

PHYSICS

Joint Entrance Screening Test (JEST - 2020)

(Booklet Series-D)

PLEASE READ THE INSTRUCTIONS CAREFULLY

1. This paper contains three parts: Part A, Part B and Part C
2. Part A and Part B contains multiple choice questions and only one of the four options in each question is correct. Questions in Part C has integers between 0 and 9999 as answers.
3. Part A contains 25 questions, and each carries 1 (one) mark for the correct answer and $-1/3$ (negative one third) mark for incorrect answer. Part B contains 15 questions and carry 3 (three) marks each for correct answer and -1 (negative one) mark for incorrect answer. Part C contains 10 questions and each carries 3 (three marks). The answer to these questions must be answered by integers of 4 digits each. Note that if the answer is, e.g. 5, you must fill in 0005. If it is 0, you must fill in 0000. If the zeros are not filled in (where required), the answer will not be credited. There are **NO NEGATIVE MARKS** for these questions.
4. Only non-programmable scientific calculator is allowed and exchange of calculators among the candidates is not permitted. Use of other items like electronic diary, writing pads, pencil box, beeper, cameras, mobile phones, palmtops, laptops, pagers etc. are not permitted inside the examination hall.

PART A: 1 Mark Questions

Q1. The persons named $A, B, C, D, E, F, G, H, I, J$ have come for an interview. They are being called one by one to the interview panel at random. What is the probability that C gives interview before A and A gives before F ?

- (a) $\frac{1}{6}$ (b) $\frac{1}{10}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

Ans. : (a)

Solution: $P = \frac{1}{3!} = \frac{1}{6}$

Q2. The ratio of maximum to minimum resistance that can be obtained with N number of $3-\Omega$ resistors is

- (a) N (b) N^2 (c) N^3 (d) N^4

Ans. 2: (b)

Solution: Resistance in series is maximum and minimum in parallel.

$$R_s = 3 + 3 + 3 + 3 + \dots + 3N = 3N$$

$$\frac{1}{R_p} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \dots = \frac{N}{3} \Rightarrow R_p = \frac{3}{N}$$

$$\Rightarrow \frac{R_s}{R_p} = \frac{3N}{3/N} = N^2$$

Q3. Consider three infinitely long, straight and coplanar wires which are placed parallel to each other. The distance between the adjacent wires is d . Each wire carries a current I in the same direction. Consider points on either side of the middle wire where the magnetic field vanishes. What is the distance of these points from the middle wire?

- (a) $\frac{2d}{3}$ (b) $\frac{2d}{\sqrt{3}}$ (c) $\frac{d}{3}$ (d) $\frac{d}{\sqrt{3}}$

Ans. 3: (d)

Solution: Magnetic field due to wire 1 at point P is

$$B_1 = \frac{\mu_0 I}{2\pi(d-r)} \text{ (pointing inward)}$$

Magnetic field due to wire 2 at point P is

$$B_2 = \frac{\mu_0 I}{2\pi r} \text{ (pointing outward)}$$

Magnetic field due to wire 3 at point P is

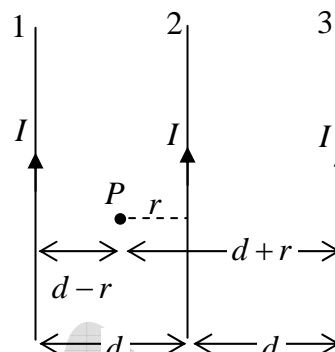
$$B_3 = \frac{\mu_0 I}{2\pi(d+r)} \text{ (pointing outward)}$$

Magnetic field due to wires at point P will vanish if

$$B_1 = B_2 + B_3 \Rightarrow \frac{\mu_0 I}{2\pi(d-r)} = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi(d+r)}$$

$$\Rightarrow \frac{1}{(d-r)} = \frac{1}{r} + \frac{1}{(d+r)} \Rightarrow \frac{1}{d-r} = \frac{d+2r}{r(d+r)} \Rightarrow rd+r^2 = d^2+2rd-rd-2r^2$$

$$\Rightarrow r^2 = d^2 - 2r^2 \Rightarrow 3r^2 = d^2 \Rightarrow r = \frac{d}{\sqrt{3}}$$



