which can cause this transition is

- (a) $\frac{\Delta c^2}{2h}$ (b) $\frac{\Delta c^2}{h}$ (c) $\frac{\Delta^2 c^2}{2Mh}$ (d) $\frac{\Delta c^2}{2Mh} (\Delta + 2M)$

Ans. : (d)

Solution: The conservation law of energy and momentum give

$$Mc^2 + h\nu = \left[(M + \Delta)^2 c^4 + p^2 c^2 \right]^{1/2} \text{ and } \frac{h\nu}{c} = p$$

$$M^2 c^4 + h^2 \nu^2 + 2Mc^2 h\nu = M^2 c^4 + \Delta^2 c^4 + 2M\Delta c^4 + h^2 \nu^2 \Rightarrow 2Mc^2 h\nu = \Delta^2 c^4 + 2M\Delta c^4$$

$$\Rightarrow 2Mc^2 h\nu = 2M\Delta c^4 \left(1 + \frac{\Delta}{2M} \right) \Rightarrow \nu = \frac{\Delta c^2}{h} \left(1 + \frac{\Delta}{2M} \right) \Rightarrow \nu = \frac{\Delta c^2}{2Mh} (\Delta + 2M).$$

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Q7. The first absorption spectrum of $^{12}\text{C}^{16}\text{O}$ is at 3.842 cm^{-1} while that of $^{13}\text{C}^{16}\text{O}$ is at 3.673 cm^{-1} . The ratio of their moments of inertia is

- (a) 1.851 (b) 1.286 (c) 1.046 (d) 1.038

Ans. : (c)

Solution: For $^{12}\text{C}^{16}\text{O}$: $2B_1 = 3.842 \text{ cm}^{-1} \Rightarrow B_1 = 1.921 \text{ cm}^{-1}$

For $^{13}\text{C}^{16}\text{O}$: $2B_2 = 3.673 \text{ cm}^{-1} \Rightarrow B_2 = 1.8365 \text{ cm}^{-1}$

$$\text{Where, } B = \frac{h}{8\pi^2 IC} \therefore \frac{I_2}{I_1} = \frac{B_1}{B_2} = \frac{1.921}{1.8365} = 1.046$$

Q8. The spin-orbit interaction in an atom is given by $H = a\vec{L} \cdot \vec{S}$, where \vec{L} and \vec{S} denote the orbital and spin angular momenta, respectively, of the electron. The splitting between the levels ${}^2P_{3/2}$ and ${}^2P_{1/2}$ is

- (a) $\frac{3}{2}a\hbar^2$ (b) $\frac{1}{2}a\hbar^2$ (c) $3a\hbar^2$ (d) $\frac{5}{2}a\hbar^2$

Ans. : (a)

Solution: Given $H = a\vec{L} \cdot \vec{S}$ where $\vec{J} = \vec{L} + \vec{S}$.

$$\therefore J^2 = L^2 + S^2 + 2\vec{L} \cdot \vec{S} \Rightarrow \vec{L} \cdot \vec{S} = \frac{1}{2}[J^2 - L^2 - S^2] \therefore H = \frac{a}{2}[J^2 - L^2 - S^2]$$

For ${}^2P_{3/2}$: $S = \frac{1}{2}$ which gives $S^2 = S(S+1)\hbar^2 = \frac{3}{4}\hbar^2$

$L = 1$ which gives $L^2 = L(L+1)\hbar^2 = 2\hbar^2$

$J = \frac{3}{2}$ which gives $J^2 = J(J+1)\hbar^2 = \frac{15}{4}\hbar^2 \therefore H_1 = \frac{a}{2}\left[\frac{15}{4} - 2 - \frac{3}{4}\right]\hbar^2 = \frac{a}{2}\hbar^2$

For ${}^2P_{1/2}$: $S = \frac{1}{2}$ which gives $S^2 = S(S+1)\hbar^2 = \frac{3}{4}\hbar^2$

$L = 1$ which gives $L^2 = L(L+1)\hbar^2 = 2\hbar^2$

$J = \frac{1}{2}$ which gives $J^2 = J(J+1)\hbar^2 = \frac{3}{4}\hbar^2 \therefore H_2 = \frac{a}{2}\left[\frac{3}{4} - 2 - \frac{3}{4}\right]\hbar^2 = -a\hbar^2$

$\therefore \Delta H = H_1 - H_2 = \frac{a}{2}\hbar^2 + a\hbar^2 = \frac{3}{2}a\hbar^2$

Q9. The spectral line corresponding to an atomic transition from $J = 1$ to $J = 0$ states splits in a magnetic field of 0.1 Tesla into three components separated by $1.6 \times 10^{-3} \text{ \AA}$. If the zero field spectral line corresponds to 1849 \AA , what is the g-factor corresponding to the $J = 1$

state? (You may use $\frac{hc}{\mu_0} \approx 2 \times 10^4 \text{ cm}$)

- (a) 2 (b) 3/2 (c) 1 (d) 1/2

Ans. : (c)

Solution: The Zeeman splitting is $\Delta E = (gM_J)(\mu_B B) = g\mu_B B$ for $M_J = +1$

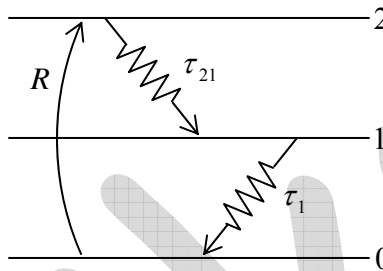
Given, Zeeman splitting separations, $\Delta\lambda = \frac{\lambda^2}{c} \Delta\nu = 1.6 \times 10^{-3} \text{ \AA}$

$$\Rightarrow \Delta \nu = \frac{c}{\lambda^2} \Delta \lambda = \frac{3 \times 10^8}{(1849 \times 10^{-10})^2} \times 1.6 \times 10^{-1} = 0.1404 \times 10^{10}$$

$$\therefore g = \frac{\Delta E}{\mu_B B} = \frac{6.625 \times 10^{-34} \times 0.1404 \times 10^{10}}{9.27 \times 10^{-24} \times 0.1} = 1.00$$

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Q10. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate R is 10^{20} atoms $\text{cm}^{-3} \text{s}^{-1}$ and the decay routes are as shown with $\tau_{21} = 20$ ns and $\tau_1 = 1 \mu\text{s}$, the equilibrium populations of states 2 and 1 are, respectively,

- (a) 10^{14} cm^{-3} and $2 \times 10^{12} \text{ cm}^{-3}$ (b) $2 \times 10^{12} \text{ cm}^{-3}$ and 10^{14} cm^{-3} .
 (c) $2 \times 10^{12} \text{ cm}^{-3}$ and $2 \times 10^6 \text{ cm}^{-3}$ (d) zero and 10^{20} cm^{-3}

Ans. : (b)

Solution: $\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}}$ and $\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1}$.

