GATE 2022 Physics (PH)

GATE 2022 General Aptitude

Q.1 – Q.5 Carry ONE mark each.

Q1. You should _________ when to say _________.

(a) no / no (b) no / know (c) know / know (d) know / no

Ans. 1: (d)

Solution: You should know when to say no.

Q2. Two straight lines pass through the origin $(x_0, y_0) = (0,0)$. One of them passes through the point $(x_1, y_1) = (1,3)$ and the other passes through the point $(x_2, y_2) = (1,2)$. What is the area enclosed between the straight lines in the interval $[0,1]$ on the x-axis?

(a) 0.5 (b) 1.0 (c) 1.5 (d) 2.0

Ans. 2: (a)

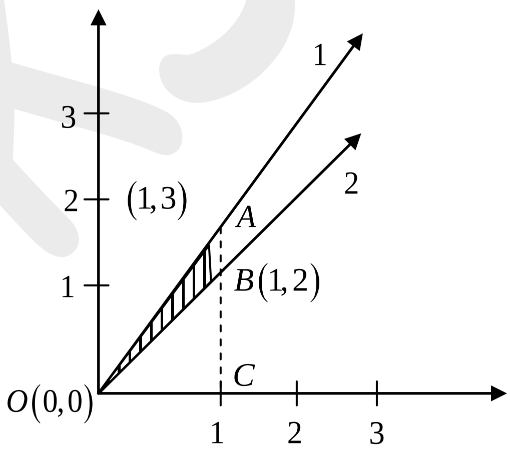
Solution: From fig, it is obvious that,

$$\text{area}(\Delta OBA) = \text{area}(\Delta OCA) - \text{area}(\Delta OCB)$$

$$= \frac{1}{2}\times OC\times CA - \frac{1}{2}\times OC\times CB$$

$$= \frac{1}{2}\times 1\times 3 - \frac{1}{2}\times 1\times 2$$

$$= \frac{1}{2} = 0.5$$



Q3. If $p:q = 1:2$; $q:r = 4:3$; $r:s = 4:5$ and u is 50% more than s, what is the ratio $p:u$?

(a) 2:15 (b) 16:15 (b) 1:5 (d) 16:45

Ans. 3: (d)

Solution: It is given that u is 50% more that s. That is,

$$u = \left(1 + \frac{50}{100}\right)s$$

or, $s:u = 2:3$

Given ,

$$\frac{p}{q} = \frac{1}{2};\ \frac{q}{r} = \frac{4}{3};\ \frac{r}{s} = \frac{4}{5};\ \frac{s}{u} = \frac{2}{5}$$

Multiplying then together, we get

$$\frac{p}{q}\times\frac{q}{r}\times\frac{r}{s}\times\frac{s}{u} = \frac{1}{2}\times\frac{4}{3}\times\frac{4}{5}\times\frac{2}{3} = \frac{16}{45}$$

Q4. Given the statements:

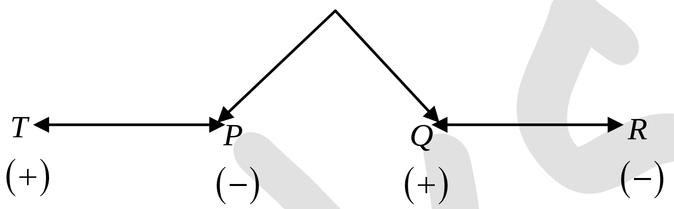
- P is the sister of Q.
- Q is the husband of R.
- R is the mother of S.
- T is the husband of P.

Based on the above information, T is ______ of S.

(a) the grandfather (b) an uncle (c) the father (d) a brother

Ans. 4: (b)

Solution: Based are given data, the relationship diagram will be

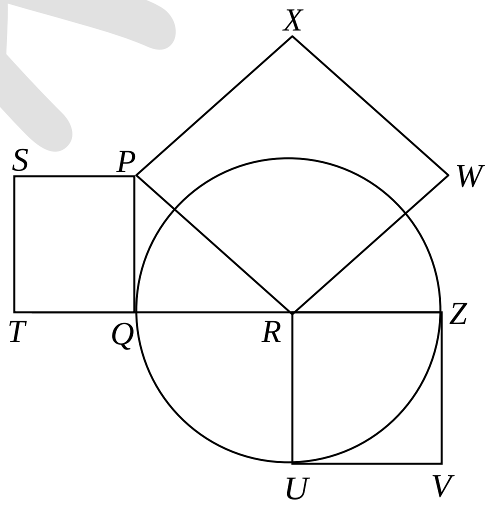


Where $\leftrightarrow$ denotes a couple

$+/-$ denote Gender.

Hence, T is uncle of S, as is obvious from the diagram.

Q5. In the following diagram, the point R is the center of the circle. The lines PQ and ZV are tangential to the circle. The relation among the areas of the squares, PXWR, RUVZ and SPQT is



(a) Area of SPQT= Area of RUVZ= Area of PXWR

(b) Area of SPQT= Area of PXWR − Area of RUVZ

(c) Area of PXWR= Area of SPQT- Area of RUVZ

(d) Area of PXWR= Area of RUVZ- Area of SPQT

Ans. 5: (b)

Solution: PQ is tangent of the circle of radius QR, Hence $PQ \perp r\ OR$

So, ΔPQR is right triangle.

Hence, $(PR)^2 = (QR)^2 + (PQ)^2$

$QR = RZ$ (being radius of the circle)

So, $(PR)^2 = (RZ)^2 + (PQ)^2$

That is, $\text{area}(P\times WR) = \text{area}(RZUV) + \text{area}(SRQT)$

$\therefore \quad \text{area}(SPQT) = \text{area}(P\times WR) - \text{area}(RZUV)$

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Q. 6 – Q. 10 Carry TWO marks each.

Q6. Healthy eating is a critical component of healthy aging. When should one start eating healthy? It turns out that it is never too early. For example, babies who start eating healthy in the first year are more likely to have better overall health as they get older.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(a) Healthy eating is important for those with good health conditions, but not for others

(b) Eating healthy can be started at any age, earlier the better

(c) Eating healthy and better overall health are more correlated at a young age, but not older age

(d) Healthy eating is more important for adults than kids

Ans. 6: (b)

Solution: (a) Incorrect option as passage doesn't mention healthy eating for those with good health condition. Extraneous assumption makes it incorrect choice.

(c) Passage takes about healthy again, it doesn't talk about its better correlation is young age then old age.

(d) Passage tractions that earliest the start of eating healthy better it is for healthy in old age. So no point of eating more important in adults then in kids.

So, Only option consistent with given paragraph is (b).

Q7. P invested ₹ 5000 per month for 6 months of a year and Q invested ₹ x per month for 8 months of the year in a partnership business. The profit is shared in proportion to the total investment made in that year. If at the end of that investment year, Q receives $\frac{4}{9}$ of the total profit, what is the value of x (in ₹)?

(a) 2500 (b) 3000 (c) 4687 (d) 8437

Ans. 7: (b)

Solution:

	Investment/month	Month	Total Investment
P	£5000	6	£30000
Q	£x	8	£8x

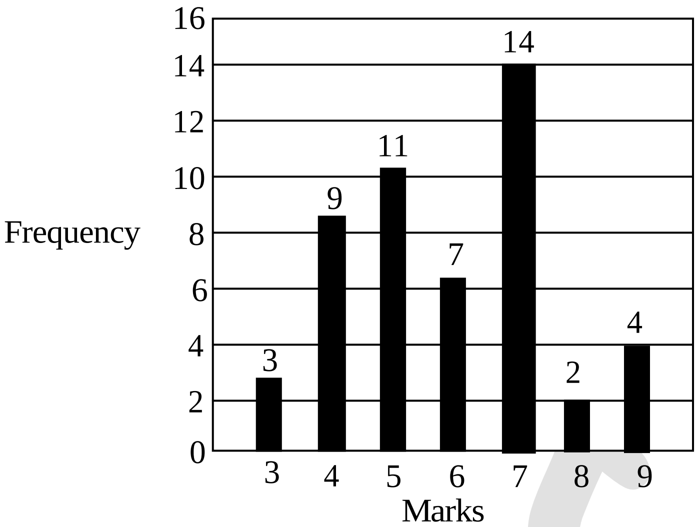
Ratio of profit of $P \& Q$:

$P:Q = 30,000:8x$

$\therefore Q's$ share $= \frac{8x}{30,000+8x}$

Which is $\frac{4}{9}$ of total profit, that is $\frac{8x}{30,000+8x} = \frac{4}{9}$ or, $x = 3000$.

Q8.



The above frequency chart shows the frequency distribution of marks obtained by a set of students in an exam.

From the data presented above, which one of the following is CORRECT?

(a) mean $>$ mode $>$ median

(b) mode $>$ median $>$ mean

(c) mode $>$ mean $>$ median

(d) median $>$ mode $>$ mean

Ans. 8: (b)

Solution: Mean (marks) $= \dfrac{3\times3+4\times9+5\times11+6\times7+7\times14+8\times2+3\times4}{3+9+11+7+14+2+4} = 5.84$

Mode $=$ marks obtained by maximum number of students $=7$

Median: Arrange the marks of all students then halve it

Students	3....3	4....4	5....5	6....6	7....7	8....8	9....9
	3	9	11	7	14	2	4

So, median $=6$

Hence, Mode $=7$; Median $=6$; Mean $=5.84$

That is, mode $>$ medium $>$ mean

Q9. In the square grid shown on the left, a person standing at $P2$ position is required to move to $P5$ position. The only movement allowed for a step involves, "two moves along one direction followed by one move in a perpendicular direction". The permissible directions for movement are shown as dotted arrows in the right.

For example, a person at a given position Y can move only to the positions marked X on the right. Without occupying any of the shaded squares at the end of each step, the minimum number of steps required to go from $P2$ to $P5$ is

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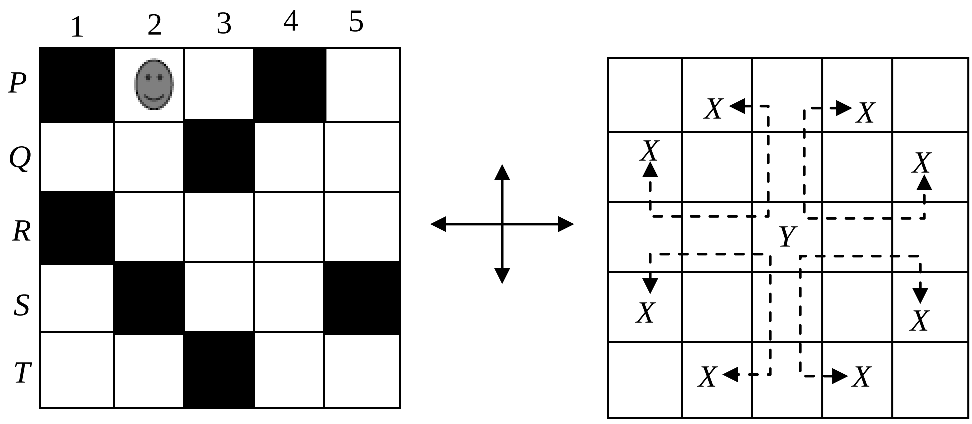


Example: Allowed steps for a Person at Y

(a) 4 (b) 5 (c) 6 (d) 7

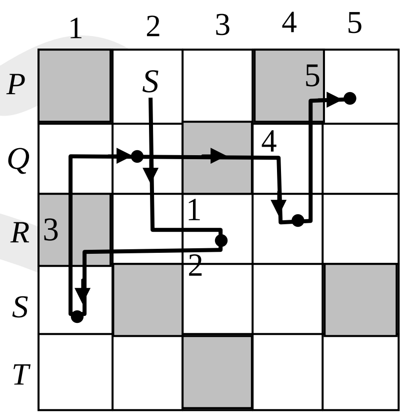
Ans. 9: (b)

Solution: Only blokes from which the person can take a final step are

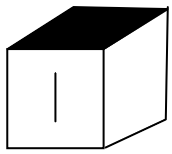
$Q3$ and $R4$

But $Q3$ is not allowed.

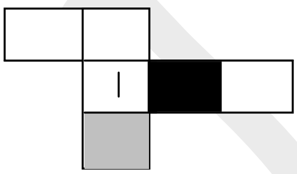
Routes and steps are shown fig.



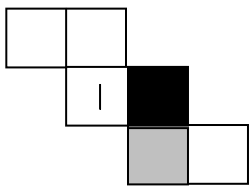
Q10. Consider a cube made by folding a single sheet of paper of appropriate shape. The interior faces of the cube are all blank. However, the exterior faces that are not visible in the above view may not be blank. Which one of the following represents a possible unfolding of the cube?



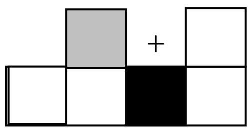
(a)



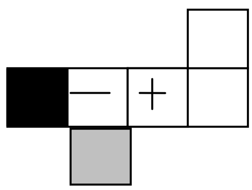
(b)



(c)



(d)



Ans. 10: (d)

Solution: The way cube is unfolded is case of (a) and (b), vertical line (1) cannot be adjacent to black- shaded surface. So, choice (a) & (b) are incorrect.