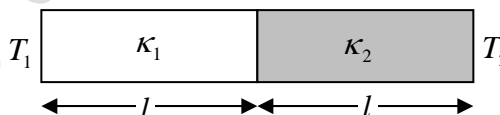
- Q4. A rod consists of two equal sections of length l each with coefficient of thermal conductivity κ_1 and κ_2 , respectively. One end of the rod is kept at a fixed temperature T_1 and the other end at a temperature T_2 ($T_1 > T_2$). If $\kappa_2 = 2\kappa_1$ then the temperature at the interface is

(a) $T = \frac{(2T_1 - T_2)}{3}$

(b) $T = \frac{(2T_1 + T_2)}{3}$

(c) $T = \frac{(T_1 + 2T_2)}{3}$

(d) $T = \frac{(T_1 - 2T_2)}{3}$



Ans. 4: (c)

Solution: Heat In = Heat Out ; $k_1 l (T - T_1) A = k_2 l (T_2 - T) A \Rightarrow T - T_1 = 2(T_2 - T) = 2T_2 - 2T$

$$3T = T_1 + 2T_2 \Rightarrow T = \frac{T_1 + 2T_2}{3}$$

- Q5. If $f(t)$ is a real and even function of t , which one of the following statements is true about its Fourier transform $F(\omega)$ (here * indicates complex conjugation)?

(a) $F^*(\omega) = -F(\omega)$

(b) $F^*(\omega) = F(\omega)$

(c) $F(-\omega) = F(\omega)$

(d) $F(-\omega) = F^*(\omega)$

Ans. 5: (b)

Solution: Let both form Fourier transformation pairs.

$$f(t) \Leftrightarrow F(\omega)$$

$$F(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} f(t) dt = \int_{-\infty}^{\infty} (\cos \omega t + i \sin \omega t) f(t) dt = \int_{-\infty}^{\infty} (\cos \omega t) f(t) dt,$$

Odd term goes to zero.

$$F(\omega) = \text{Real and Even} \Rightarrow F^*(\omega) = F(\omega)$$

or

Let $f(t) = \text{Real and Even}$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} f(t) (\cos \omega t + i \sin \omega t) dt = \int_{-\infty}^{\infty} f(t) \cos \omega t dt$$

$$\text{So } F^*(\omega) = F(\omega)$$

$$\text{Ex: } f(t) = \cos \omega_0 t$$

Q6. Consider an ideal gas whose entropy is given by

$$S = \frac{n}{2} \left[\sigma + 5R \ln \frac{U}{n} + 2R \ln \frac{V}{n} \right],$$

where n is the number of moles, σ is a constant, R is the universal gas constant, U is the internal energy and V is the volume of the gas. The specific heat at constant pressure is then given by

- (a) $\frac{5}{2}nR$ (b) $\frac{7}{2}nR$ (c) $\frac{3}{2}nR$ (d) nR

Ans. 6: (b)

$$\text{Solution: } TdS = dU - PdV \Rightarrow dS = \frac{1}{T}dU - \frac{P}{T}dV$$

$$\Rightarrow \left(\frac{\partial S}{\partial U} \right)_V = \frac{1}{T} \Rightarrow \frac{n}{2} \frac{5R}{U} = \frac{1}{T} \Rightarrow U = \frac{5}{2}nRT$$

$$\Rightarrow C_V = \frac{5}{2}nR \quad \text{and} \quad C_P = \frac{7}{2}nR$$

Q7. A 16-bit analog to digital converter works in the range 0–1 Volt. The least count of the converter is

- (a) 0.30 mV (b) 15.26 mV (c) 5.44 nV (d) $15.26\text{ }\mu\text{V}$

Ans. 7: (d)

Solution: Least count = $\frac{V_{\max}}{2^n} = \frac{1\text{V}}{2^{16}} = \frac{1\text{V}}{65536} \approx 15.26\text{ }\mu\text{V}$

Q8. A particle in a spherically symmetric potential is known to be in an eigenstate of \vec{L}^2 and L_z with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$, respectively. What is the value of $\langle l, m | L_x^2 | l, m \rangle$?