Under equilibrium condition $\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$

$$\Rightarrow N_2 = \tau_{21} R = 10^{20} \times 20 \times 10^{-9} = 2 \times 10^{12} \text{ cm}^{-3}$$

$$\Rightarrow N_1 = \frac{\tau_1 N_2}{\tau_{21}} = \frac{10^{-6} \times 2 \times 10^{12} \text{ cm}^{-3}}{20 \times 10^{-9}} = 10^{14} \text{ cm}^{-3}$$

Q11. Consider a hydrogen atom undergoing a $2P \rightarrow 1S$ transition. The lifetime t_{sp} of the $2P$ state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV. Assuming that the refractive index of the medium $n_0 = 1$, the ratio of Einstein coefficients for stimulated and spontaneous emission $B_{21}(\omega)/A_{21}(\omega)$ is given by

- (a) $0.683 \times 10^{12} \text{ m}^3 \text{J}^{-1} \text{s}^{-1}$ (b) $0.146 \times 10^{-12} \text{ Jsm}^{-3}$.
 (c) $6.83 \times 10^{12} \text{ m}^3 \text{J}^{-1} \text{s}^{-1}$ (d) $1.463 \times 10^{-12} \text{ Jsm}^{-3}$.

Ans. : (a)

Solution: $n_0 = 1$, $\Delta E = 10.2 \text{ eV}$ and $\frac{B_{21}}{A_{21}} = \frac{\pi^2 c^3}{\hbar \omega^3 n_0^3} = \frac{\hbar^2 \pi^2 c^3}{(\Delta E)^3 n_0^3} = 0.67 \times 10^{12}$.

Q12. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0.98$. The mirrors are separated by a distance $d = 20 \text{ cm}$ and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes $\delta\nu$ and the width $\Delta\nu_p$ of each mode of the laser cavity are:

- (a) $\delta\nu = 75 \text{ kHz}$, $\Delta\nu_p = 24 \text{ kHz}$ (b) $\delta\nu = 100 \text{ kHz}$, $\Delta\nu_p = 100 \text{ kHz}$
 (c) $\delta\nu = 750 \text{ MHz}$, $\Delta\nu_p = 2.4 \text{ MHz}$ (d) $\delta\nu = 2.4 \text{ MHz}$, $\Delta\nu_p = 750 \text{ MHz}$

Ans. : (c)

Solution: Mode separation $\delta\nu = \frac{c}{2dn_0} = 750 \text{ MHz}$

where $c = 3 \times 10^8 \text{ m/sec}$, $d = 20 \times 10^{-2} \text{ m}$ and $n_0 = 1$.

Width of each mode $\delta\nu_p = \frac{1}{2\pi t_c}$ where $t_c = \frac{2n_0 d}{c \ln\left(\frac{1}{R_1 R_2 e^{-2\alpha d}}\right)}$.

Note: In this question, there is no need to calculate $\delta\nu_p$. Since in the given options there is only one option with $\delta\nu = 750 \text{ MHz}$ and i.e., option (c). You can calculate $\delta\nu_p$ without calculator so use your common sense.

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Q13. A muon (μ^-) from cosmic rays is trapped by a proton to form a hydrogen-like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom will be

- (a) 5.62 \AA (b) 6.67 \AA (c) 3.75 \AA (d) 13.3 \AA

Ans. : (b)

Solution: In case of muonic atom, the reduce mass is $m' = \frac{m_\mu m_p}{m_\mu + m_p} = 180 m_e$

$$E'_n = \left(\frac{m'}{m_e}\right) \frac{E_1}{n^2} = 180 \frac{E_1}{n^2} \quad \text{where, } E_1 = -13.6 \text{ eV}$$

For ground state of muonic atom $n=1$, $E'_1 = 180E_1$

For first excited state of muonic atom $n=2$, $E'_2 = 45E_1$

The longest wavelength of the photon corresponds to the transition between first and ground state of muonic atom.

The energy difference between first excited and ground state is

$$\Delta E = E'_2 - E'_1 = -135E_1 = 1836 \text{ eV} = 2938 \times 10^{-19} \text{ J}$$

In term of wavelength

$$\Delta E = hv = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2938 \times 10^{-19}} = 6.67 \times 10^{-10} \text{ m} = 6.67 \text{ \AA}$$

Q14. Consider the hydrogen-deuterium molecule HD. If the mean distance between the two atoms is 0.08 nm and the mass of the hydrogen atom is $938 \text{ MeV}/c^2$, then the energy difference ΔE between the two lowest rotational states is approximately

- (a) 10^{-1} eV (b) 10^{-2} eV (c) $2 \times 10^{-2} \text{ eV}$ (d) 10^{-3} eV

Ans. : (b)

Solution: Rotational energy expression $E = \frac{h^2}{8\pi^2 I} J(J+1) = AJ(J+1)$

Difference between two lowest energy levels is $\Delta E = 2A$

where $A = \frac{h^2}{8\pi^2 I} = \frac{\hbar^2}{2I}$ and $I = \mu r^2$

$$\text{Here, } \mu = \frac{M_H M_D}{M_H + M_D} = \frac{M_H \times 2M_H}{M_H + 2M_H} = \frac{2}{3} M_H, \quad \mu = \frac{2}{3} \times 938 \text{ MeV}/c^2$$

$$\hbar = 1.01 \times 10^{-34} \text{ (J-sec)} = 1.01 \times 10^{-34} \times \frac{1}{1.6} \times 10^{19} \text{ (eV-sec)} = 6.3 \times 10^{-16} \text{ (eV-sec)}$$

$$\therefore \Delta E = \frac{\hbar^2}{I} = \frac{(6.3 \times 10^{-16} \text{ eV-sec})^2}{\frac{2}{3} \times 938 \times 10^6 \text{ eV}/c^2 \times (0.08 \times 10^{-9} \text{ m})^2} = 9.2 \times 10^{-3} \text{ eV} \cong 10^{-2} \text{ eV}$$