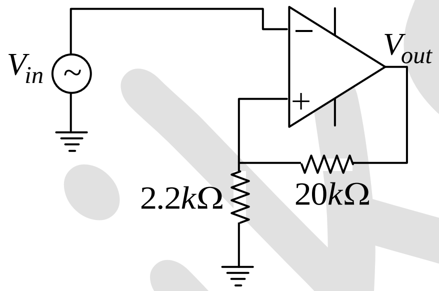
Also, Whenever black-shaded surface is in sight, vertical line (1) shade will be visible.

So, option (c) not a possible unfolding

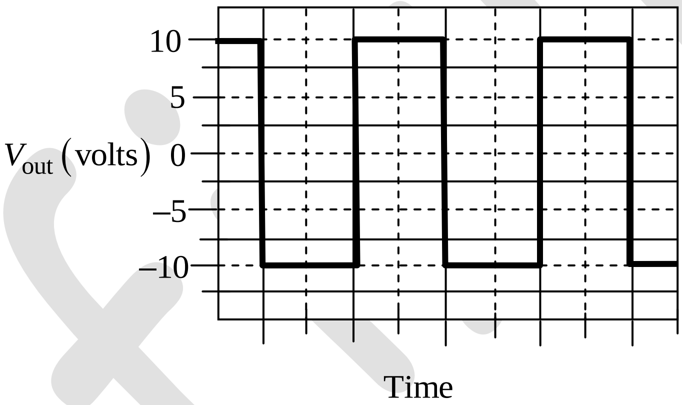
Hence, correct choice is (d).

Q.11 – Q.35 Carry ONE mark Each

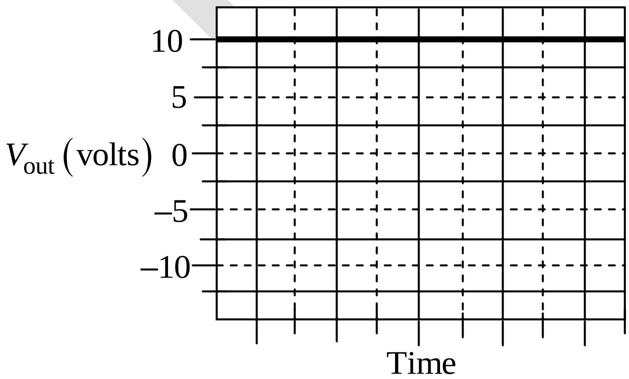
Q11. For the Op-Amp circuit shown below, choose the correct output waveform corresponding to the input $V_{in} = 1.5\sin 20\pi t$ (in Volts). The saturation voltage for this circuit is $V_{sat} = \pm 10V$.



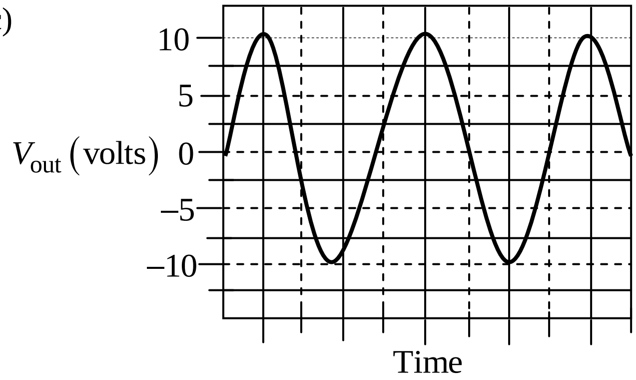
(a)



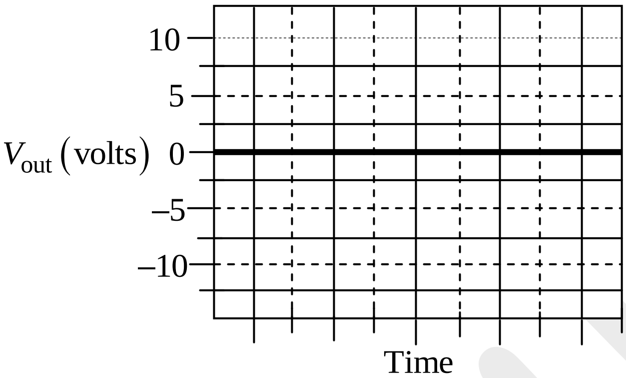
(b)



(c)



(d)



Ans. 11:(a)

Solution:

Given circuit is an Schmitt Trigger circuit. In this output will be always saturated i.e. limited between $+V_{sat}$ to $-V_{sat}$

Q12. Match the order of $\beta-$ decays given in the left column to appropriate clause in the right column. Here $X\left(I^{\pi}\right)$ and $Y\left(I^{\pi}\right)$ are nuclei with intrinsic spin I and parity π.

1. $X\left(\frac{1^+}{2}\right)\rightarrow Y\left(\frac{1^+}{2}\right)$	(i) First forbidden β–decay
2. $X\left(\frac{1^-}{2}\right)\rightarrow Y\left(\frac{5^+}{2}\right)$	(ii) Second forbidden β–decay
3. $X\left(3^+\right)\rightarrow Y\left(0^+\right)$	(iii) Third forbidden β-decay
4. $X\left(4^-\right)\rightarrow Y\left(0^+\right)$	(iv) Allowed β-decay

(a) 1 – i, 2 – ii, 3 – iii, 4 – iv

(b) 1 – iv, 2 – i, 3 – ii, 4 – iii

(c) 1 – i, 2 – iii, 3 – ii, 4 – iv

(d) 1 – iv, 2 – ii, 3 – iii, 4 – i

Ans. 12: (b)

Solution: (1) $\Delta I = 0,\ \Delta\pi = \text{No}$

Allowed β - decay

(2) $\Delta I = 2, \Delta\pi =$ YES

First forbidden β - decay

(3) $\Delta I = 3,\ \Delta\pi = \text{No}$

Second forbidden β - decay

(4) $\Delta I = 4,\ \Delta\pi =$ YES

Third forbidden β - decay

Q13. What is the maximum number of free independent real parameters specifying an n-dimensional orthogonal matrix?

(a) $n(n-2)$ (b) $(n-1)^2$ (c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

Ans. 13: (c)

Solution: Consider a 2×2 orthogonal matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. As the matrix is orthogonal

$\Rightarrow A^T A = I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = I$$

$$\begin{bmatrix} a & c \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = I$$

$$\begin{bmatrix} a^2+c^2 & ab+cd \\ ab+cd & b^2+d^2 \end{bmatrix} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus we have

$a^2+c^2 = 1$ and $b^2+d^2 = 1$, Two constraints on diagonal elements.

This will make two parameters components dependent.

And $ab+cd=1$, one constraint ($\frac{1}{2}C$ No. of diagonal elements) on the value of diagonal elements. This will make one more component dependent.

Thus total no. of m dependent

Components $= 4-3 = 1$

$$= \left[\frac{n(n-1)}{2}\right] \quad \{n = 2 \text{ for } 2\times 2 \text{ matrix}\}.$$

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Generalizing for $n \times n$ matrix.

Total components $= n^2$

n-diagonal elements are dependent because of n-constrain

$\frac{1}{2}(n^2 - n)$- diagonal elements are dependent.

$\Rightarrow \frac{n^2 - n}{2} + n = \frac{n(n+1)}{2}$ elements are dependent

Hence $n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}$ elements ax independent.

Q14. An excited state of Ca atom is $[Mg]3p^5 4s^2 3d^1$. The spectroscopic terms corresponding to the total orbital angular momentum are

(a) S, P, and D (b) P, D and F (c) P and D (d) S and P

Ans. 14: (b)

Solution: We ignore the electrons in the [Mg] core and the electrons in the 4s block as well. We have to consider only the p electron, (l_1 = 1) and d electron (l_2 = 2). Thus, total orbital angular momentum L = | l_1+ l_2 |… | l_1 - l_2 | = 3, 2, 1 i.e P, D and F

Q15. On the surface of a spherical shell enclosing a charge free region, the electrostatic potential values are as follows: One quarter of the area has potential ϕ_0, another quarter has potential $2\phi_0$ and the rest has potential $4\phi_0$. The potential at the centre of the shell is

(You can use a property of the solution of Laplace's equation.)

(a) $\frac{11}{4}\phi_0$ (b) $\frac{11}{2}\phi_0$ (c) $\frac{7}{3}\phi_0$ (d) $\frac{7}{4}\phi_0$

Ans. 15: (a)

Solution:

$$V_{in}(r,\theta) = \sum A_\alpha r^\alpha P_\alpha(\cos\theta) = A_0 r^0 P_0(\cos\theta) + A_1 r^1 P_1(\cos\theta) + ...$$

$$V_{in}(r,\theta) = A_0 + A_1 r P_1(\cos\theta) + ...$$

$V_{in}(0,\theta) = A_0 + A_1 \times 0 + ...$ All other terms are zero.

Thus potential at centre is only decided by A_0 ; $A_0 = ?$

Also, it can be shown that surface area of sphere from $0 = 0$ to $\frac{\pi}{3}$ and $\frac{\pi}{3} = \frac{\pi}{2}$ and $\frac{\pi}{2}$ to π is respectively $\pi R^2, \pi R^2$ and $2\pi R^2$

Let's apply the boundary condition, which is $V(R,\theta)=\begin{Bmatrix} \phi_0, & 0<\theta<\frac{\pi}{3} \\ 2\phi_0, & \frac{\pi}{3}<\theta<\frac{\pi}{2} \\ 4\phi_0, & \frac{\pi}{2}<\theta<\pi \end{Bmatrix}$

Thus $V(R,\theta)=\sum A_\alpha R^\alpha P_\alpha(\cos\theta)$

Now $A_\alpha=\frac{2\alpha+1}{2R^\alpha}\int_0^\pi V(R,\theta)P_\alpha(\cos\theta)\sin\theta\,d\theta$ {Griffith page 140 3rd edition}

As we only want A_0

$$\Rightarrow A_0=\frac{2\times 0+1}{2R^0}\int_0^\pi V(R,\theta)P_0(\cos\theta)\sin\theta d\theta$$

$$P_0(\cos\theta)=1$$

$$A_0=\frac{1}{2}\left[\int_0^{\pi/3}\phi_0(\sin\theta)d\theta+\int_{\pi/3}^{\pi/2}2\phi_0(\sin\theta)d\theta+\int_{\pi/2}^{\pi}(\phi_0)\sin\theta\,d\theta\right]$$

$$A_0=\left[\phi_0\times-|\cos\theta|_0^{\pi/3}+2\phi_0\times-|\cos\theta|_{\pi/3}^{\pi/2}+4\phi_0\times-|\cos\theta|_{\pi/2}^{\pi}\right]$$

$$\Rightarrow A_0=\frac{1}{2}\left[\phi_0\left(\frac{1}{2}-1\right)-2\phi_0\left(-\frac{1}{2}+0\right)+4\phi_0\right]$$

$$\Rightarrow A_0=\frac{1}{2}\left[+\frac{\phi_0}{2}+\phi_0+4\phi_0\right]=\frac{1}{2}\left[\frac{(1+2+8)\phi_0}{2}\right]=\frac{11}{4}\phi_0=\frac{11}{4}\phi_0.$$

As $V_{in}(0,\theta)=A_0$, Potential at centre $=\frac{11}{4}\phi_0$

Q16. A point charge q is performing simple harmonic oscillations of amplitude A at angular frequency ω. Using Larmor's formula, the power radiated by the charge is proportional to

(a) $q\omega^2A^2$ (b) $q\omega^4A^2$ (c) $q^2\omega^2A^2$ (d) $q^2\omega^4A^2$

Ans. 16: (d)

Solution:

$$p(t)=q\,x(t)=qA\omega\,\&\,\omega t$$

$$\dot{p}=-qA\omega\sin\omega t\Rightarrow\ddot{p}=-qA\omega^2\cos^2\omega t$$

$$\langle P\rangle\propto(\ddot{p})^2\propto q^2A^2\omega^4$$

Q17. Which of the following relationship between the internal energy U and the Helmholtz's free energy F is true?

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(a) $U = -T^2\left[\frac{\partial\left(\frac{F}{T}\right)}{\partial T}\right]_V$ (b) $U = +T^2\left[\frac{\partial\left(\frac{F}{T}\right)}{\partial T}\right]_V$

(c) $U = +T\left[\frac{\partial F}{\partial T}\right]_V$ (d) $U = -T\left[\frac{\partial F}{\partial T}\right]_V$

Ans. 17: (a)

Solution:

$$F = U - TS \quad (1)$$

$$dF = -SdT - PdV \quad (2)$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V \quad (3)$$

$$F = U + T\left(\frac{\partial F}{\partial T}\right)_V$$

$$U = F - T\left(\frac{\partial F}{\partial T}\right)_V \quad (4)$$

Now $\left[\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right]_V = \frac{1}{T}\left(\frac{\partial F}{\partial T}\right)_V - \frac{F}{T^2}$

$$= -\frac{1}{T^2}\left[F - T\left(\frac{\partial F}{\partial T}\right)_V\right]$$

$$U = F - T\left(\frac{\partial F}{\partial T}\right)_V = -T^2\left[\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right]_V$$

Q18. If nucleons in a nucleus are considered to be confined in a three-dimensional cubical box, then the first four magic numbers are

(a) 2,8,20,28 (b) 2,8,16,24 (c) 2,8,14,20 (d) 2,10,16,28

Ans. 18: (c)

Solution: $E = \left(n_x^2 + n_y^2 + n_z^2\right)\frac{\pi^2\hbar^2}{2mL^2}$

n_x	n_y	n_z	
(1	1	1)	(2)
(1	1	2)	(2)
(2	1	1)	(2)

$(1$	2	$1)$	(2)	(8)	
$(1$	2	$2)$	(2)		
$(2$	1	$2)$	(2)		
$(2$	2	$1)$	(2)		(14)
$(1$	1	$3)$	(2)		
$(1$	3	$1)$	(2)		
$(3$	1	$1)$	(2)		(20)

Q19. Consider the ordinary differential equation

$$y''-2xy'+4y=0$$

and its solution $y(x)=a+bx+cx^2$. Then

(a) $a=0, c=-2b\neq 0$

(b) $c=-2a\neq 0, b=0$

(c) $b=-2a\neq 0, c=0$

(d) $c=2a\neq 0, b=0$

Ans. 19:(b)

Solution: $y''-2xy'+4y=0$ (1)

Given solution $y(x)=a+bx+cx^2$

$y'=b+2cx$

$y''=2c$

Put the value of y, y' and $y''m$ (1), we get

$2c-2x[b+2cx]+4\left[a+bx+cx^2\right]=0$

$2c-2bx-4cx^2+4a+4bx+4cx^2=0$

$2bx+(2c+4a)=0$

$\Rightarrow b=0$ and $2c+4a=0$

$\Rightarrow c=-2a$

Thus, $b=0$ and $c=-2a$

Q20. For an Op-Amp based negative feedback, non-inverting amplifier, which of the following statements are true?