- (a) $\frac{\hbar^2}{2}(l^2 + l + m^2)$ (b) $\hbar^2(l^2 + l - m^2)$
 (c) $\frac{\hbar^2}{2}(l^2 + l - m^2)$ (d) $\frac{\hbar^2}{2}(l^2 + l + m^2)$

Ans. 8: (d)

Solution: $L_x^2 = \frac{(L_+ + L_-)^2}{4} = \frac{L_+^2 + L_-^2 + L_+L_- + L_-L_+}{4} = \frac{L_+^2 + L_-^2 + 2(L^2 - L_z^2)}{4}$

$$\langle l, m | L_x^2 | l, m \rangle = \frac{1}{4} [\langle L_+^2 \rangle + \langle L_-^2 \rangle + 2\langle L^2 \rangle - 2\langle L_z^2 \rangle]$$

$$\Rightarrow \langle l, m | L_x^2 | l, m \rangle = \frac{1}{4} \times [0 + 0 + 2l(l+1)\hbar^2 - 2m^2\hbar^2] = (l^2 + l - m^2) \frac{\hbar^2}{2}$$

Q9. A particle of mass m carrying angular momentum l moves in a central potential

$$V(r) = -\frac{ke^{-ar}}{r}, \text{ where } k, a \text{ are positive constants. If the particle undergoes circular motion,}$$

what is the equation determining its radius r_0 ?

- (a) $\frac{l^2}{mr_0} = kar_0 e^{-ar_0}$ (b) $\frac{l^2}{mr_0} = ke^{-ar_0} (1 + ar_0)$
 (c) $\frac{l^2}{2mr_0} = ke^{-ar_0}$ (d) $\frac{l^2}{2mr_0} = ke^{-ar_0} (1 + ar_0)$

Ans. 9: (b)

Solution: $V_{\text{eff}} = \frac{l^2}{2mr^2} - \frac{Ke^{-ar}}{r}$

For circular motion $\frac{\partial V_{eff}}{\partial r} = 0$ at $r = r_0$

$$\left(\frac{-l^2}{mr^3} + \frac{Ke^{-ar}}{r^2} + \frac{\alpha Ke^{-ar}}{r} \right) \Bigg|_{r=r_0} = 0 \Rightarrow \frac{-l^2}{mr_0^3} + \frac{Ke^{-ar_0}}{r_0^2} + \frac{\alpha Ke^{-ar_0}}{r_0} = 0$$

$$\Rightarrow \frac{l^2}{mr_0^3} = \frac{Ke^{-ar_0}}{r_0^2} + \frac{\alpha Ke^{-ar_0}}{r_0} \Rightarrow \frac{l^2}{mr_0^3} = Ke^{-ar_0} \left[\frac{1}{r_0^2} + \frac{\alpha}{r_0} \right] \Rightarrow \frac{l^2}{mr_0} = Ke^{-ar_0} [1 + \alpha r_0]$$

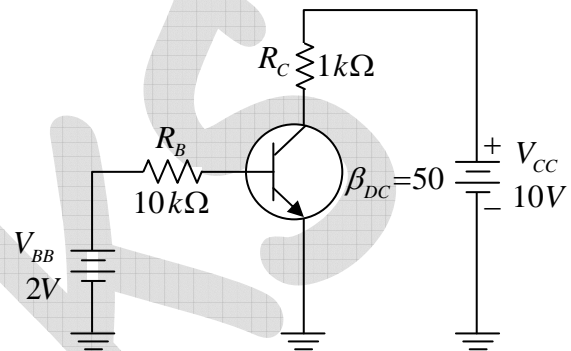
Q10. Calculate the collector current and determine whether or not the transistor in figure shown below is in saturation. Assume $V_{CE}(sat) = 0.2V$