Q15. The electronic energy levels in a hydrogen atom are given by $E_n = -13.6/n^2 \text{ eV}$. If a selective excitation to the $n=100$ level is to be made using a laser, the maximum allowed frequency line-width of the laser is

- (a) 6.5 MHz (b) 6.5 GHz (c) 6.5 Hz (d) 6.5 kHz

- Q19. The spectroscopic symbol for the ground state of ${}_{13}\text{Al}$ is ${}^2P_{1/2}$. Under the action of a strong magnetic field (when $L-S$ coupling can be neglected) the ground state energy level will split into
- (a) 3 levels (b) 4 levels (c) 5 levels (d) 6 levels

Ans. : (c)

Solution: In extremely strong magnetic field coupling between L-S breaks down. J is no longer a valid quantum number.

The ground state energy level will have $(2S+1)(2L+1)=6$ states, but 2 states are degenerate. Therefore, total energy levels will be 5.

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- Q20. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is
- (a) 10 (b) 3 (c) 6 (d) 9

Ans. : (a)

Solution: For p state: $l = 1, s = 1/2 : j = 1/2 \text{ \& } 3/2$.

This gives two spectral terms ${}^2P_{3/2}$ & ${}^2P_{1/2}$

For s state: $l = 0, s = 1/2 : j = 1/2$: This gives spectral terms ${}^2S_{1/2}$

The terms ${}^2P_{3/2}$ and ${}^2S_{1/2}$ corresponding to $J = 3/2$ & $J = 1/2$ will break into $2J+1$ Zeeman levels, which is 4 and 2 respectively.

$$\text{For } {}^2P_{3/2} \quad M_j = -3/2 \quad -1/2 \quad +1/2 \quad +3/2$$

$$\text{For } {}^2S_{1/2} \quad M_j = -1/2 \quad +1/2$$

The selection rule is $\Delta M_j = 0, \pm 1$ ($M_j = 0 \rightarrow M_j = 0$ if $\Delta J = 0$)

$\Delta M_j = 0$ gives π component, $\Delta M_j = \pm 1$ gives σ component

Number of π component = 2, Numbers of σ^+ components = 2

Number of σ^- components = 2

The terms ${}^2P_{1/2}$ and ${}^2S_{1/2}$ corresponding to $J = 1/2$ & $J = 1/2$ will break into $2J+1$ Zeeman levels, which is 2 & 2 respectively.

$$\text{For } {}^2P_{1/2} \quad M_j = -1/2 \quad +1/2, \text{ For } {}^2S_{1/2} \quad M_j = -1/2 \quad +1/2$$

The selection rule is $\Delta M_J = 0, \pm 1$ ($M_J = 0 \rightarrow M_J = 0$ If $\Delta J = 0$)

$\Delta M_J = 0$ gives π component, $\Delta M_J = \pm 1$ gives σ component

Number of π component = 2, Numbers of σ^+ components = 1

Number of σ^- components = 1

Thus, total number of Zeeman component = 10

Q21. A double slit interference experiment uses a laser emitting light of two adjacent frequencies ν_1 and ν_2 ($\nu_1 < \nu_2$). The minimum path difference between the interfering beams for which the interference pattern disappears is

- (a) $\frac{c}{\nu_2 + \nu_1}$ (b) $\frac{c}{\nu_2 - \nu_1}$ (c) $\frac{c}{2(\nu_2 - \nu_1)}$ (d) $\frac{c}{2(\nu_2 + \nu_1)}$

Ans. : (c)

Solution: The condition of maximum intensity for interfering laser beam is: $d \sin \theta = n\lambda$

The condition of dark intensity for interfering laser beam is: $d \sin \theta = \left(n + \frac{1}{2}\right)\lambda$

For interference pattern to vanish, the minimum path difference should be $\lambda/2$

The spectral bandwidth of laser is defined as, $\Delta\nu = \frac{c\Delta\lambda}{\lambda^2} \Rightarrow \frac{c}{\Delta\nu} = \frac{\lambda^2}{\Delta\lambda}$

For two closely spaced line of wavelength λ_1 and λ_2

$$\frac{c}{\Delta\nu} = \frac{\lambda^2}{\Delta\lambda} = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} = \frac{1}{\frac{1}{\lambda_2} - \frac{1}{\lambda_1}} = \frac{c}{\frac{c}{\lambda_2} - \frac{c}{\lambda_1}} = \frac{c}{\nu_2 - \nu_1}$$

Since, for interference pattern to vanish for two closely spaced line of wavelength λ_1 and

λ_2 , the minimum path difference should be $= \frac{\lambda}{2} = \frac{c}{2\Delta\nu} = \frac{c}{2(\nu_2 - \nu_1)}$

Q22. How much does the total angular momentum quantum number J change in the transition of $Cr(3d^6)$ atom as it ionize to $Cr^{2+}(3d^4)$?

- (a) Increases by 2 (b) Decreases by 2 (c) Decreases by 4 (d) Does not change

Ans. : (c)

Solution: In $Cr(3d^6)$ state

$$M_L = -2 \quad -1 \quad 0 \quad +1 \quad +2$$

↑	↓	↑	↑	↑	↑
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In this configuration, $S = 4 \times \frac{1}{2} = 2$ and $L = 2$

This is the case of more than half filled subshell, thus state with highest J value will have the lowest energy.

The highest J - value is $J = L + S = 4$

Thus, the ground state spectral term for this configuration is 5D_4 .

Now in $Cr^{2+}(3d^4)$

$$M_L = -2 \quad -1 \quad 0 \quad +1 \quad +2$$



In this configuration, $S = 4 \times \frac{1}{2} = 2$ and $L = 2$

Since this is the case of less than half filled subshell, thus, state with lowest J value will have the lowest energy.

The lowest J - value is $J = L - S = 2 - 2 = 0$.

Thus the ground state spectral term for this configuration is the J - value decreases from $J = 4$ to $J = 0$.

Thus correct answer is option (c).