(a) Closed loop gain $<$ Open loop gain

(b) Closed loop bandwidth $<$ Open loop bandwidth

(c) Closed loop input impedance $>$ Open loop input impedance

(d) Closed loop output impedance $<$ Open loop output impedance

Ans. 20: (a),(c), (d)

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Q21. From the pairs of operators given below, identify the ones which commute. Here l and j correspond to the orbital angular momentum and the total angular momentum, respectively.

(a) l^2, j^2 (b) j^2, j_z (c) j^2, l_z (d) l_z, j_z

Ans. 21: (a), (b), (d)

Solution: The commutator relation between $J_1^2, L^2, S^2,\ J_z,\ \vec{L}.\vec{S},\ \vec{S}_z,\ \vec{L}_z$ are as follows,

(i) J^2, L^2, S^2, Jz commutes with $\vec{L}.\vec{S}$ but not L_z and S_z

Thus option (c) is incorrect.

Q22. For normal Zeeman lines observed $\|$ and $\perp$ to the magnetic field applied to an atom, which of the following statements are true?

(a) Only π-lines are observed $\|$ to the field

(b) σ-lines $\perp$ to the field are plane polarized

(c) π-lines $\perp$ to the field are plane polarized

(d) Only σ-lines are observed $\|$ to the field

Ans. 22: (b), (c) (d)

Q23. Pauli spin matrices satisfy

(a) $\sigma_\alpha\sigma_\beta - \sigma_\beta\sigma_\alpha = i \in_{\alpha\beta\gamma} \sigma_\gamma$ (b) $\sigma_\alpha\sigma_\beta - \sigma_\beta\sigma_\alpha = 2i \in_{\alpha\beta\gamma} \sigma_\gamma$

(c) $\sigma_\alpha\sigma_\beta + \sigma_\beta\sigma_\alpha = \in_{\alpha\beta\gamma} \sigma_\gamma$ (d) $\sigma_\alpha\sigma_\beta + \sigma_\beta\sigma_\alpha = 2\delta_{\alpha\beta}$

Ans. 23: (b), (d)

Solution: General anti commutator relation.

Let us verify option (b)

$$\sigma_x\sigma_y - \sigma_y\sigma_x = 2i\sigma_z$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_x\sigma_y - \sigma_y\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} = 2i\sigma_z$$

Let us verify the relation in option (a).

$$\sigma_x\sigma_y + \sigma_y\sigma_x = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Thus, option (b) and (d) are correct option.

Q24. For the refractive index $n = n_r(\omega) + in_{im}(\omega)$ of a material, which of the following statements are correct?

(a) n_r can be obtained from n_{im} and vice versa

(b) n_{im} could be zero

(c) n is an analytic function in the upper half of the complex ω plane

(d) n is independent of ω for some materials

Ans. 24: (a), (c)

Q25. Complex function $f(z) = z + |z-a|^2$ (a is a real number) is

(a) continuous at (a, a)

(b) complex-differentiable at (a, a)

(c) complex-differentiable at $(a, 0)$

(d) analytic at $(a, 0)$

Ans. 25: (a), (c)

Solution: $f(z) = z + |z-a|^2$

$$= x + iy + |x - a + iy|^2$$

$$= x + (x-a)^2 + y^2 + iy$$

Continuity at (a, a)

Parallel to x-axis

$$\underset{z \to a}{\text{Lt}} f(z) = \underset{x \to a}{\text{Lt}} \left[f(z) \right]_{y=a}$$

$$= \underset{x \to a}{\text{Lt}} \left[x + (x-a)^2 + a^2 + ia \right]$$

$$= a + a^2 + ia \qquad (1)$$

Parallel to y-axis

$$\underset{z \to a}{\text{Lt}} f(z) = \underset{y \to a}{\text{Lt}} \left[f(z) \right]_{x=a}$$

$$= \underset{y \to a}{\text{Lt}} \left[x + (x-a)^2 + y^2 + iy \right]_a$$

$$= \underset{y \to a}{\text{Lt}} \left[a + 0 + y^2 + iy \right] = a + a^2 + ia \qquad (2)$$

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Along line having slope (m) passing through (a,a)

Equation of line is $y = a + m(x-a)$

$$\underset{z \to a}{\text{Lt}} f(z) = \underset{x \to a}{\text{Lt}} \left[x + (x-a)^2 + (a + m(x-a))^2 + i(a + m(x-a)) \right]$$

$$= a + 0 + (a + m0)^2 + i(a + m0)$$

$$= a + a^2 + ia \qquad (3)$$

As $(1) = (2) = (3)$

Hence function is continuous at (a,a)

Differentiability at $(a,0)$

By definition, the derivative of the function at $(a,0)$ is

$$f'(a,0) = \underset{\Delta z \to 0}{\text{Lt}} \frac{f(a + i0 + \Delta z) - f(a + i0)}{\Delta z}$$

$$= \frac{f(a + \Delta z) - f(a)}{\Delta z}$$

Since $f(z) = z + |z-a|^2$

$$f(a) = a + |a-a|^2 = a$$

$$f(a + \Delta z) = a + \Delta z + |a + \Delta z - a|^2$$

$$= a + \Delta z + |\Delta z|^2$$

Thus $f'(a,0) = \underset{\Delta z \to 0}{\text{Lt}} \dfrac{a + \Delta z + |\Delta z|^2 - a}{\Delta z}$

$$= \underset{\Delta z \to 0}{\text{Lt}} \Delta z \frac{[1 + \Delta z^*]}{\Delta z} = \underset{\Delta z \to 0}{\text{Lt}} (1 + \Delta z^*)$$

Parallel to x-axis:- $\Delta y = 0, \Delta x \to 0$

$$f'(a,0) = \underset{\Delta x \to 0}{\text{Lt}} [1 + \Delta x - i\Delta y]_{\Delta y = 0}$$

$$= \underset{\Delta x \to 0}{\text{Lt}} [1 + \Delta x] = 1 + 0 = 1 \qquad (4)$$

n to y-axis:- $\Delta x = 0, \Delta y \to 0$

$$f'(a,0) = \underset{\Delta y \to 0}{Lt} [1 + 0 - i\Delta y] = 1 - i \cdot 0 = 1 \qquad (5)$$

Along line having slope $'m'$ passing through $(a,0)$

Equation of line is $y=m(x-a)$

$\Delta y = m\Delta x$

$$f'(a,0)=\underset{\Delta x\to 0}{\text{Lt}}\left[1+\Delta x-im\Delta x\right]$$

$$=1+0-im0=1 \qquad (6)$$

As $(4)=(5)=(6)$

Hence function is differentiable at $(a,0)$

Thus ‘a’ and ‘c’ are correct options.

Q26. If $g(k)$ is the Fourier transform of $f(x)$ then which of the following are true?

(a) $g(-k)=+g^*(k)$ implies $f(x)$ is real

(b) $g(-k)=-g^*(k)$ implies $f(x)$ is purely imaginary

(c) $g(-k)=+g^*(k)$ implies $f(x)$ is purely imaginary

(d) $g(-k)=-g^*(k)$ implies $f(x)$ is real

Ans. 26: (a), (b)

Solution: As per statements

$$g(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-ikx}dx \qquad (1)$$

taking complex conj of (1)

$$g^*(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f^*(x)e^{ikx}dx \qquad (2)$$

Replacing k by $-k$ $m(1)$

$$g(-k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{ikx}dx \qquad (3)$$

Now if $g(-k)=g^*(k)$ {condition ‘a’}

$\Rightarrow (2)=(3)$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f^*(x)e^{ikx}dx=\frac{1}{\sqrt{2\pi}}\int f(x)e^{ikx}dx$$

$\Rightarrow f(x)=f^*(x)$

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Hence $f(x)$ must be real and not purely imaginary

{Condition 'b'}

$g(-k) = -g^*(k)$

$$\Rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ikx} dx = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f^*(x) e^{ikx} dx$$

$\Rightarrow f(x) = -f^*(x)$. Thus $f(x)$ must be purely imaginary and not real.

Thus 'a' and 'b' are correct options.

Q27. The ordinary differential equation

$$(1-x^2)y'' - xy' + 9y = 0$$

has a regular singularity at

(a) -1 (b) 0 (c) $+1$ (d) no finite value of x

Ans. 27: (a), (c)

Solution: $(1-x^2)y'' - xy' + 9y = 0$

Dividing by $1-x^2$

$$y'' - \frac{x}{1-x^2}y' + \frac{9}{1-x^2}y = 0$$

Compare with $y'' + p(x)y' + Q(x)y = 0$

$$P(x) = \frac{-x}{1-x^2},\ Q(x) = \frac{9}{1-x^2}$$

At 1 and -1 both $P(x)$ and $Q(x)$ diverge first condition satisfied.

At $x = 1$

$$(x-1)P(x) = (x-1)\frac{-x}{1-x^2} = (x-1)\cdot\frac{x}{x^2-1} = \frac{x}{x+1} = \frac{1}{2} \text{ finite.}$$

At $x = -1$

$$(x+1)P(x) = (x+1)\cdot\frac{x}{x^2-1} = \frac{x}{x-1} = \frac{-1}{-1-1} = \frac{1}{2}$$

Thus $(x-x_0)P(x)$ remains finite .

At $x = 1$

$$(x-1)^2 Q(x) = (x-1)^2 \frac{9}{(1-x)(1+x)} = -9\cdot\frac{x-1}{x+1} = -9\times\frac{0}{2} = 0$$

At $x = -1$

$$\left(x+1\right)^2 Q\left(x\right)=\left(x+1\right)^2\cdot\frac{-9}{\left(1+x\right)\left(1-x\right)}=-9\cdot\frac{\left(x+1\right)}{\left(x-1\right)}=-9\times\frac{0}{-2}=0$$

Thus $\left(x-x_0\right)^2 Q\left(x\right)$ remains finite.

Thus both 1 and -1 are regular singular points.

Q28. For a bipolar junction transistor, which of the following statements are true?

(a) Doping concentration of emitter region is more than that in collector and base region

(b) Only electrons participate in current conduction

(c) The current gain β depends on temperature

(d) Collector current is less than the emitter current

Ans. 28: (a), (c), (d)

Q29. Potassium metal has electron concentration of $1.4\times10^{28}m^{-3}$ and the corresponding density of states at Fermi level is $6.2\times10^{46}\text{Joule}^{-1}m^{-3}$. If the Pauli paramagnetic susceptibility of Potassium is $n\times10^{-k}$ in standard scientific form, then the value of k (an integer) is __________

(Magnetic moment of electron is $9.3\times10^{-24}\text{Joule}\,T^{-1}$; permeability of free space is $4\pi\times10^{-7}Tm\,A^{-1}$)

Ans. 29: 6 to 6

Solution: Given

$n_e=1.4\times10^{28}m^{-3}$, $D(E_F)=6.2\times10^{46}J\,m^{-3}$, $\mu_B=9.3\times10^{-24}\text{Joule}\,T^{-1}$,

$\mu_0=4\pi\times10^{-7}Tm\,A^{-1}$

We know that, $\chi_{Pauli}=\mu_0\mu_B^2D(E_F)$

$$\chi_{Pauli}=\mu_0\mu_B^2D(E_F)=4\pi\times10^{-7}\times\left(9.3\times10^{-24}\right)^2\times6.2\times10^{46}$$

$$\chi_{Pauli}=4\pi\times\left(9.3\right)^2\times6.2\times10^{-7-48+46}=6.735\times10^{-6}$$

$k=6$

Q30. A power supply has internal resistance R_S and open load voltage $V_S=5V$. When a load resistance R_L is connected to the power supply, a voltage drop of $V_L=4V$ is measured across the load. The value of $\frac{R_L}{R_S}$ is __________ (Round off to the nearest integer)

Ans. 30: 4 to 4

Solution:

$$V_L=\frac{R_L}{R_L+R_S}\times5V=4V\Rightarrow5R_L=4R_L+4R_S\quad\Rightarrow\frac{R_L}{R_S}=4$$

Q31. Electric field is measured along the axis of a uniformly charged disc of radius $25\,cm$. At a distance d from the centre, the field differs by 10% from that of an infinite plane having the same charge density. The value of d is _______ cm.