(a) $6.5 mA$, not in saturation

(b) $11.5 mA$, in saturation

(c) $11.5 mA$, not in saturation

(d) $6.5 mA$, in saturation



Ans. 10: (a)

Solution: Emitter base junction is forward bias.

$$I_B = \frac{2V - 0.7V}{10k} = 0.13mA \Rightarrow I_C = \beta I_B = 6.5mA$$

$$V_B = V_{BE} = 0.7V \text{ and } V_C = V_{CC} - I_C R_C = 10V - (6.5 \times 1)V = 3.5V$$

$$\Rightarrow V_{CB} = V_C - V_B = 3.5V - 0.7V = 2.8V = +ve \text{ (Collector base junction reverse bias)}$$

Thus transistor is in active region.

Q11. Charges are placed as follows: q at $(a, a, 0)$ and $(-a, -a, 0)$ and $-q$ at $(a, -a, 0)$ and $(-a, a, 0)$. At large distances, how does the electrostatic potential behave as a function of the distance r from the centre $(0, 0, 0)$?

(a) $1/r^3$

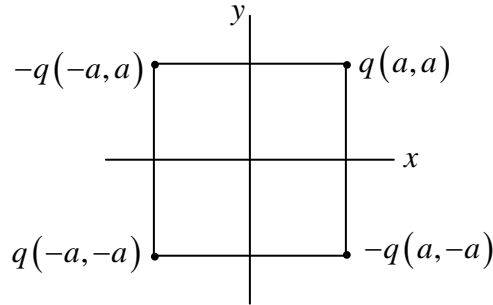
(b) $1/r^2$

(c) $1/r$

(d) $1/r^4$

Ans. 11: (a)

Solution:



According to multipole expansion $Q_{mono} = q - q + q - q = 0$

$$\bar{p} = q(a\hat{x} + a\hat{y}) - q(-a\hat{x} + a\hat{y}) + q(-a\hat{x} - a\hat{y}) - q(a\hat{x} - a\hat{y}) = 0$$

Thus $V \propto \frac{1}{r^3}$

Q12. An $n \times n$ Hermitian matrix A is not a multiple of the identity matrix. Which one of the following statements is always true?

(a) $nTr(A^2) = (TrA)^2$

(b) $nTr(A^2) < (TrA)^2$

(c) $nTr(A^2) > (TrA)^2$

(d) $Tr(A^2) = n(TrA)^2$

Ans. 12: (c)

Solution: Let $n = 3$ and $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow Tr(A) = 3$

$$A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}_{3 \times 3} \Rightarrow Tr(A^2) = 9 \Rightarrow nTr(A^2) = 3 \times 9 = 27 \text{ and } (TrA)^2 = 9$$

Thus $nTr(A^2) > (TrA)^2$.

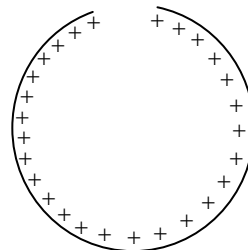
Q13. A ring of radius $0.5m$ has a gap of $0.002\pi m$. If the ring carries a charge of $+1.0C$ distributed uniformly along it, then the electric field at the centre of the ring is

(a) $7.5 \times 10^7 NC^{-1}$

(b) $7.2 \times 10^7 NC^{-1}$

(c) $6.2 \times 10^7 NC^{-1}$

(d) $6.5 \times 10^7 NC^{-1}$



Ans. 13: (b)

Solution: The electric field at the centre is due to missing segment.