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Q23. An atomic transition ${}^1P \rightarrow {}^1S$ in a magnetic field 1 Tesla shows Zeeman splitting. Given that the Bohr magneton $\mu_B = 9.27 \times 10^{-24} \text{ J/T}$, and the wavelength corresponding to the transition is 250 nm, the separation in the Zeeman spectral lines is approximately

- (a) 0.01 nm (b) 0.1 nm (c) 1.0 nm (d) 10 nm

Ans. : (a)

Solution: This is the case of Normal Zeeman effect. The Zeeman separation in terms of frequency, $\Delta\nu = \frac{\mu_B B}{h}$, where μ_B is Bohr magneton

$$\text{In terms of wavelength, } \Delta\lambda = \frac{\lambda^2}{c} \Delta\nu = \frac{\lambda^2}{c} \times \frac{\mu_B B}{h}$$

$$\Delta\lambda = \frac{(250 \times 10^{-9} \text{ m})^2 \times (9.27 \times 10^{-24} \text{ J/T})(1\text{T})}{(3 \times 10^8 \text{ m/s})(6.625 \times 10^{-34} \text{ Js})} \cong 3 \times 10^{-12} \text{ m} \cong 0.003 \text{ nm}$$

None of the answer is matching correctly. But best suitable answer is option (a)

- Q24. If the leading anharmonic correction to the energy of n^{th} vibrational level of a diatomic molecule is $-x_e \left(n + \frac{1}{2} \right)^2 \hbar \omega$ with $x_e = 0.001$, the total number of energy levels possible is approximately
- (a) 500 (b) 1000 (c) 250 (d) 750

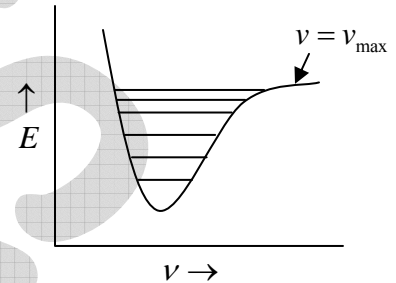
Ans. : (a)

Solution: The energy of anharmonic oscillator is $E_v = \left(v + \frac{1}{2} \right) \hbar \omega - x_e \left(v + \frac{1}{2} \right)^2 \hbar \omega$

where $v = 0, 1, 2, \dots, v_{\text{max}}$ is vibrational quantum number

$$\text{Now, } \left. \frac{dE_v}{dv} \right|_{v=v_{\text{max}}} = 0 \Rightarrow \hbar \omega - 2x_e \left(v_{\text{max}} + \frac{1}{2} \right) \hbar \omega = 0$$

$$\Rightarrow 1 = 2x_e \left(v_{\text{max}} + \frac{1}{2} \right) \Rightarrow v_{\text{max}} = \frac{1}{2x_e} - \frac{1}{2} \cong \frac{1}{2x_e} = \frac{1}{2 \times 0.001} = 500$$



- Q25. The effective spin-spin interaction between the electron spin \vec{S}_e and the proton spin \vec{S}_p in the ground state of the Hydrogen atom is given by $H' = a\vec{S}_e \cdot \vec{S}_p$. As a result of this interaction, the energy levels split by an amount
- (a) $\frac{1}{2} a\hbar^2$ (b) $2a\hbar^2$ (c) $a\hbar^2$ (d) $\frac{3}{2} a\hbar^2$

Ans. : (c)

Solution: The Hamiltonian is given as $H = a\vec{S}_e \cdot \vec{S}_p$ where S_e and S_p are electron and proton spin.

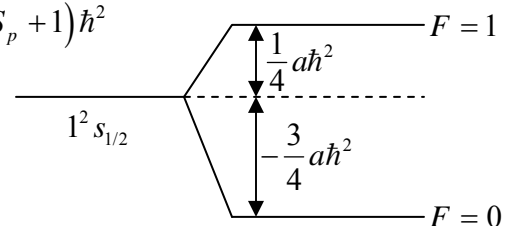
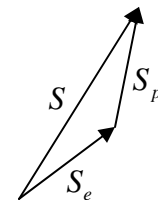
The coupling between S_e and S_p gives net resultant spin angular momentum

$$\vec{S} = \vec{S}_e + \vec{S}_p, \quad S^2 = S_e^2 + S_p^2 + 2\vec{S}_e \cdot \vec{S}_p \Rightarrow \vec{S}_e \cdot \vec{S}_p = \frac{1}{2} (S^2 - S_e^2 - S_p^2)$$

$$\therefore H = \frac{a}{2} (S^2 - S_e^2 - S_p^2)$$

where $S^2 = S(S+1)\hbar^2$, $S_e^2 = S_e(S_e+1)\hbar^2$, $S_p^2 = S_p(S_p+1)\hbar^2$

Since $S_e = \frac{1}{2}$ and $S_p = \frac{1}{2} \Rightarrow S = 0, 1$



For $S = 0$ (singlet state), $H_1 = \frac{a}{2} \left(0 - \frac{3}{4} - \frac{3}{4} \right) \hbar^2 = -\frac{3}{4} a \hbar^2$

For $S = 1$ (Triplet state), $H_2 = \frac{a}{2} \left(2 - \frac{3}{4} \hbar^2 - \frac{3}{4} \hbar^2 \right) = \frac{1}{4} a \hbar^2$

$\therefore \Delta H = H_2 - H_1 = \frac{1}{4} a \hbar^2 + \frac{3}{4} a \hbar^2 \Rightarrow \Delta H = a \hbar^2$

NET/JRF (JUNE-2015)

Q26. Of the following term symbols of the np^2 atomic configurations, 1S_0 , 3P_0 , 3P_1 , 3P_2 and 1D_2 which is the ground state?