(Round off to one decimal place)

Ans. 31: 2.4 to 2.6

Solution:

$$E_{\text{disc}} = \frac{\sigma}{2\in_0}\left[1-\frac{d}{\sqrt{R^2+d^2}}\right],\ E_{\text{infinitesheet}} = \frac{\sigma}{2\in_0}$$

$$E_{\text{disc}} = 10\%\,E_{\text{inf.}} \Rightarrow \frac{\sigma}{2\in_0}\left[1-\frac{d}{\sqrt{R^2+d^2}}\right] = \frac{90}{100}\times\frac{\sigma}{2\in_0} \Rightarrow 1-\frac{d}{\sqrt{R^2+d^2}} = \frac{9}{10}$$

$$\Rightarrow \frac{d}{\sqrt{R^2+d^2}} = 1-\frac{9}{10} = \frac{1}{10} \Rightarrow 100d^2 = R^2+d^2$$

$$\Rightarrow 99d^2 = R^2 \Rightarrow d = \frac{R}{\sqrt{99}} = \frac{25}{\sqrt{99}}\,cm = 2.5\,cm$$

Q32. In a solid, a Raman line observed at $300\,cm^{-1}$ has intensity of Stokes line four times that of the anti-Stokes line. The temperature of the sample is _______ K.

(Round off to the nearest integer) $\left(1cm^{-1} \equiv 1.44\,K\right)$

Ans. 32: 311 to 312

Solution: Temperature dependence of intensity of Stoke and Anti-stoke lines are given by:

$$I_S \propto \frac{1}{1-e^{-\frac{h\upsilon_j}{k_BT}}}$$

$$I_{AS} \propto \frac{1}{e^{\frac{h\upsilon_j}{k_BT}}-1}$$

where, ν_j is the frequency shift of the Raman line, taking the ratio of the two intensities,

$$\frac{I_S}{I_{AS}} = \frac{\dfrac{1}{1-e^{-\frac{h\nu_j}{k_BT}}}}{\dfrac{1}{e^{\frac{h\nu_j}{k_BT}}-1}} = \frac{e^{\frac{h\nu}{k_BT}}-1}{1-e^{-\frac{h\nu_j}{k_BT}}} \Rightarrow \frac{h\nu_j}{k_BT} = \ln\left[\frac{I_S}{I_{AS}}\right]$$

$$T = \frac{hc\bar{\nu}}{k_B\ln\left[\frac{I_S}{I_{AS}}\right]} = \frac{6.626\times10^{-34}\,Js\times3\times10^{8}\,m/s\times300\times100m^{-1}}{1.38\times10^{-23}\,J/K\times\ln 4}$$

$$= 31.17\times10^{1}K = 311.7K$$

Hence, T = 311 to 312

Q33. An electromagnetic pulse has a pulse width of $10^{-3}s$. The uncertainty in the momentum of the corresponding photon is of the order of $10^{-N}\,kg\,m\,s^{-1}$, where N is an integer. The value of N is ____________ (speed of light $=3\times10^{8}\,ms^{-1}, h=6.6\times10^{-34}\,Js$)

Ans. 33: 39 to 40

Solution: Solution: We have $\Delta t=10^{-3}\,\text{sec}, h=6.6\times10^{-34}, c=3\times10^{8}\,m/s$

The uncertainty in energy is

$$\Delta E.\Delta t=\frac{\hbar}{2}\Rightarrow \Delta E=\frac{\hbar}{2\Delta t}$$

The uncertainty in the momentum is given by.

$$\Delta p=\frac{\Delta E}{c}=\frac{\hbar}{2c\Delta t}=\frac{1.05\times10^{-34}}{2\times3\times10^{8}\times10^{-3}}=0.175\times10^{-39}=1.75\times10^{-40}\,kg\,m/s$$

Q34. The wave function of a particle in a one-dimensional infinite well of size $2a$ at a certain time is $\psi(x)=\frac{1}{\sqrt{6a}}\left[\sqrt{2}\sin\left(\frac{\pi x}{a}\right)+\sqrt{3}\cos\left(\frac{\pi x}{2a}\right)+\cos\left(\frac{3\pi x}{2a}\right)\right]$. Probability of finding the particle in $n=2$ state at that time is ______% (Round off to the nearest integer)

Ans. 34: 33 to 34

Solution: We have

$$\psi(x)=\frac{1}{\sqrt{6a}}\left[\sqrt{2}\sin\frac{\pi x}{a}+\sqrt{3}\cos\frac{\pi x}{2a}+\cos\frac{3\pi x}{2a}\right]$$

$$=\frac{1}{\sqrt{6}}\sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}+\frac{\sqrt{3}}{\sqrt{12}}\frac{\sqrt{2}}{\sqrt{a}}\cos\frac{\pi x}{2a}+\frac{1}{\sqrt{12}}\sqrt{\frac{2}{a}}\cos\frac{3\pi x}{2a}\Bigg]$$

$$=\frac{1}{\sqrt{6}}|\phi_2\rangle+\frac{\sqrt{3}}{\sqrt{12}}|\phi_1\rangle+\frac{1}{\sqrt{12}}|\phi_3\rangle$$

The wave function of the particle in such a potential is given by.

$$|\psi_1\rangle=\sqrt{\frac{2}{a}}\cos\frac{\pi x}{2a};\quad |\psi_2\rangle=\sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}$$

$$|\psi_3\rangle=\sqrt{\frac{2}{a}}\cos\frac{3\pi x}{2a}$$

The normalization constant is obtained as follows.

$$|\psi\rangle=A\left(\frac{1}{\sqrt{6}}|\phi_2\rangle+\frac{1}{\sqrt{4}}|\phi_1\rangle+\frac{1}{\sqrt{12}}|\phi_3\rangle\right)$$

$$\langle\psi|\psi\rangle=A^2\left(\frac{1}{6}\langle\phi_2|\phi_2\rangle+\frac{1}{4}\langle\phi_1|\phi_1\rangle+\frac{1}{12}\langle\phi_3|\phi_3\rangle\right)=1$$

$$A^2\left(\frac{1}{6}+\frac{1}{4}+\frac{1}{12}\right)=1 \Rightarrow A^2\left(\frac{2+3+1}{12}\right)=1 \Rightarrow A=\sqrt{2}$$

Thus the normalized wave function is given by

$$|\psi\rangle=\frac{1}{\sqrt{2}}|\phi_1\rangle+\frac{1}{\sqrt{3}}|\phi_2\rangle+\frac{1}{\sqrt{6}}|\phi_3\rangle$$

The probability of finding the particle in state $n=2$ is

$$(\phi_2)=|\langle\phi_2|\psi\rangle|^2=\left|\frac{1}{\sqrt{3}}\langle\phi_2|\phi_2\rangle\right|^2=\frac{1}{3}=33.33\%$$

Q35. A spectrometer is used to detect plasma oscillations in a sample. The spectrometer can work in the range of $3\times10^{12}\text{ rad }s^{-1}$ to $30\times10^{12}\text{ rad }s^{-1}$. The minimum carrier concentration that can be detected by using this spectrometer is $n\times10^{21}\,m^{-3}$. The value of n is ____________ (Round off to two decimal places)

(Charge of an electron $=-1.6\times10^{-19}\,C$, mass of an electron $=9.1\times10^{-31}\,kg$ and $\in_0=8.85\times10^{-12}\,C^2N^{-1}m^{-1}$)

Ans. 35: 2.70 to 2.96

Solution: $\omega_p=\sqrt{\frac{n_0e^2}{\in_0 m}} \Rightarrow 3\times10^{12}=\sqrt{\frac{n_0\times\left(1.6\times10^{-19}\right)^2}{8.85\times10^{-12}\times9.1\times10^{-31}}}$

$$n_0=\frac{9\times10^{24}\times8.85\times10^{-12}\times9.1\times10^{-31}}{2.56\times10^{-38}}$$

$$=283.1\times10^{19}\,m^{-3}$$

$$n_0=2.83\times10^{21}\,m^{-3}$$

$$\simeq2.83\times10^{21}\,m^{-3}$$

Q.36 – Q.65 Carry TWO marks Each

Q36. Consider a non-interacting gas of spin 1 particles, each with magnetic moment μ, placed in a weak magnetic field B, such that $\frac{\mu B}{k_BT}<<1$. The average magnetic moment of a particle is

(a) $\frac{2\mu}{3}\left(\frac{\mu B}{k_BT}\right)$ (b) $\frac{\mu}{2}\left(\frac{\mu B}{k_BT}\right)$ (c) $\frac{\mu}{3}\left(\frac{\mu B}{k_BT}\right)$ (d) $\frac{3\mu}{4}\left(\frac{\mu B}{k_BT}\right)$

Ans. 36: (a)

Solution: In quantum mechanical treatment, single-dipole partition function is (see RK Patharia, Article 3.9)

$$Q_1(\beta) = \frac{\sinh\left\{\left(1+\frac{1}{2J}\right)x\right\}}{\sinh\left(\frac{1}{2J}x\right)} \qquad (1)$$

Where $x = \beta(g\mu_B J)B$. The mean magnetic moment of the system is then given by

$$M_z = N\langle\mu_z\rangle = \frac{N}{\beta}\frac{\partial \ln Q_1(\beta)}{\partial\beta} = Ng\mu_B JB_j(x)$$

$$\frac{M_z}{N} = \langle\mu_z\rangle = g\mu_B JB_j(x)$$

Where $B_J(x)$ is the Brillouin function. For

$$x <<< 1, i.e., \frac{\mu_B B}{k_B T} <<< 1$$

$$B_j(x) \approx \frac{1}{3}\left(1+\frac{1}{J}\right)x + ...$$

Therefore,

$$\langle\mu_z\rangle = \frac{(g\mu_B J)^2}{3k_B T}\left(1+\frac{1}{J}\right)B = g^2\frac{2}{3}\mu_B\left(\frac{\mu_B B}{k_B T}\right)$$

Where, last term is written for J=1

Q37. Water at $300K$ can be brought to $320K$ using one of the following processes.

Process 1: Water is brought in equilibrium with a reservoir at $320K$ directly.

Process 2: Water is first brought in equilibrium with a reservoir at $310K$ and then with the reservoir at $320K$.

Process 3: Water is first brought in equilibrium with a reservoir at $350K$ and then with the reservoir at $320K$.

The corresponding changes in the entropy of the universe for these processes are $\Delta S_1, \Delta S_2$ and ΔS_3, respectively. Then

(a) $\Delta S_2 > \Delta S_1 > \Delta S_3$ (b) $\Delta S_3 > \Delta S_1 > \Delta S_2$

(c) $\Delta S_3 > \Delta S_2 > \Delta S_1$ (d) $\Delta S_1 > \Delta S_2 > \Delta S_3$

Ans. 37: (b)

Solution: Here initial temperature (T_i) of water is $300K$ & final temperature (T_f) is $320K$.

Process1: $\Delta S_{\text{Water}} = C_W \ln\left(\frac{T_f}{T_i}\right) = C_W \ln\left(\frac{320}{300}\right) = 0.06454C_W$

$$\Delta S_{\text{reseservoir}} = \frac{-C_W \Delta T}{320} = -\frac{C_W(320-300)}{320} = -0.0625C_W$$

$$\Delta S_{\text{Uniucesse}} = \Delta S_1 = \Delta S_{water} + \Delta S_{reservoir}$$

$$= 0.06454C_W - 0.0625C_W$$

$$= 0.00204C_W$$

Process-2: ΔS_{water} will be same as initial and final equilibrium states are same.

$$\therefore \Delta S_{\text{water}} = C_W \ln\frac{320}{300} = 0.06454C_W$$

$$\Delta S_{reservoir} = -C_W\left[\frac{10}{310}+\frac{10}{320}\right]$$

$$= -C_W[0.03226+0.03125]$$

$$= -0.06351C_W$$

$$\Delta S_2 = \Delta S_{water} + \Delta S_{reservoir} = 0.00103C_W$$

Process 3: $\Delta S_{water} = 0.06454C_W$

$$\Delta S_{reseruor} = -C_W\left[\frac{50}{350}\right] + C_W\left(\frac{30}{320}\right)$$

$$= -0.142857C_W + 0.09375C_W$$

$$= -0.04911C_W$$

$$\Delta S_3 = 0.06454C_W - 0.04911C_W$$

$$= 0.01543C_W$$

$$\therefore \Delta S_3 > \Delta S_1 > \Delta S_2$$

Q38. A student sets up Young's double slit experiment with electrons of momentum p incident normally on the slits of width w separated by distance d. In order to observe interference fringes on a screen at a distance D from the slits, which of the following conditions should be satisfied?