$$\text{Charge per unit length } \lambda = \frac{1C}{2\pi \times \frac{1}{2} - 0.002\pi} = \frac{1}{0.998\pi} C/m$$

$$\therefore \frac{\text{Arc Length}}{\text{Circumference}} = \frac{\text{Centre Angle}}{360^\circ} \Rightarrow \frac{l}{2\pi r} = \frac{\theta}{360^\circ}$$

$$\Rightarrow \frac{l}{2\pi r} = \frac{\theta}{360^\circ} \Rightarrow \theta = 360^\circ \times \frac{l}{2\pi r}$$

$$\Rightarrow \theta = 360^\circ \times \frac{0.002\pi}{2\pi \times 0.5} = 0.72^\circ$$

$$\therefore \alpha + \theta + \alpha = 180^\circ \Rightarrow \alpha = \frac{180^\circ - \theta}{2} = \frac{180^\circ - 0.72^\circ}{2} = 89.64^\circ \Rightarrow \alpha = 89.64^\circ$$

and $\alpha + \theta = 90.36^\circ$.

$$\text{Electric field due to elemental length } dl \text{ at center is } dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2}.$$

Consider all elemental length on the arc, then at the centre horizontal component ($dE \cos \alpha$) will

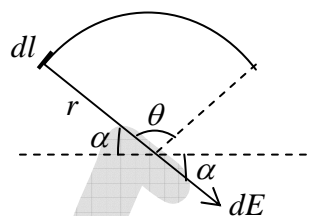
cancel out and the vertical component ($dE \sin \alpha$) will add up.

$$\text{Thus resultant field } E = \int dE \sin \alpha = \frac{\lambda}{4\pi\epsilon_0} \int_{\alpha}^{\alpha+\theta} \sin \alpha \frac{rd\alpha}{r^2} \quad \because dl = rd\alpha$$

$$\Rightarrow E = \frac{\lambda}{4\pi\epsilon_0 r} [-\cos(\alpha + \theta) + \cos \alpha] \Rightarrow E = \frac{1}{0.998\pi} \times \frac{9 \times 10^9}{0.5} [-\cos(90.36) + \cos(89.64)]$$

$$\Rightarrow E = \frac{1}{0.998\pi} \times \frac{9 \times 10^9}{0.5} [0.0063 + 0.0063]$$

$$\Rightarrow E = \frac{0.1134}{1.566} \times 10^9 = \frac{11.34}{1.566} \times 10^7 = 7.2 \times 10^7 \text{ V/m}$$



Ans. 16: (b)

$$\begin{aligned} \text{Solution: } \psi(x) &= 2\sqrt{3}e^{-x}(1 - e^{-x}); & x \geq 0 \\ &= 0; & x \leq 0 \end{aligned}$$

$$|\psi^2| = (2\sqrt{3}(e^{-x} - e^{-2x}))(2\sqrt{3}(e^{-x} - e^{-2x})) = 4 \times 3(e^{-2x} + e^{-4x} - e^{-3x} - e^{-3x})$$

$$\Rightarrow |\psi^2| = 12(e^{-4x} + e^{-2x} - 2e^{-3x})$$

$$\int_0^{\infty} |\psi|^2 dx = - \left[12 \left[\frac{-1}{4} - \frac{1}{2} + \frac{2}{3} \right] \right] = -12 \left[\frac{-3 - 6 + 8}{12} \right] = 1$$

For most probable position; $\frac{d|\psi|^2}{dx} = 0$

$$\Rightarrow -4e^{-4x} - 2e^{-2x} + 6e^{-3x} = 0 \Rightarrow e^{-2x} [-4e^{-2x} - 2 + 6e^{-x}] = 0$$

$$\Rightarrow -4e^{-2x} - 2 + 6e^{-x} = 0 \Rightarrow 4e^{-2x} - 6e^{-x} + 2 = 0 \quad (\text{Let } e^{-x} = y)$$

$$\Rightarrow 4y^2 - 6y + 2 = 0 \Rightarrow 2y^2 - 3y + 1 = 0 \Rightarrow 2y^2 - 2y - y + 2 = 0$$

$$\Rightarrow 2y(y-1) - 1(y-1) = 0 \Rightarrow (y-1)(2y-1) = 0$$

$$\Rightarrow y = 1 \text{ and } y = \frac{1}{2}$$

$$\Rightarrow e^{-x} = 1 \text{ and } e^{-x} = \frac{1}{2}$$

$$x_p = 0.693$$

$$\langle x \rangle = \int x |\psi|^2 dx = 12 \left[\int_0^{\infty} x e^{-4x} dx + \int_0^{\infty} x e^{-2x} dx - 2 \int_0^{\infty} x e^{-3x} dx \right]$$