- (a) 3P_0 (b) 1S_0 (c) 3P_2 (d) 3P_1

Ans. (a)

Solution: According to Hund's rules

- (i) State with highest multiplicity has lowest energy
- (ii) State with same multiplicity, the state with highest L will have lowest energy
- (iii) State with same multiplicity and L value. The state with lowest J has lowest energy (only if subshell is less than half filled) from the given states 1S_0 , 3P_0 , 3P_1 , 3P_2 , 1D_2

The 3P_0 will have the lowest energy

Q27. A diatomic molecule has vibrational states with energies $E_v = \hbar\omega \left(v + \frac{1}{2} \right)$ and rotational states with energies $E_j = B j(j+1)$, where v and j are non-negative integers. Consider the transitions in which both the initial and final states are restricted to $v \leq 1$ and $j \leq 2$ and subject to the selection rules $\Delta v = \pm 1$ and $\Delta j = \pm 1$. Then the largest allowed energy of transition is

- (a) $\hbar\omega - 3B$ (b) $\hbar\omega - B$ (c) $\hbar\omega + 4B$ (d) $2\hbar\omega + B$

Ans. (c)

Solution: $E = \hbar\omega \left(v + \frac{1}{2} \right) + B J(J+1)$

For vibrational transition with $\Delta v = \pm 1$ and rotational transition with $\Delta J = \pm 1$ we get

$\Delta E = E_{\text{initial}} - E_{\text{final}} = \hbar\omega + 2B(J+1)$

where J is lowest quantum number

According to question in rotational states, two transition is possible, one for $J = 0 \rightarrow J = 1$ and second for $J = 1 \rightarrow J = 2$

But, second transition will have photon of higher energy

\therefore For $J = 1 \rightarrow J = 2$

$$\Delta E = \hbar\omega + 2B(1+1) = \hbar\omega + 4B$$

NET/JRF (DEC-2015)

Q28. The LS configurations of the ground state of ^{12}Mg , ^{13}Al , ^{17}Cl and ^{18}Ar are respectively,

(a) 3S_1 , $^2P_{1/2}$, $^2P_{1/2}$ and 1S_0

(b) 3S_1 , $^2P_{3/2}$, $^2P_{3/2}$ and 3S_1

(c) 1S_0 , $^2P_{1/2}$, $^2P_{3/2}$ and 1S_0

(d) 1S_0 , $^2P_{3/2}$, $^2P_{1/2}$ and 3S_1

Ans. : (c)

Solution: $^{12}Mg : 1s^2 2s^2 2p^6 3s^2$

Ground state: 1S_0

$^{13}Al : 1s^2 2s^2 2p^6 3s^2 3p^1$

the terms are $2p_{1/2}$ and $2p_{3/2}$. Since its less than half filled. Thus

Ground state: $2p_{1/2}$

$^{17}Cl : 1s^2 2s^2 2p^6 3s^2 3p^5$

The terms are $2p_{1/2}$ and $2p_{3/2}$. Since it shell is more than half filled.

Thus Ground state: $2p_{3/2}$

$^{18}Ar : 1s^2 2s^2 2p^6 3s^2 3p^6$

Ground state: 1S_0

Q29. For a two level system, the population of atoms in the upper and lower levels are 3×10^{18} and 0.7×10^{18} , respectively. If the coefficient of stimulated emission is $3.0 \times 10^5 m^3 / W \cdot s^3$ and the energy density is $9.0 J / m^3 \cdot Hz$, the rate of stimulated emission will be

(a) $6.3 \times 10^{16} s^{-1}$

(b) $4.1 \times 10^{16} s^{-1}$

(c) $2.7 \times 10^{16} s^{-1}$

(d) $1.8 \times 10^{16} s^{-1}$

Ans. : **None of the answer is matching.**

Solution: The rate of stimulated emission is

$$\frac{dN_2}{dt} = N_2 B_{21} u(\omega)$$

$$\text{where } N_2 = 3 \times 10^{18}, B_{21} = 3 \times 10^5 \frac{m^3}{W - s^3} = 3 \times 10^5 \frac{m^3}{J - s^2}$$

$$\text{and } u(\omega) = 9.0 \text{ J} / m^3 - \text{Hz} = 9.0 \frac{J - s}{m^3}$$

$$\frac{dN_2}{dt} = 3 \times 10^{18} \times 3 \times 10^5 \frac{m^3}{J - s^2} \times 9 \frac{J - s}{m^3} = 8.1 \times 10^{24} s^{-1}$$

Q30. The first ionization potential of K is 4.34 eV , the electron affinity of Cl is 3.82 eV and the equilibrium separation of KCl is 0.3 nm . Then energy required to dissociate a KCl molecule into a K and a Cl atom is

- (a) 8.62 eV (b) 8.16 eV (c) 4.28 eV (d) 4.14 eV

Ans. : (c)

Solution: Energy required to dissociate KCl is $(KCl \rightarrow K^+ + Cl^-)$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} = \left(9 \times 10^9 \frac{Nm^2}{c^2} \right) \frac{(1.6 \times 10^{-19} c)^2}{0.3 \times 10^{-9} m} = 7.7 \times 10^{-19} \text{ J} = 4.79 \text{ eV}$$

The bond dissociation energy is the energy required to dissociate a molecule into its component atom $KCl \rightarrow K + Cl$

To find the energy required to dissociate KCl into K and Cl , we must add an electron to the K^+ ion, which releases the atomic potassium ionization energy. Remove one electron from Cl^- ion which requires the atomic chlorine electron affinity energy

Given ionization energy of $K = E_{ie} = 4.34 \text{ eV}$

Electron affinity of $Cl = E_{ai} = 3.82 \text{ eV}$

Thus the energy gained in the transformation from ion to atom is

$$= E_{ie} - E_{ai} = 4.34 - 3.82 = 0.52 \text{ eV}$$

Thus dissociation energy $= 4.79 - 0.52 = 4.27 \text{ eV}$

NET/JRF (JUNE-2016)

Q31. The ground state electronic configuration of ^{22}Ti is $[\text{Ar}]3d^24s^2$. Which state, in the standard spectroscopic notations, is not possible in this configuration?