(a) $\frac{\hbar}{p} > \frac{Dw}{d}$ (b) $\frac{\hbar}{p} > \frac{dw}{D}$ (c) $\frac{\hbar}{p} > \frac{d^2}{D}$ (d) $\frac{\hbar}{p} > \frac{d^2}{\sqrt{Dw}}$

Ans. 38: (b)

Solution: $\Delta x = \omega$

$$\Delta p = 2p\sin\theta$$

$$\Delta x\,\Delta p \sim \hbar$$

$$\omega \times 2p\sin\theta \sim \hbar$$

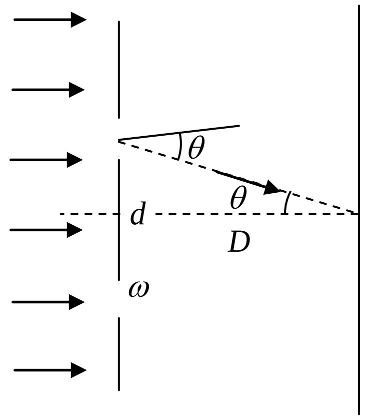
$\sin\theta \sim \theta \sim \frac{d/2}{D}$

$\omega \times 2p\frac{d}{2D} \sim \hbar$

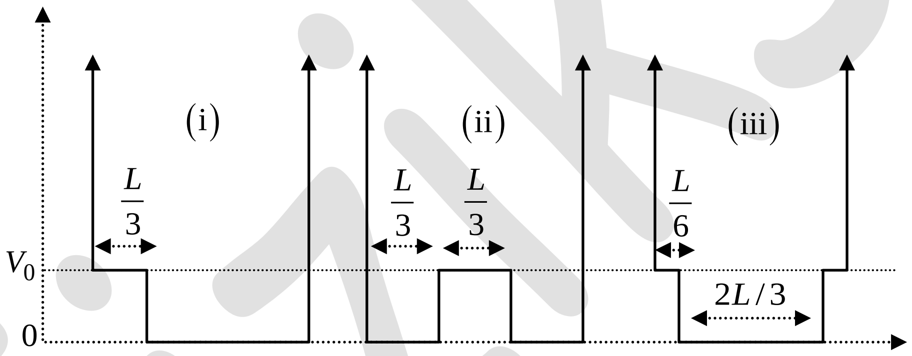
$\frac{d\omega}{D} \sim \frac{\hbar}{p}$

More accurately

$\frac{d\omega}{D} < \frac{\hbar}{p}$



Q39. Consider a particle in three different boxes of width L. The potential inside the boxes vary as shown in figures (i), (ii) and (iii) with $V_0 << \frac{\hbar^2\pi^2}{2mL^2}$. The corresponding ground-state energies of the particle are E_1, E_2 and E_3, respectively. Then



(a) $E_2 > E_1 > E_3$ (b) $E_3 > E_1 > E_2$ (c) $E_2 > E_3 > E_1$ (d) $E_3 > E_2 > E_1$

Ans. 39: (a)

Solution: The ground state wave function is given by

$$\psi = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}$$

Let us determine the correction in energy due to the potential in first box.

$$H' = \begin{cases} v_0 & 0 < x < L/3 \\ 0 & \text{otherwise} \end{cases}$$

The ground state energy correction in first order is

$$E_1^{(1)} = \langle\psi_1|H'|\psi_1\rangle = \frac{2}{L}\int_0^{L/3} v_0 \sin^2\frac{\pi x}{L}dx = \frac{2v_0}{L}\frac{L}{24}\left[4 - \frac{3\sqrt{3}}{\pi}\right]$$

$$= 0.195v_0$$

Similarly, let us determine the correction in energy due to potential in second box.

$$H' = \begin{cases} v_0 & L/3 < x < 2L/3 \\ 0 & \text{otherwise} \end{cases}$$

The ground state energy correction in first order is

$$E_2^{(1)} = \langle \psi_1 | H' | \psi \rangle = \frac{2}{L}\int_{a/3}^{2L/3} v_0 \sin^2 \frac{\pi x}{L} dx$$

$$= \frac{2v_0}{L}\frac{L}{12}\left[2 + \frac{3\sqrt{3}}{\pi}\right] = \frac{v_0}{6}\left[2 + \frac{3\sqrt{3}}{\pi}\right] = 0.609v_0$$

Similarly let us determine the correction in energy due to potential in third box

$$H' \begin{cases} v_0 & 0 < x < L/6 \\ v_0 & 5L/6 < x < L \\ 0 & \text{otherwise} \end{cases}$$

The ground state energy correction in first order

$$E_3^{(1)} = \langle \psi_1 | H' | \psi_1 \rangle = \int_0^{L/6} \psi_1^*(x) H' \psi_1(x) dx$$

$$+ \int_{5L/6}^{L} \psi_1^*(x) H' \psi_1(x) dx$$

$$= 2\int_0^{L/6} \psi_1^*(x) H' \psi_1(x) dx = 2\frac{2v_0}{L}\int_0^{L/6} \sin^2 \frac{\pi x}{L} dx$$

$$= \frac{4v_0}{L} \cdot \frac{L}{24}\left[2 - \frac{3\sqrt{3}}{\pi}\right] = \frac{v_0}{6}\left[2 - \frac{3\sqrt{3}}{\pi}\right] = -0.0575v_0$$

Thus the order of ground state energy in three boxes is given by

$$E_2 > E_1 > E_3$$

Q40. In cylindrical coordinates (s, φ, z) which of the following is a Hermitian operator?

(a) $\frac{1}{i}\frac{\partial}{\partial s}$ (b) $\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{s}\right)$ (c) $\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{2s}\right)$ (d) $\left(\frac{\partial}{\partial s} + \frac{1}{s}\right)$

Ans. 40: (c)

Solution: Let us choose operator given is option (c).

$$A = \frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{2s}\right)$$

$$\langle A\phi | \varphi \rangle = \int \left(-\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{2s}\right)\phi^*(s)\right)\varphi(s) ds$$

$$=-\frac{1}{i}\left(\int_0^\infty \frac{\partial}{\partial s}\phi^*(s)\varphi(s)sds+\frac{1}{2s}\int_0^\infty \phi^*(s)\varphi(s)sds\right.$$

$$=-\frac{1}{i}\left[\left[\phi^*(s)\varphi(s)s\right]_0^\infty \int \phi^*(s)\frac{d\varphi}{ds}(s)sds-\int \phi^*(s)\varphi(s)ds+\frac{1}{2}\int \phi^*(s)\varphi(s)ds\right]$$

$$=-\frac{1}{i}\left[-\int \phi^*(s)\frac{d\varphi}{ds}(s)sds-\frac{1}{2s}\int \phi^*(s)\varphi(s)sds\right]$$

$$=\int \phi^*(s)\left[\frac{1}{i}\left[\frac{\partial}{\partial s}+\frac{1}{2s}\right]\varphi(s)sds=\langle \phi(s)|A\varphi(s)\rangle\right.$$

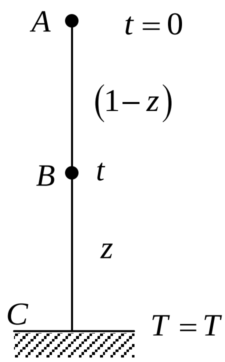
Thus, operator A is Hermition.

Q41. A particle of mass $1kg$ is released from a height of $1m$ above the ground. When it reaches the ground, what is the value of Hamilton's action for this motion in Js? (g is the acceleration due to gravity; take gravitation potential to be zero on the ground)

(a) $-\frac{2}{3}\sqrt{2g}$ (b) $\frac{5}{3}\sqrt{2g}$ (c) $3\sqrt{2g}$ (d) $-\frac{1}{3}\sqrt{2g}$

Ans. 41: (d)

Solution:



At point B

$$L=\frac{1}{2}m\dot{z}^2-mgz$$

$$u=0\rightarrow \dot{z}=0+gt=gt$$

$$(1-z)=0+\frac{1}{2}gt^2$$

$$z=1-\frac{1}{2}gt^2$$

Time taken to reach the point C

$$0=1-\frac{1}{2}gT^2\Rightarrow T=\sqrt{\frac{2}{g}}$$

Action $A=\int_0^T L\,dt$

$$=\int_0^{\sqrt{2/g}}\left[\frac{1}{2}mg^2t^2-mg\left(1-\frac{1}{2}gt^2\right)\right]dt$$

$$=\int_0^{\sqrt{2/g}}\left[mg^2t^2-mg\right]dt$$

$$= \left[\frac{1}{3}\times 1\times g^2 t^3 - 1\times gt\right]_0^{\sqrt{2/g}}$$

$$= \frac{2}{3}\sqrt{2g} - \sqrt{2g}$$

$$= -\frac{1}{3}\sqrt{2g}$$

Q42. If $(\dot{x}\dot{y} + axy)$ is a constant of motion of a two-dimensional isotropic harmonic oscillator with Lagrangian

$$L = \frac{m(\dot{x}^2 + \dot{y}^2)}{2} - \frac{k(x^2 + y^2)}{2}$$

then α is

(a) $+\frac{k}{m}$ (b) $-\frac{k}{m}$ (c) $-\frac{2k}{m}$ (d) 0

Ans. 42: (a)

Solution: $L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{k}{2}(x^2 + y^2)$

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{k}{2}(x^2 + y^2)$$

$$\dot{x} = \frac{\partial H}{\partial p_x} = \frac{p_x}{m}$$

$$p_x = m\dot{x}; \qquad p_y = m\dot{y}$$

$$A = \dot{x}\dot{y} + \alpha xy = \frac{p_x p_y}{m^2} + \alpha xy$$

$[A, H] = 0$ if A is constant of motion

$$\frac{\partial A}{\partial x}\frac{\partial H}{\partial p_x} - \frac{\partial A}{\partial p_x}\frac{\partial H}{\partial x} + \frac{\partial A}{\partial y}\frac{\partial H}{\partial p_y} - \frac{\partial A}{\partial p_y}\frac{\partial H}{\partial y} = 0$$

$$(\alpha y)\left(\frac{p_x}{m}\right) - \frac{p_y}{m^2}(kx) + (\alpha x)\frac{p_y}{m} - \frac{p_x}{m^2}(ky) = 0$$

$$\frac{\alpha}{m}(yp_x + xp_y) - \frac{k}{m^2}(xp_y + yp_x) = 0$$

$$(xp_y + yp_x)\left(\frac{\alpha}{m} - \frac{k}{m^2}\right) = 0 \Rightarrow \alpha = +\frac{k}{m}$$

Second method

$$\frac{d}{dt}(\dot{x}y+\alpha xy)=0$$

$$\ddot{x}y+\dot{x}\dot{y}+\alpha(\dot{x}y+x\dot{y})=0 \quad (1)$$

Equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$$

$$m\ddot{x}+kx=0$$

Similarly, $$\left.\begin{aligned}\ddot{x}&=-\frac{k}{m}x\\ \ddot{y}&=-\frac{k}{m}y\end{aligned}\right] \quad (2)$$

Substitute these values into Equation (1)

$$-\frac{k}{m}(x\dot{y}+y\dot{x})+\alpha(\dot{x}y+x\dot{y})=0$$

$$(x\dot{y}+y\dot{x})\left(-\frac{k}{m}+\alpha\right)=0$$

$$\alpha=\frac{k}{m}$$

Q43. In a two-dimensional square lattice, frequency ω of phonons in the long wavelength limit changes linearly with the wave vector k. Then the density of states of phonons is proportional to

(a) ω (b) ω^2 (c) $\sqrt{\omega}$ (d) $\frac{1}{\sqrt{\omega}}$

Ans. 43: (a)

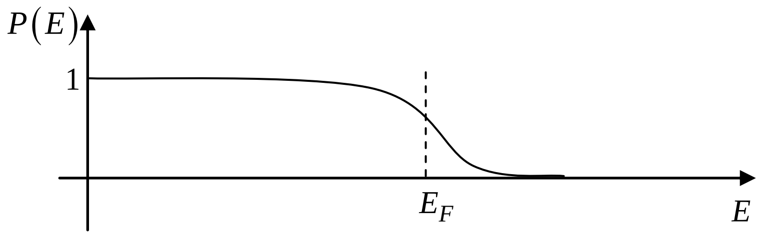
Solution: Density of states $D(E)\propto E^{\left(\frac{d}{s}-1\right)}$

From question $d=2$ and $s=1$

So, $D(E)\propto E^{\left(\frac{2}{1}-1\right)} \Rightarrow D(E)\propto E=\hbar\omega \Rightarrow D(E)\propto\omega$

Q44. At $T=0\,K$, which of the following diagram represents the occupation probability $P(E)$ of energy states of electrons in a BCS type superconductor?