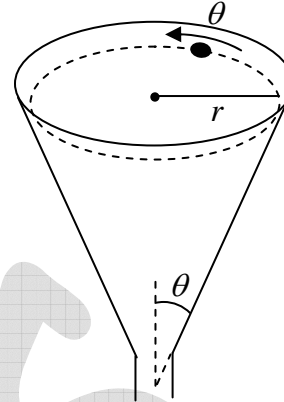
$$\Rightarrow \langle x \rangle = 12 \left(\frac{1}{4^2} + \frac{1}{2^2} - \frac{2}{3^2} \right) = 12 \left(\frac{1}{16} + \frac{1}{4} - \frac{2}{9} \right) = 12 \left(\frac{9 + 36 - 32}{144} \right) \Rightarrow \langle x \rangle = \frac{13}{12} = 1.08$$

$$x_p = 0.693$$

$$\frac{\langle x \rangle}{x_p} = \frac{1.08}{0.693} = 1.563$$

Q17. A particle is to slide along the horizontal circular path on the inner surface of the funnel as shown in the figure. The surface of the funnel is frictionless. What must be the speed of the particle (in terms of r and θ) if it is to execute this motion?

- (a) $\sqrt{rg \sin \theta}$
- (b) $\sqrt{rg \cos \theta}$
- (c) $\sqrt{rg \tan \theta}$
- (d) $\sqrt{rg \cot \theta}$



Ans. 17: (d)

Solution: $L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2 + \dot{z}^2) - mgz$

$$\frac{r}{z} = \tan \theta \Rightarrow z = r \cot \theta$$

$$L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2 + \dot{r}^2 \cot^2 \theta) - mgr \cot \theta = 0$$

For circular motion, $\dot{r} = 0$, $\ddot{r} = 0$

$$mr\dot{\phi}^2 = mg \cot \theta \Rightarrow \dot{\phi} = \sqrt{\frac{g \cot \theta}{r}} \Rightarrow v = r\dot{\phi} = \sqrt{gr \cot \theta}$$

Q18. Two rails of a railroad track are insulated from each other and from the ground, and are connected by a millivoltmeter. What is the reading of the millivoltmeter when a train travels at the speed 90 km/hr down the track? Assume that the vertical component of the earth's magnetic field is 0.2 gauss and that the tracks are separated by two meters. Use

$$1 \text{ gauss} = 10^{-4} \text{ Tesla} = 10^{-4} \text{ V} \cdot \text{sec} / \text{m}^2$$

- (a) 10
- (b) 1
- (c) 0.2
- (d) 180

Ans. 18: (b)

Solution: Magnetic flux $\phi_m = Blx \Rightarrow \text{e.m.f } \varepsilon = -\frac{d\phi_m}{dt} = -Blv$

$$\Rightarrow |\varepsilon| = 0.2 \times 10^{-4} \times 2 \text{ m} \times \frac{90 \times 10^3}{3600} \text{ Volts} \Rightarrow |\varepsilon| = 1 \text{ mV}$$

Q19. A particle of mass m moves in a one-dimensional potential $V(x) = F_0|x|$, where F_0 is a positive constant. Given the initial conditions, $x(0) = x_0 > 0$ and $\dot{x}(0) = 0$, which one of the following statements is correct?

(a) The particle undergoes simple harmonic motion about the origin with frequency

$$\omega = 2\pi \sqrt{\frac{F_0}{mx_0}}$$

(b) The angular frequency of oscillations of the particle is $\omega = \frac{1}{2}\pi \sqrt{\frac{F_0}{2mx_0}}$

(c) The particle begins from rest and is accelerated along the positive x -axis such that

$$x(t) = \frac{x_0 + F_0 t^2}{2m}$$

(d) The angular frequency of oscillations of the particle is independent of its mass

Ans. 19: (b)

Solution: The force is constant so it is not small oscillation but motion is periodic so we can find time period as well as angular frequency.