- (a) 1F_3 (b) 1S_0 (c) 1D_2 (d) 3P_0

Ans. : (a)

Solution: The spectroscopic term for d^2 are obtained as

$$l_1 = 2, l_2 = 2 \Rightarrow L = 4, 3, 2, 1, 0$$

$$s_1 = \frac{1}{2}, s_2 = \frac{1}{2} \Rightarrow S = 0, 1$$

Now, $S = 0, L = 4 \Rightarrow J = 4, ^1G_4$

$S = 0, L = 2 \Rightarrow J = 2, ^1D_2$

$S = 0, L = 0 \Rightarrow J = 0, ^1S_0$

$S = 1, L = 3 \Rightarrow J = 4, 3, 2, ^3F_4, ^3F_3, ^3F_2$

$S = 1, L = 1 \Rightarrow J = 2, 1, 0, ^3P_2, ^3P_1, ^3P_0$

Thus 1F_3 is not possible spectroscopic term of $[\text{Ar}]3d^24s^2$

Q32. In a normal Zeeman Effect experiment using a magnetic field of strength 0.3 T , the splitting between the components of a 660 nm spectral line is

- (a) 12 pm (b) 10 pm (c) 8 pm (d) 6 pm

Ans. : (d)

Solution:
$$\Delta\lambda = \frac{\lambda^2}{c} \frac{eB}{4\pi m} = \frac{(660 \times 10^{-9})^2}{3 \times 10^8} \times \frac{1.6 \times 10^{-19} \times 0.3}{4\pi \times 9.1 \times 10^{-31}} = 6.09 \times 10^{-12}\text{ m} = 6\text{ pm}$$

Q33. The separation between the energy levels of a two-level atom is 2 eV . Suppose that 4×10^{20} atoms are in the ground state and 7×10^{20} atoms are pumped into the excited state just before lasing starts. How much energy will be released in a single laser pulse?

- (a) 24.6 J (b) 22.4 J (c) 98 J (d) 48 J

Ans. : (d)

Solution: $N_2 - N_1 = 3 \times 10^{20}$

Energy of laser pulse,
$$E = \left(\frac{N_2 - N_1}{2} \right) h\nu = \frac{3 \times 10^{20}}{2} \times 2 \times 1.6 \times 10^{-19}\text{ J} \Rightarrow E = 48\text{ J}$$

Q36. A two level system in a thermal (black body) environment can decay from the excited state by both spontaneous and thermally stimulated emission. At room temperature (300 K), the frequency below which thermal emission dominates over spontaneous emission is nearest to

- (a) 10^{13} Hz (b) 10^8 Hz (c) 10^5 Hz (d) 10^{11} Hz

Ans. : (d)

Solution: At thermal equilibrium, the ratio of the number of spontaneous to stimulated emission is given by

$$\frac{A_{21}}{B_{21}u(\omega)} = e^{\frac{h\omega}{kT}} - 1; \quad \text{where } \frac{h}{kT} = \frac{1.054 \times 10^{-34}\text{ J.S}}{1.38 \times 10^{-23}\text{ J/K} \times 300\text{ K}} = 2.55 \times 10^{-14}\text{ sec}$$

Now, for $\nu = 10^{13}\text{ Hz} \Rightarrow \omega = 6.3 \times 10^{13}\text{ rad/sec}$,

$$\frac{A_2 1}{B_{21}u(\omega)} = e^{1.6} - 1 = 5 - 1 \cong 4$$

For $\nu = 10^{11}\text{ Hz} \Rightarrow \omega = 6.3 \times 10^{11}\text{ rad/sec}$,

$$\therefore \frac{A_{21}}{B_{21}u(\omega)} = e^{1.6 \times 10^{-3}} - 1 = 1.03 - 1 \cong 0.03$$

This ratio is less than one, thus stimulated thermal emission dominates over spontaneous emission below all frequency of 10^{11} Hz

NET/JRF (JUNE-2017)

Q37. An atomic spectral line is observed to split into nine components due to Zeeman shift. If the upper state of the atom is 3D_2 then the lower state will be

- (a) 3F_2 (b) 3F_1 (c) 3P_1 (d) 3P_2

Ans. : (c)

Solution: The possible zeeman component for different transitions are:

(i) ${}^3D_2 \rightarrow {}^3F_2$: Total zeeman components are 12

(ii) ${}^3D_2 \rightarrow {}^3P_1$: Total zeeman components are 9

(iii) ${}^3D_2 \rightarrow {}^3P_2$: Total zeeman components are 12

3F_1 state is not allowed because for 3F , possible 'J' values are 2, 3 and 4.

Only ${}^3D_2 \rightarrow {}^3P_1$ transition gives 9 components. Therefore, ground state is 3P_1 .

Thus, correct option is (c).

- Q38. If the coefficient of stimulated emission for a particular transition is $2.1 \times 10^{19} m^3 W^{-1} s^{-3}$ and the emitted photon is at wavelength 3000 \AA , then the lifetime of the excited state is approximately
- (a) $20 ns$ (b) $40 ns$ (c) $80 ns$ (d) $100 ns$

Ans. : (c)

Solution: $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$. Life time $= \tau = \frac{1}{A_{21}} = \frac{c^3}{8\pi h\nu^3 B_{21}} = \frac{\lambda^3}{8\pi h B_{21}}$

$$\Rightarrow \tau = \frac{(3000 \times 10^{-10})^3}{8\pi (6.6 \times 10^{-34} J.s.) (2.1 \times 10^{19} m^3 W^{-1} s^{-3})}$$

$$= \frac{2.7 \times 10^{-20}}{3.5 \times 10^{-13}} \approx 7.7 \times 10^{-8} \text{ sec} \approx 80 ns$$

- Q39. If the binding energies of the electron in the K and L shells of silver atom are $25.4 keV$ and $3.34 keV$, respectively, then the kinetic energy of the Auger electron will be approximately
- (a) $22 keV$ (b) $9.3 keV$ (c) $10.5 keV$ (d) $18.7 keV$

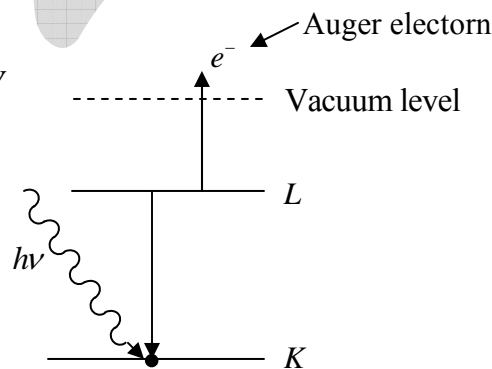
Ans. : (d)

Solution: $K.E.$ Auger electron is $K.E = (E_K - E_L) - E_L$

$$= E_K - 2E_L$$

$$= (25.4 - 2 \times 3.34) keV$$

$$= 18.7 keV$$



NET/JRF (DEC - 2017)

Q40. Consider a system of identical atoms in equilibrium with blackbody radiation in a cavity at temperature T . The equilibrium probabilities for each atom being in the ground state $|0\rangle$ and an excited state $|1\rangle$ are P_0 and P_1 respectively. Let n be the average number of photons in a mode in the cavity that causes transition between the two states. Let $W_{0\rightarrow 1}$ and $W_{1\rightarrow 0}$ denote, respectively, the squares of the matrix elements corresponding to the atomic transitions $|0\rangle \rightarrow |1\rangle$ and $|1\rangle \rightarrow |0\rangle$. Which of the following equations hold in equilibrium?