(a)



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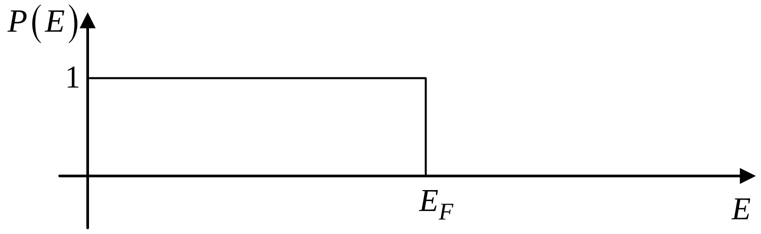
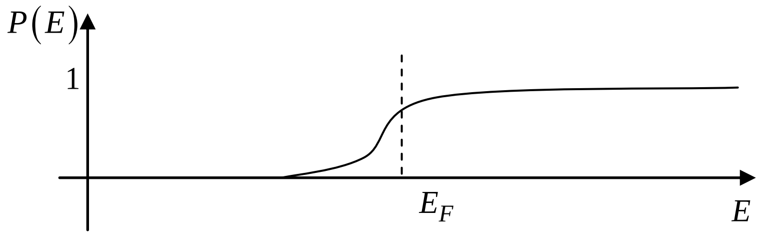
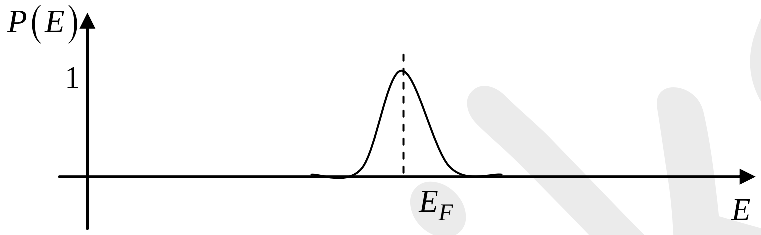
(b) 

(c) 

(d) 

Ans. 44: (a)

Q45. For a one-dimensional harmonic oscillator, the creation operator $\left(a^{\dagger}\right)$ acting on the n^{th} state $|\psi_n\rangle$ where $n = 0,1,2,\ldots,$ gives $a^{\dagger}|\psi_n\rangle = \sqrt{n+1}|\psi_{n+1}\rangle$. The matrix representation of the position operator $x = \sqrt{\frac{\hbar}{2m\omega}}\left(a + a^{\dagger}\right)$ for the first three rows and columns is

(a) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{3} \end{pmatrix}$

(b) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(c) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}$

(d) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 1 & 0 & \sqrt{3} \\ 0 & 0 & 0 \\ \sqrt{3} & 0 & 1 \end{pmatrix}$

Ans. 45: (c)

Solution: The position operator is given by

$$\langle x\rangle = \left\langle m\left|\sqrt{\frac{\hbar}{2m\omega}}\left(a^{\dagger}+a\right)\right|n\right\rangle$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n+1}\delta_{m,n+1} + \sqrt{n}\delta_{m,n-1}\right)$$

For $m = n$, i.e., all diagonal elements in the matrix must be zero.

For $m=0, n=1$ the value of $\langle x\rangle$ is

$$\langle x\rangle_{01}=\sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{2}\delta_{0,2}+\sqrt{1}\delta_{1,0}\right]=\sqrt{\frac{\hbar}{2m\omega}}$$

For $m=1, n=0$, the value of $\langle x\rangle$ is

$$\langle x\rangle_{10}=\sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{0+1}\delta_{1,1}+\sqrt{0}\delta_{1,-1}\right]=\sqrt{\frac{\hbar}{2m\omega}}$$

For $m=0, n=2$, the value of $\langle x\rangle$ is

$$\langle x\rangle_{0,2}=\sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{0+2}\delta_{0,3}+\sqrt{2}\,\delta_{0,1}\right]=0$$

For $m=1, n=2$, the value of $\langle x\rangle_{12}$ is

$$\langle x\rangle_{12}=\sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{2+1}\,\delta_{1,3}+\sqrt{2}\delta_{1,1}\right]=\sqrt{\frac{\hbar}{2m\omega}}\sqrt{2}$$

For $m=2, n=1$, the value of $\langle x\rangle_{21}$ is

$$\langle x\rangle_{21}=\sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{1+1}\delta_{212}+\sqrt{1}\delta_{2,0}\right]=\sqrt{\frac{\hbar}{2m\omega}}\sqrt{2}$$

For $m=2, n=0$, the value of $\langle x\rangle_{20}$ is

$$\langle x\rangle_{20}=\sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{1+0}\delta_{2,1}+\sqrt{0}\delta_{2,-1}\right]=0$$

The matrix representation of position vector is

$$\langle x\rangle=\begin{bmatrix}\langle 0|x|0\rangle & \langle 0|x|1\rangle & \langle 0|x|2\rangle\\ \langle 1|x|0\rangle & \langle 1|x|1\rangle & \langle 1|x|2\rangle\\ \langle 2|x|0\rangle & \langle 2|x|1\rangle & \langle 2|x|2\rangle\end{bmatrix}$$

$$\langle x\rangle=\sqrt{\frac{\hbar}{2m\omega}}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & \sqrt{2}\\ 0 & \sqrt{2} & 0\end{bmatrix}$$

and general from is

$$\langle x\rangle=\sqrt{\frac{\hbar}{2m\omega}}\begin{bmatrix}0 & 1 & 0 & 0 & 0\\ 1 & 0 & \sqrt{2} & 0 & 0\\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0\\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4}\\ 0 & 0 & 0 & \sqrt{4} & 0\end{bmatrix}$$

Q46. A piston of mass m is fitted to an airtight horizontal cylindrical jar. The cylinder and piston have identical unit area of cross-section. The gas inside the jar has volume V and is held at pressure $P = P_{atmosphere}$. The piston is pushed inside the jar very slowly over a small distance. On releasing, the piston performs an undraped simple harmonic motion of low frequency. Assuming that the gas is ideal and no heat is exchanged with the atmosphere, the frequency of the small oscillations is proportional to

(a) $\sqrt{\frac{P}{\gamma mV}}$ (b) $\sqrt{\frac{P\gamma}{Vm}}$ (c) $\sqrt{\frac{P}{mV^{\gamma-1}}}$ (d) $\sqrt{\frac{\gamma P}{mV^{\gamma-1}}}$

Ans. 46: (b)

Solution: Let initial pressure is $P_1 = P$

Initial volume is V

When pressure changes slightly by ΔP, i.e piston is pushed in side, the volume is reduced by ΔV.

Further given that no heat exchange is there

$$PV^{\gamma} = (P+\Delta P)(V-\Delta V)^{\gamma}$$

$$= P\left[1+\frac{\Delta P}{P}\right]V^{\gamma}\left[1-\frac{\Delta V}{V}\right]^{\gamma}$$

$$= PV^{\gamma}\left[1+\frac{\Delta P}{P}\right]\left[1-\gamma\frac{\Delta V}{V}+....\right]$$

$$= PV^{\gamma}\left[1+\frac{\Delta P}{P}-\gamma\frac{\Delta V}{V}-\gamma\frac{\Delta P\,\Delta V}{PV}\right] \quad (1)$$

As $\Delta P \,\&\, \Delta V$ are using small, $\Delta P\,\Delta V$ can be neglected.

$$\therefore\ 1 = \left[1+\frac{\Delta P}{P}-\gamma\frac{\Delta V}{V}\right]$$

$$\Delta P = \gamma P\frac{\Delta V}{V} \quad (2)$$

Applied external force F, that caused a displacement x (a volume change of gas by $\Delta V = Ax$) is given by

$$F = -A\Delta P = -\gamma AP\frac{\Delta V}{V} = \frac{-\gamma A^2 P}{V}x \quad (3)$$

Acceleration produced in piston is

$$a = \frac{F}{m} = -\frac{\gamma A^2 P}{mV}x$$

$a \propto x$

Therefore $\omega = \sqrt{\dfrac{\gamma A^2 P}{mV}}$

$\omega \propto \sqrt{\dfrac{P\gamma}{mV}}$

Q47. A paramagnetic salt of mass m is held at temperature T in a magnetic field H. If S is the entropy of the salt and M is its magnetization, then $dG = -SdT - M\,dH$, where G is the Gibbs free energy. If the magnetic field is changed adiabatically by $\Delta H \to 0$ and the corresponding infinitesimal changes in entropy and temperature are ΔS and ΔT, then which of the following statements are correct

(a) $\Delta S = -\dfrac{1}{T}\left(\dfrac{\partial G}{\partial T}\right)_H \Delta T$

(b) $\Delta S = 0$

(c) $\Delta T = -\dfrac{\left(\dfrac{\partial M}{\partial T}\right)_H}{\left(\dfrac{\partial S}{\partial T}\right)_H}\Delta H$

(d) $\Delta T = 0$

Ans. 47: (b), (c)

Solution: The magnetic interaction energy $= -M.dH$ (1)

$dU = T\,dS - M.dH$ (2)

$dG - SdT - M.dH$ (3)

$S = -\left(\dfrac{\partial G}{\partial T}\right)_H, \; M = -\left(\dfrac{\partial G}{\partial H}\right)$

$\because G$ is a perfect differential,

$$\frac{\partial}{\partial T}\left(\frac{\partial G}{\partial H}\right) = \frac{\partial}{\partial H}\left(\frac{\partial G}{\partial T}\right)$$

$$\left(\frac{\partial M}{\partial H}\right)_H = \left(\frac{\partial S}{\partial H}\right)_T \quad (4)$$

Now $\left(\dfrac{\partial T}{\partial H}\right)_S = -\left(\dfrac{\partial T}{\partial S}\right)_H \left(\dfrac{\partial S}{\partial H}\right)_T$

$$= -\left(\frac{\partial T}{\partial S}\right)_H \left(\frac{\partial M}{\partial T}\right)_H$$

$$= \frac{-\left(\frac{\partial M}{\partial T}\right)_H}{\left(\frac{\partial S}{\partial T}\right)_H} \quad (5)$$

$\therefore$ (5) implies

$$\Delta T = -\frac{\left(\frac{\partial M}{\partial T}\right)_H}{\left(\frac{\partial S}{\partial T}\right)_H}\Delta H \quad (6)$$

This is indeed the process of adiabatic unitization.

Where negative ΔH implies negative $\Delta T \,\&\, \Delta S = 0$.

Q48. A particle of mass m is moving inside a hollow spherical shell of radius a so that the potential is

$$V(r) = \begin{cases} 0 \text{ for } r < a \\ \infty \text{ for } r \geq a \end{cases}$$

The ground state energy and wave function of the particle are E_0 and $R(r)$, respectively. Then which of the following options are correct?

(a) $E_0 = \frac{\hbar^2 \pi}{2ma^2}$

(b) $-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = E_0 R \ (r < a)$

(c) $-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d^2R}{dr^2} = E_0 R \quad (r < a)$

(d) $R(r) = \frac{1}{r}\sin\left(\frac{\pi r}{a}\right) \quad (r < a)$

Ans. 48: (a), (b), (d)

Solution: The Schrödinger equation for a particle moving in radial potential is given by

$$H\varphi = E\varphi \Rightarrow \frac{-\hbar^2}{2m}\frac{1}{r^2}\left(\frac{\partial}{\partial r}r^2\frac{dR}{dr}\right) + V_{eff}R(r) = E_0 R(r)$$

where $V_{eff} = V + \frac{\ell(\ell+1)}{2mr^2}\hbar^2 = 0$

as $V = 0, \ell = 0$.

Thus Schrödinger equation to

$$\frac{-\hbar^2}{2m}\frac{1}{r^2}\left(\frac{\partial}{\partial r}r^2\frac{dR}{dr}\right) = E_0 R(r)$$

or $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2m}{\hbar^2}ER(r) = 0$

Defining $R(r)=\frac{U(r)}{r}$ and substituting in above equation, we get

$$\frac{d^2U(r)}{dr^2}+k^2R(r)=0, \qquad r^2=\frac{2mE}{\hbar^2}r<a$$

The solution of above equation is

$$U(r)=A\sin kr+B\cos kr$$

Applying Boundary condition

$$U(r=0)=0; U(r=a)=0$$

$$U(r=0)=A\sin 0+B\cos 0=0 \Rightarrow B=0$$

Thus the wave function is given by

$$U(r)=A\sin kr$$

Applying Boundary condition $U(r=a)=0$

$$U(r=a)=A\sin ka=0 \Rightarrow ka=n\pi \Rightarrow k\frac{n\pi}{a}$$

or $$\frac{2mE}{\hbar^2}=\frac{n^2\pi^2}{a^2} \Rightarrow E=\frac{n^2\pi^2\hbar^2}{2ma^2}$$

and the Radial wave function is given by

$$R(r)=\frac{U(r)}{r}=\frac{A}{r}\sin\frac{\pi nr}{a}$$

For ground state the energy and wave function of the particle are

$$E=\frac{\pi^2\hbar^2}{2ma^2}; R(r)=\frac{1}{r}\sin\frac{\pi r}{a}.$$

Q49. A particle of unit mass moves in a potential $V(r)=-V_0e^{-r^2}$. If the angular momentum of the particle is $L=0.5\sqrt{V_0}$, then which of the following statements are true?

(a) There are two equilibrium points along the radial coordinate

(b) There is one stable equilibrium point at r_1 and one unstable equilibrium point at $r_2>r_1$

(c) There are two stable equilibrium points along the radial coordinate

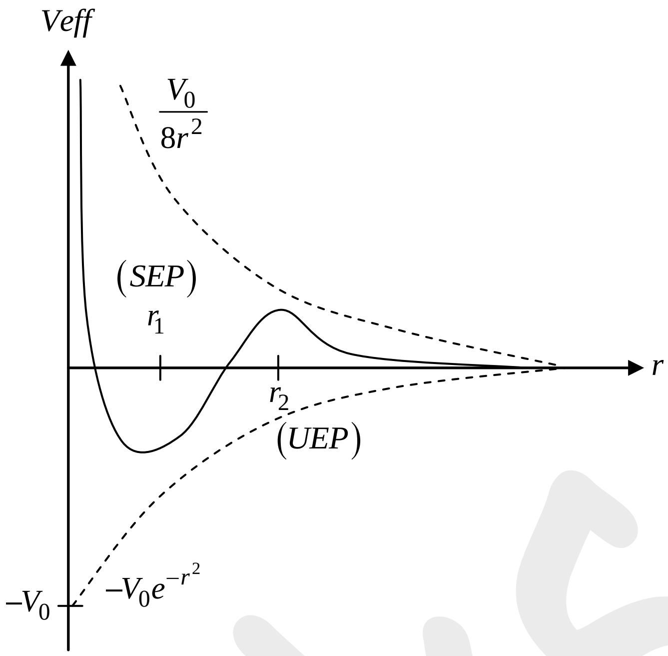
(d) There is only one equilibrium point along the radial coordinate

Ans. 49: (a), (b)

Solution: $V_{eff}=\frac{l^2}{2mr^2}-V_0e^{-r^2}$

$$l=0.5\sqrt{V_0}, m=1$$

$$V_{eff}=\frac{V_0}{\theta r^2}-V_0e^{-r^2}$$



Here $r_1 < r_2$

These equilibrium points are corresponding to circular orbits of radius r_1 and r_2 respectively.