(a) $P_0 n W_{0\rightarrow 1} = P_1 W_{1\rightarrow 0}$

(b) $P_0 n W_{0\rightarrow 1} = P_1 n W_{1\rightarrow 0}$

(c) $P_0 n W_{0\rightarrow 1} = P_1 W_{1\rightarrow 0} - P_1 n W_{1\rightarrow 0}$

(d) $P_0 n W_{0\rightarrow 1} = P_1 W_{1\rightarrow 0} + P_1 n W_{1\rightarrow 0}$

Ans. : (d)

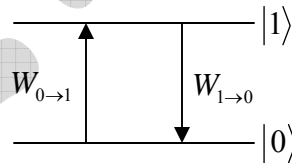
Solution: In equilibrium condition the number of upward transitions must be equal to the number of downward transition.

Rate of upward transition = $P_0 n W_{0\rightarrow 1}$

Rate of downward transition = Rate of spontaneous transitions + Rate of stimulated transition = $P_1 W_{1\rightarrow 0} + P_1 n W_{1\rightarrow 0}$

Thus, at thermal equilibrium

$P_0 n W_{0\rightarrow 1} = P_1 W_{1\rightarrow 0} + P_1 n W_{1\rightarrow 0}$



\therefore Correct option is (d)

Q41. The Zeeman shift of the energy of a state with quantum numbers L, S, J and m_j is

$$H_z = \frac{m_j \mu_B B}{J(J+1)} \left(\langle \vec{L} \cdot \vec{J} \rangle + g_s \langle \vec{S} \cdot \vec{J} \rangle \right)$$

where B is the applied magnetic field, g_s is the g -factor for the spin and

$\frac{\mu_B}{h} = 1.4 \text{ MHz} - G^{-1}$, where h is the Planck constant. The approximate frequency shift of

the $S = 0, L = 1$ and $m_j = 1$ state, at a magnetic field of $1G$, is

(a) 10 MHz

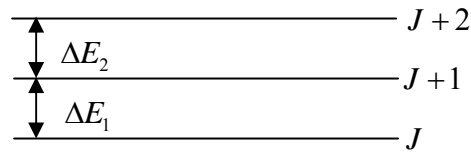
(b) 1.4 MHz

(c) 5 MHz

(d) 2.8 MHz

Ans. : (b)

Solution: $\Delta v = \frac{eB}{4\pi m} = \frac{e\hbar}{2m} \cdot \frac{B}{h} = \frac{\mu_B}{h} B$
 $= (1.4 \text{ MHz } G^{-1})(1G) = 1.4 \text{ MHz}$



Q42. The separations between the adjacent levels of a normal multiplet are found to be 22 cm^{-1} and 33 cm^{-1} . Assume that the multiplet is described well by the $L-S$ coupling scheme and the Lande's interval rule, namely $E(J) - E(J-1) = AJ$, where A is a constant. The term notations for this multiplet is

- (a) ${}^3P_{0,1,2}$ (b) ${}^3F_{2,3,4}$ (c) ${}^3G_{3,4,5}$ (d) ${}^3D_{1,2,3}$

Ans. : (d)

Solution: $\Delta E_1 = E(J+1) - E(J) = A(J+1) = 22 \text{ cm}^{-1}$

$\Delta E_2 = E(J+2) - E(J+1) = A(J+2) = 33 \text{ cm}^{-1}$

$\therefore \frac{A(J+1)}{A(J+2)} = \frac{22}{33} \Rightarrow J = 1$

Thus, the J values for all those three levels are 1, 2, 3. The corresponding terms are ${}^3D_{1,2,3}$.

Q43. If the fine structure splitting between the $2^2P_{3/2}$ and $2^2P_{1/2}$ levels in the hydrogen atom is 0.4 cm^{-1} , the corresponding splitting in Li^{2+} will approximately be

- (a) 1.2 cm^{-1} (b) 10.8 cm^{-1} (c) 32.4 cm^{-1} (d) 36.8 cm^{-1}

Ans. : (c)

Solution: $\Delta E \propto z^4$

$\therefore \frac{(\Delta E)_{Li}}{(\Delta E)_H} = \frac{(z_{Li})^4}{(z_H)^4} = \frac{3^4}{1^4} = 81$

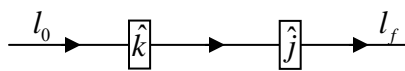
$\Rightarrow (\Delta E)_{Li} = 81(\Delta E_H) = 81 \times 0.4 \text{ cm}^{-1}$

$(\Delta E)_{Li} = 32.4 \text{ cm}^{-1}$

Thus, correct option is (c).

NET/JRF (JUNE-2018)

Q44. Two Stern-Gerlach apparatus S_1 and S_2 are kept in a line (x -axis). The directions of their magnetic fields are along the positive z and y -axes, respectively. Each apparatus only transmits particles with spins aligned in the direction of its magnetic field. If an initially unpolarized beam of spin $\frac{1}{2}$ particles passes through this configuration, the ratio of intensities $I_0 : I_f$ of the initial and final beams is



- (a) 16:1 (b) 2:1 (c) 4:1 (d) 1:0

Ans. : (c)

Solution: $I_f = \frac{I_0}{4}$ $\frac{I_0}{I_f} = \frac{4}{1}$

Q45. A photon of energy 115.62 keV ionizes a K -shell electron of a Be atom. One L -shell electron jumps to the K -shell to fill this vacancy and emits a photon of energy 109.2 keV in the process. If the ionization potential for the L -shell is 6.4 keV , the kinetic energy of the ionized electron is

- (a) 6.42 keV (b) 12.82 keV (c) 20 eV (d) 32 eV

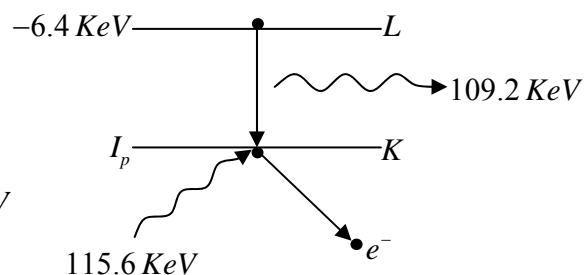
Ans. : (c)

Solution: Binding energy of K -shell electron

$$I_p = 6.4 \text{ KeV} + 109.2 \text{ KeV} = 115.6 \text{ KeV}$$

Thus, K.E. of ionized electron is

$$= 115.62 \text{ KeV} - 115.6 \text{ KeV} = 0.02 \text{ KeV} = 20 \text{ eV}$$



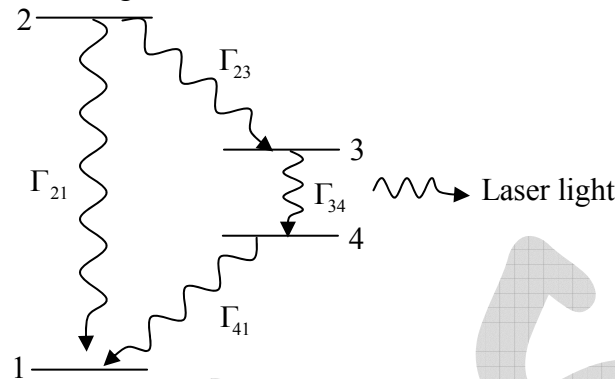
Q46. The value of the Lande g -factor for a fine-structure level defined by the quantum number $L = 1, J = 2$ and $S = 1$, is

- (a) $\frac{11}{6}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{3}{2}$

Ans. : (d)

Solution: $g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} = 1 + \frac{2(2+1) + 2 - 2}{2 \times 2(2+1)} = 1 + \frac{1}{2} = \frac{3}{2}$

Q47. The electronic energy level diagram of a molecule is shown in the following figure,



Let Γ_{ij} denote the decay rate for a transition from the level i to j . The molecules are optically pumped from level 1 to 2. For the transition from level 3 to level 4 to be a lassing transition, the decay rates have to satisfy

- (a) $\Gamma_{21} > \Gamma_{23} > \Gamma_{41} > \Gamma_{34}$ (b) $\Gamma_{21} > \Gamma_{41} > \Gamma_{23} > \Gamma_{34}$
 (c) $\Gamma_{41} > \Gamma_{23} > \Gamma_{21} > \Gamma_{34}$ (d) $\Gamma_{41} > \Gamma_{21} > \Gamma_{34} > \Gamma_{23}$

Ans. : (c)

Solution: The state 3 is metastable state, therefore Γ_{34} would be lowest to enhance population inversion $\Gamma_{41} > \Gamma_{23}$. Thus correct sequence is $\Gamma_{41} > \Gamma_{23} > \Gamma_{21} > \Gamma_{34}$

NET/JRF (DEC - 2018)

Q48. At low temperatures, in the Debye approximation, the contribution of the phonons to the heat capacity of a two dimensional solid is proportional to

- (a) T^2 (b) T^3 (c) $T^{1/2}$ (d) $T^{3/2}$

Ans. : (a)

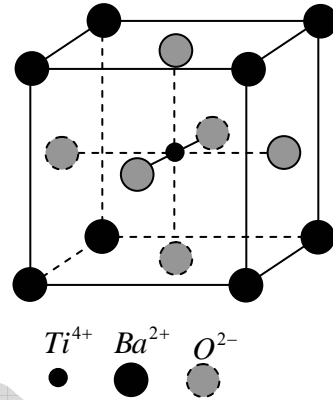
Solution: The dispersion relation of phonons is $\omega = AK$

The phonon specific heat in d -dimension is $C_v \propto T^d$

For 2 dimensional solid $d = 2$

$$C_v \propto T^2$$

Q49. Barium Titanate ($BaTiO_3$) crystal has a cubic perovskite structure, where the Ba^{2+} ions are at the vertices of a unit cube, the O^{2-} ions are at the centres of the faces while the Ti^{2+} is at the centre. The number of optical phonon modes of the crystal is



- (a) 12 (b) 15
(c) 5 (d) 18

Ans. : (a)

Solution: Effective number of atoms per unit cell is

$$n_{eff} = \frac{1}{8} \times 8 + \frac{1}{2} \times 6 + 1 \times 1 = 1 + 3 + 1 = 5$$

Total degree of freedom $5 \times 3 = 15$

The number of Acoustical phonon modes = 3

The number of optical phonon modes $15 - 3 = 12$

Q50. The dispersion relation of optical phonons in a cubic crystal is given by $\omega(k) = \omega_0 - ak^2$ where ω_0 and a are positive constants. The contribution to the density of states due to these phonons with frequencies just below ω_0 is proportional to

- (a) $(\omega_0 - \omega)^{1/2}$ (b) $(\omega_0 - \omega)^{3/2}$ (c) $(\omega_0 - \omega)^2$ (d) $(\omega_0 - \omega)$

Ans. : (a)

Solution: $g(k) dk = \left(\frac{L}{2\pi}\right)^3 4\pi k^2 dk = \left(\frac{L}{2\pi}\right)^3 4\pi k - k dk$

Given $\omega = \omega_0 - ak^2 \Rightarrow k = \frac{1}{\sqrt{a}}(\omega_0 - \omega)^{1/2}$ and $d\omega = -2ak dk$

$$\therefore g(\omega) d\omega = \left(\frac{L}{2\pi}\right)^3 \frac{4\pi}{2a^{3/2}} (\omega_0 - \omega)^{1/2}$$

Thus, $\rho(\omega) \propto (\omega_0 - \omega)^{1/2}$

Q51. A silicon crystal is doped with phosphorus atoms. (The binding energy of a H atom is $13.6 eV$, the dielectric constant of silicon is 12 and the effective mass of electrons in the crystal is $0.4m_e$). The gap between the donor energy level and the bottom of the conduction band is nearest to

- (a) $0.01 eV$ (b) $0.08 eV$ (c) $0.02 eV$ (d) $0.04 eV$

Ans. : (d)

$$\text{Solution: } E_d = \frac{13.6}{\epsilon^2} \times \frac{M^*}{M_e} (eV)$$

$$\therefore E_d = \frac{13.6}{(12)^2} \times 0.4 = 0.04 eV$$

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