Q50. In a diatomic molecule of mass M, electronic, rotational and vibrational energy scales are of magnitude E_e, E_R and E_V, respectively. The spring constant for the vibrational energy is determined by E_e. If the electron mass is m then

(a) $E_R \sim \frac{m}{M} E_e$ (b) $E_R \sim \sqrt{\frac{m}{M}} E_e$ (c) $E_V \sim \sqrt{\frac{m}{M}} E_e$ (d) $E_V \sim \left(\frac{m}{M}\right)^{1/4} E_e$

Ans. 50: (a), (c)

Solution: Unsolved

Q51. Electronic specific heat of a solid at temperature T is $C = \gamma T$, where γ is a constant related to the thermal effective mass (m_{eff}) of the electrons. Then which of the following statements are correct?

(a) $\gamma \propto m_{eff}$

(b) m_{eff} is greater than free electron mass for all solids

(c) Temperature dependence of C depends on the dimensionality of the solid

(d) The linear temperature dependence of C is observed at $T \ll$ Debye temperature

Ans. 51: (a), (d)

Q52. In a Hall effect experiment on an intrinsic semiconductor, which of the following statements are correct?

(a) Hall voltage is always zero

(b) Hall voltage is negative if the effective mass of holes is larger than those of electrons

(c) Hall coefficient can be used to estimate the carrier concentration in the semiconductor

(d) Hall voltage depends on the mobility of the carriers

Ans. 52: (d)

Q53. A parallel plate capacitor with spacing d and area of cross-section A is connected to a source of voltage V. If the plates are pulled apart quasistatically to a spacing of $2d$, then which of the following statements are correct?

(a) The force between the plates at spacing $2d$ is $\frac{1}{8}\left(\frac{\epsilon_0 AV^2}{d^2}\right)$

(b) The work done in moving the plates is $\frac{1}{8}\left(\frac{\epsilon_0 AV^2}{d}\right)$

(c) The energy transferred to the voltage source is $\frac{1}{2}\left(\frac{\epsilon_0 AV^2}{d}\right)$

(d) The energy of the capacitor reduces by $\frac{1}{4}\left(\frac{\epsilon_0 AV^2}{d}\right)$

Ans. 53: (a), (c), (d)

Solution:

(a) $$F = Q_0 E = \frac{Q_0^2}{2\epsilon_0 A} = \frac{C_0^2 V^2}{2\epsilon_0 A} = \left(\frac{\epsilon_0 A}{2d}\right)^2 \times \frac{V^2}{2\epsilon_0 A} = \frac{\epsilon_0 AV^2}{8d^2}$$

(b) $$W = \int \vec{F}\cdot d\vec{l} = \int_d^{2d} \frac{\epsilon_0 AV^2}{2x^2}dx = \frac{\epsilon_0 AV^2}{4d}$$

(c) Energy transferred to source must be equal to energy decrease of the capacitor.

(d) Initial energy $= \frac{1}{2}\frac{\epsilon_0 AV^2}{d}$, final energy $= \frac{1}{2}\frac{\epsilon_0 AV^2}{2d}$

change $= \frac{1}{2}\frac{\epsilon_0 A}{d}V^2 - \frac{1}{2}\frac{\epsilon_0 A}{2d}V^2 = -\frac{1}{4}\frac{\epsilon_0 AV^2}{d^2}$

Q54. A system with time independent Hamiltonian $H(q,p)$ has two constants of motion $f(q,p)$ and $g(q,p)$. Then which of the following Poisson brackets are always zero?

(a) $\{H, f+g\}$ (b) $\{H,\{f,g\}\}$ (c) $\{H+f,g\}$ (d) $\{H,H+fg\}$

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Ans. 54: (a), (b), (d)

Solution: $\{H, f\} = 0$

$\{H, g\} = 0$

(a) $\{H, f+g\} = \{H, f\} + \{H, g\} = 0$

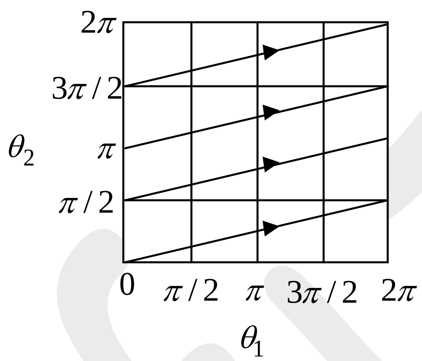
(b) $\{H, \{f, g\}\} = -\{f, \{g, H\}\} - \{g, \{H, f\}\} = -\{f, 0\} - \{g, 0\} = 0$

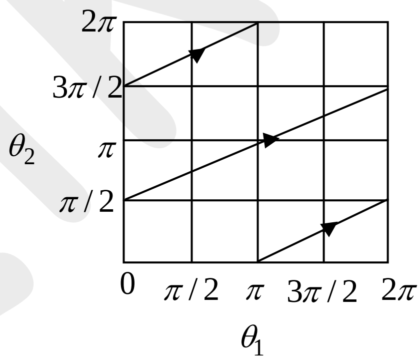
Here, Jacobi Identity is used.

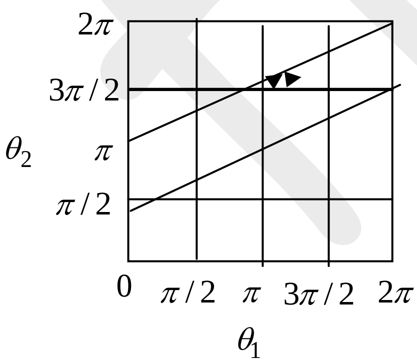
(c) $\{H + f, g\} = \{H, g\} + \{f, g\} = \{f, g\}$

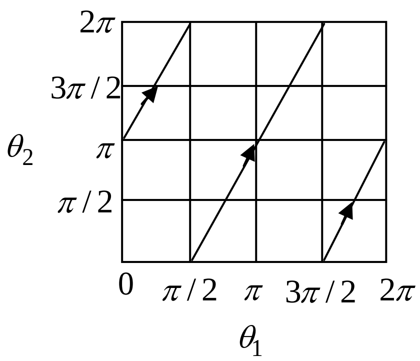
(d) $\{H, H + fg\} = \{H, H\} + \{H, fg\} = \{H, f\}g + f\{H, g\} = 0$

Q55. In the action-angle variables $(I_1, I_2, \theta_1, \theta_2)$ consider the Hamiltonian $H = 4I_1 I_2$ and $0 \le \theta_1, \theta_2 < 2\pi$. Let $\frac{I_1}{I_2} = \frac{1}{2}$. Which of the following are possible plots of the trajectories with different initial conditions in $\theta_1 - \theta_2$ plane?

(a)


(b)


(c)


(d)


Ans. 55: (b), (c)

Solution: $H = 4I_1 I_2$

$$\dot{\theta}_1 = \frac{\partial H}{\partial I_1} = 4I_2$$

$\dot{\theta}_2 = \frac{\partial H}{\partial I_2} = 4I_1$

$\frac{d\theta_1 / dt}{d\theta_2 / dt} = \frac{4I_2}{4I_1} = 2$

$\frac{d\theta_2}{d\theta_1} = \frac{1}{2}$

Slope of $\theta_1 - \theta_2$ curve $= \frac{1}{2}$

Q56. A particle of mass m in the $x-y$ plane is confined in an infinite two-dimensional well with vertices $(0,0),(0,L),(L,L),(L,0)$. The eigen-functions of this particle are $\psi_{n_x n_y} = \frac{2}{L}\sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)$. If perturbation of the form $V = Cxy$, where C is a real constant, is applied, then which of the following statements are correct for the first excited state?

(a) The unperturbed energy is $\frac{3\pi^2\hbar^2}{2mL^2}$

(b) The unperturbed energy is $\frac{5\pi^2\hbar^2}{2mL^2}$

(c) First order energy shift due to the applied perturbation is zero

(d) The shift (δ) in energy due to the applied perturbation is determined by an equation of the form $\begin{vmatrix} a-\delta & b \\ b & a-\delta \end{vmatrix} = 0$, where a and b are real, non-zero constants

Ans. 56: (b), (d)

Solution: We have,

$$\psi_{n_x, n_y} = \frac{2}{L}\sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)$$

and its corresponding energies are,

$$E_{n_x, n_y} = \left(n_x^2 + n_y^2\right)\frac{\pi^2\hbar^2}{2mL^2}$$

For ground state $n_x = n_y = 1$, the ground state energy is given by

$$E_{11} = \left(1^2 + 1^2\right)\frac{\pi^2\hbar^2}{2mL^2} = 2\frac{\pi^2\hbar^2}{2mL^2}$$

The first order correction in ground state energy is

$$E_{11}^{(1)} = \langle \psi_{11}(x,y) | H' | \psi_{11}(x,y) \rangle$$

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$$= \frac{2}{L}\frac{2}{L} C \int_0^L x \sin^2 \frac{\pi x}{L} dx \int_0^L y \sin^2 \frac{\pi y}{L} dy$$

$$= \left(\frac{2}{L}\right)^2 \left(\frac{L}{4}\right)^2 \left(\frac{L^2}{4}\right) L = \frac{CL^2}{4}$$ as the energy state energy is odd function .

The first excited state has energy,

$$(n_x, n_y) = \begin{cases} (2,1) \\ (1,2) \end{cases}$$

$$E_{21} = E_{12} = (2^2 + 1^2) \frac{\pi^2 \hbar^2}{2ma^2} = 5\frac{\pi^2 \hbar^2}{2ma^2}$$

The wave function of the particles are

$$n_x = 1, n_y = 2, E_{12} = \frac{5\pi^2\hbar^2}{2ma^2};\ \psi_{12}^0(x,y) = \frac{2}{a} \sin\frac{\pi x}{a} \sin\frac{2\pi y}{a}$$

$$n_x = 2, n_y = 1, E_{21} = \frac{5\pi^2\hbar^2}{2ma^2};\ \psi_{21}^0(x,y) = \frac{2}{a} \sin\frac{2\pi x}{a} \sin\frac{\pi y}{a}$$

The perturbed matrix for this Hamiltonian is given by

$$H_p = \begin{bmatrix} \langle 1,2|H'|1,2\rangle & \langle 1,2|H'|2,1\rangle \\ \langle 2,1|H'|1,2\rangle & \langle 2,1|H'|2,1\rangle \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

The values of inner product are

$$\langle 1,2|H'|1,2\rangle = \frac{4}{L^2} C \int_0^L x \sin^2 \frac{\pi x}{L} dx \int_0^L y \sin^2 \frac{2\pi y}{L} dy = \frac{CL^2}{4}$$

$$\langle 1,1|H'|2,1\rangle = \frac{4}{L^2} C \int_0^L x \sin^2 \frac{2\pi x}{L} dx \int_0^L y \sin^2 \frac{\pi y}{L} dy = \frac{CL^2}{4}$$

$$\langle 2,1|H'|1,2\rangle = \frac{4}{L^2} C \int_0^L x \sin\frac{2\pi x}{L} \sin\frac{\pi x}{L} dx \int_0^L y \sin\frac{\pi y}{L} \sin\frac{2\pi y}{L} dL = \frac{25bCL^2}{81\pi^4}$$

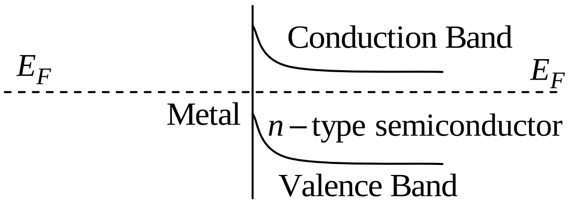
$$\langle 2|H'|2,1\rangle = \frac{4}{L^2} C \int_0^L x \sin\frac{\pi x}{L} \sin\frac{2\pi x}{L} dx \int_0^L 9 \sin\frac{2\pi y}{L} \sin\frac{\pi y}{L} dy \ = \frac{25bCL^2}{81\pi^4}$$

Thus the eigen value of perturbed matrix is determined ground secular equation.

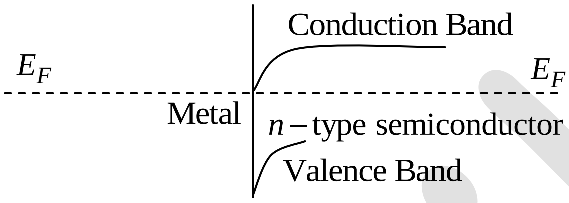
$$|H - \delta I| = \begin{vmatrix} a-\delta & b \\ b & a-\delta \end{vmatrix} = 0$$

Q57. A junction is formed between a metal on the left and an n-type semiconductor on the right. Before forming the junction, the Fermi level E_F of the metal lies below that of the semiconductor. Then which of the following schematics are correct for the bands and the $I-V$ characteristics of the junction?

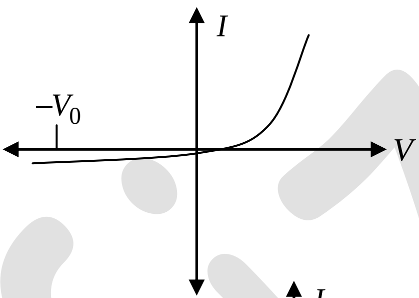
(a)



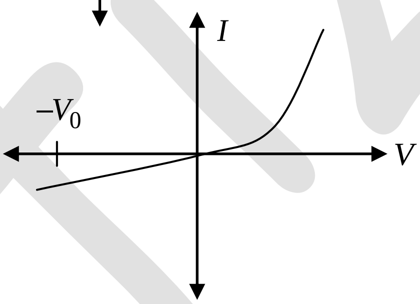
(b)



(c)



(d)



Ans. 57: (a), (c)

Q58. A plane polarized electromagnetic wave propagating in $y-z$ plane is incident at the interface of two media at Brewster's angle. Taking $z=0$ as the boundary between the two media, the electric field of the reflected wave is given by

$$\vec{E}_R = A_R \cos\left[k_0\left\{\frac{\sqrt{3}}{2}y - \frac{1}{2}z\right\} - \omega t\right]\hat{x}$$

then which among the following statements are correct?

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(a) The angle of refraction is $\frac{\pi}{6}$

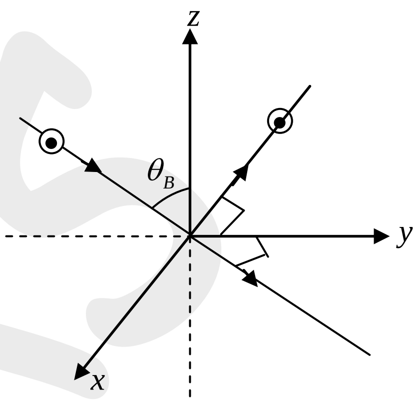
(b) Ratio of permittivity of the medium of refraction $(\in_2)$ with respect to the medium on incidence $(\in_1)$, $\frac{\in_2}{\in_1} = 3$

(c) The incident wave can have components of its electric field in $y-z$ plane

(d) The angle of reflection is $\frac{\pi}{6}$

Ans. 58: (a), (b), (c)

Solution:



$$\vec{k} = \frac{\sqrt{3}}{2}k_0\hat{y} - \frac{k_0}{2}\hat{z}$$

$$\tan\theta_B = \frac{k_y}{k_z} = \frac{\sqrt{3}k_0/2}{2k_0} = \sqrt{3}$$

$$\tan_B = \tan\left(\frac{\pi}{3}\right) \Rightarrow \theta_B = \frac{\pi}{3}$$

$$\theta_R + \theta_B + \frac{\pi}{2} = \pi$$

$$\theta_R + \frac{\pi}{3} + \frac{\pi}{2} = \pi \Rightarrow \theta_R = \pi - \frac{\pi}{2} - \frac{\pi}{3} = +\frac{\pi}{6}$$

$$\tan\theta_B = \frac{n_2}{n_1} = \frac{\sqrt{\in_{r_2}}}{\sqrt{\in_{r_1}}} = \sqrt{3} \Rightarrow \frac{\in_{r_2}}{\in_{r_1}} = \sqrt{3} \Rightarrow \frac{\in_2}{\in_1} = 3$$

Q59. The minimum number of two-input NAND gates required to implement the following Boolean expression is ________________

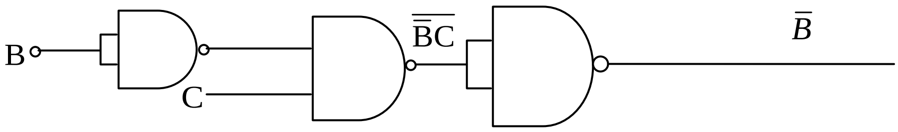
$$Y = \left[A\bar{B}(C+BD) + \bar{A}\bar{B}\right]C$$

Ans. 59: 3 to 3

Solution:

$$Y = A\bar{B}(C+BD)C + \bar{A}\bar{B}C + A\bar{B}BDC + \bar{A}\bar{B}C$$

$$= A\bar{B}C + \bar{A}\bar{B}C = (A+\bar{A})\bar{B}C = \bar{B}$$



Q60. In a nucleus, the interaction $V_{so}\vec{l}\cdot\vec{s}$ is responsible for creating spin-orbit doublets. The energy difference between $p_{1/2}$ and $p_{3/2}$ states in units of $V_{so}\frac{\hbar^2}{2}$ is ____________ (Round off to the nearest integer)

Ans. 60: 3 to 3

Solution: We know, $\vec{j}=\vec{l}+\vec{s}\Rightarrow j^2+s^2+2\left(\vec{l}\cdot\vec{s}\right)\ \therefore \vec{l}\cdot\vec{s}=\frac{1}{2}\left(j^2-l^2-s^2\right)$

Thus, $H_{so}\left|\psi\right\rangle=E\left|\psi\right\rangle=V_{so}\frac{\hbar^2}{2}\left[j\left(j+1\right)-l\left(l+1\right)-s\left(s+1\right)\right]$

For $p_{1/2}$: s = ½; l = 1; j = ½

$$E_1=V_{so}\frac{\hbar^2}{2}\times\left[\frac{3}{4}-2-\frac{3}{4}\right]=V_{so}\frac{\hbar^2}{2}\times-2=-V_{so}\hbar^2$$

For $p_{3/2}$: s = ½; l = 1; j = $_{3/2}$

$$E_2=V_{so}\frac{\hbar^2}{2}\times\left[\frac{15}{4}-2-\frac{3}{4}\right]=V_{so}\frac{\hbar^2}{2}\times\frac{4}{4}=V_{so}\frac{\hbar^2}{2}$$

Thus, the energy difference $\Delta E=E_2-E_1=V_{so}\frac{3\hbar^2}{2}$

Q61. Two identical particles of rest mass m_0 approach each other with equal and opposite velocity $v=0.5c$, where c is the speed of light. The total energy of one particle as measured in the rest frame of the other is $E=\alpha m_0c^2$. The value of α is _______________ (Round off to two decimal places)

Ans. 61: 1.65 to 1.70

Solution: $v_{AE}=0.5c$

$v_{BE}=-0.5c$

$$v_{AB}=\frac{v_{AE}-v_{BE}}{1-\frac{v_{AE}v_{BE}}{c^2}}=\frac{0.5c-\left(-0.5c\right)}{1-\frac{\left(0.5c\right)\left(-0.5c\right)}{c^2}}$$

$$v_{AB}=\frac{c}{1+0.25}=\frac{4c}{5}$$

$$E=mc^2=\frac{m_0c^2}{\sqrt{1-\frac{v_{AB}^2}{c^2}}}=\frac{m_0c^2}{\sqrt{1-\frac{16}{25}}}$$

$$E=\frac{5}{3}m_0c^2=\alpha m_0c^2$$

$$\alpha=\frac{5}{3}=1.67$$

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Q62. In an X-Ray diffraction experiment on a solid with FCC structure, five diffraction peaks corresponding to $(111),(200),(220),(311)$ and (222) planes are observed using $1.54\,\mathring{A}$ X-rays. On using $3\,\mathring{A}$ X-rays on the same solid, the number of observed peaks will be ____________

Ans.: 1 to 1

Solution: Bragg's Law

$$2d_{hkl}\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{\lambda}{2d_{hkl}} = \frac{\lambda}{2a}\sqrt{h^2+k^2+l^2}$$

Corresponding to maximum value of $\sin\theta(=1)$, the expression of $\sqrt{h^2+k^2+l^2}$ has maximum values for the $\sin\theta$. From this condition we can find out the value of lattice parameter (a) from the peak corresponding to (222) plane. So

$$1 = \frac{1.54}{2a}\sqrt{2^2+2^2+2^2} \Rightarrow a = 2.66\,\mathring{A}$$

For $\lambda = 3\,\mathring{A}$, Bragg's Law

$$\text{Sin}\,\theta = \frac{\lambda}{2a}\sqrt{h^2+k^2+l^2} = \frac{3}{2\times 2.66}\sqrt{h^2+k^2+l^2}$$

For Peak (111) $\text{Sin}\,\theta = \frac{3}{2\times 2.66}\sqrt{1^2+1^2+1^2} = \frac{3}{2\times 2.66}\sqrt{3} = 0.976$

Peak (200) $\text{Sin}\,\theta = \frac{3}{2\times 2.66}\sqrt{2^2+0+0} = \frac{3}{2\times 2.66}\sqrt{4} = 1.27$

Peak (220) $\text{Sin}\,\theta = \frac{3}{2\times 2.66}\sqrt{2^2+2^2+0} = \frac{3}{2\times 2.66}\sqrt{8} = 1.657$

Peak (310) $\text{Sin}\,\theta = \frac{3}{2\times 2.66}\sqrt{3^2+1^2+0} = \frac{3}{2\times 2.66}\sqrt{10} = 1.85$

Peak (222) $\text{Sin}\,\theta = \frac{3}{2\times 2.66}\sqrt{2^2+2^2+2^2} = \frac{3}{2\times 2.66}\sqrt{12} = 2.029$

The maximum value of $\sin\theta$ will be 1. So for wavelength $\lambda = 3\,\mathring{A}$ only (111) peak observed.

Q63. For 1 mole of Nitrogen gas, the ratio $\left(\frac{\Delta S_I}{\Delta S_{II}}\right)$ of entropy change of the gas in processes (I) and (II) mentioned below is _______________ (Round off to one decimal place)

(I) The gas is held at $1\,\text{atm}$ and is cooled from $300K$ to $77K$.

(II) The gas is liquefied at $77K$.

(Take $C_p = 7.0\,\text{cal mol}^{-1}K^{-1}$, Latent heat $L = 1293.6\,\text{cal mol}^{-1}$)

Ans.: 0.5 to 0.7

Solution: $\Delta S_1 = \int_{300}^{77}\frac{d\theta}{T} = \int_{300}^{77}\frac{C_p dT}{T} = C_p\ln\left(\frac{77}{300}\right) = 7\ln\frac{77}{300} = -9.519839\,\text{Cal K}^{-1}$

$$\Delta S_2 = \frac{d\theta}{T} = \frac{L}{T} = \frac{1293.6}{77} = -16.8\,Cal\,K^{-1}$$

$$\frac{\Delta S_I}{\Delta S_{II}} = \frac{9.519839}{16.8} = 0.5666$$

≈ 0.6

Q64. Frequency bandwidth Δv of a gas laser of frequency $v\,Hz$ is

$$\Delta v = \frac{2v}{c}\sqrt{\frac{\alpha}{A}}$$

where $\alpha = 3.44\times10^6 m^2 s^{-2}$ at room temperature and A is the atomic mass of the lasing atom. For $^4He - ^{20}Ne$ laser (wavelength $= 633\,nm$), $\Delta v = n\times10^9\,Hz$. The value of n is ________ (Round off to one decimal place)

Ans. 64: 1.2 to 1.4

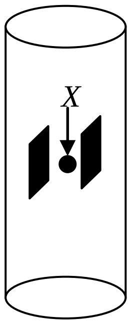
Solution: Sol^n: Frequency bandwidth Δv of a He-Ne laser is given by,

$$\Delta v = \frac{2v}{c}\sqrt{\frac{\alpha}{A}} = \frac{2}{\lambda}\sqrt{\frac{\alpha}{A}} = \frac{2}{633\times10^{-9}}\sqrt{\frac{3.44\times10^6}{20}}$$

$$= \frac{2\times414.73}{633}\times10^9 \sim 1.3\times10^9\,Hz$$

The lasing atom is Ne for which atomic mass is 20 amu.

Q65. A current of $1A$ is flowing through a very long solenoid made of winding density $3000\,\text{turns}/m$. As shown in the figure, a parallel plate capacitor, with plates oriented parallel to the solenoid axis and carrying surface charge density $6\in_0 Cm^{-2}$, is placed at the middle of the solenoid. The momentum density of the electromagnetic field at the midpoint X of the capacitor is $n\times10^{-13}\,Nsm^{-3}$. The value of n is ___________



(Round off to the nearest integer)

(speed of light $c = 3\times10^8 ms^{-1}$)

Ans. 65: 2 to 2

Solution:

$n' = 3000$ turns/m, $I = 1A$, $B_{\text{inside}} = \mu_0 n' I$

Electric field inside capacitor $E = \frac{\sigma}{\in_0}$

$$P_d = \frac{S}{c^2} = \frac{1}{c^2}\times\frac{1}{\mu_0}EB = \frac{1}{c^2\mu_0}\times\frac{\sigma}{\in_0}\times\mu_o n' I$$

$$P_d = \frac{\sigma}{c^2\in_0}n' I = \frac{\sigma\in_0}{(3\times10^8)^2\times\in_0}\times3000\times A = \frac{2000}{10^{16}} = 2\times10^{-13}\,Nsm^{-3} \quad \Rightarrow n = 2$$

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