For periodic motion we can find action angle variable.

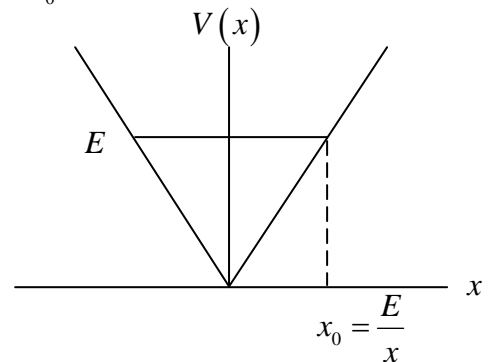
$$J = \oint p dx \Rightarrow \oint \sqrt{2m(E - F_0|x|)} dx = 4 \int_0^{\frac{E}{F_0}} \sqrt{2m(E - F_0 x)} dx$$

Put $2m(E - F_0 x) = t \Rightarrow dx = -\frac{dt}{2mF_0}$, $x = 0, t = 2mE, x = \frac{E}{F_0}, t = 0$

$$\text{So } J = -\frac{4}{2mE} \int_{2mE}^0 \sqrt{t} dt = \frac{4}{2mF_0} \int_0^{2mE} \sqrt{t} dt$$

$$J = \frac{4}{2mF_0} \frac{2}{3} (2mE)^{3/2} = \frac{8}{3F_0} (2m)^{1/2} E^{3/2}$$

Time period is given by $T = \frac{dJ}{dE} = \frac{8}{3F_0} (2m)^{1/2} \frac{3}{2} E^{1/2}$



For given condition given the initial conditions, $x(0) = x_0 > 0$ and $\dot{x}(0) = 0$, means x_0 is turning point and $E = Fx_0$.

So time period is given by $T = 4 \left(\frac{2mx_0}{F_0} \right)^{1/2}$ and $\omega = \frac{2\pi}{T} = \frac{\pi}{2} \left(\frac{F_0}{2mx_0} \right)^{1/2}$

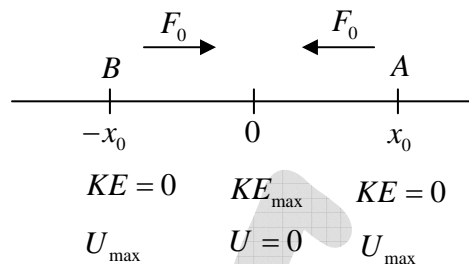
OR

$$F = -\frac{\partial V}{\partial x}$$

$$V(x) = F_0 x \quad x > 0$$

$$= -F_0 x \quad x < 0$$

$$F = \begin{cases} -F_0 & x > 0 \\ +F_0 & x < 0 \end{cases}$$



Particle will oscillate about origin but its motion is not SHM because $F \neq -kx$

Time taken by particle from A to 0 is $t_1 = \sqrt{\frac{2x_0}{a}} = \sqrt{\frac{2x_0 m}{F_0}}$

Time period $T = 4t_1 = 4\sqrt{2x_0 m / f_0}$

Frequency $n = \frac{1}{T} = \frac{1}{4} \sqrt{\frac{F_0}{2x_0}} \Rightarrow \omega = 2\pi n = \frac{1}{2} \pi \sqrt{\frac{F_0}{2x_0 m}}$

Q20. A carbon rod of resistance R_c and a metal rod of resistance R_m are connected in series. Let their linear temperature coefficients of resistivity have magnitudes α_c and α_m , respectively. The condition that the net resistance would be independent of temperature is

- (a) $R_c = R_m$ (b) $\alpha_c = \alpha_m$ (c) $\frac{R_c}{R_m} = \frac{\alpha_m}{\alpha_c}$ (d) $\frac{R_m}{R_c} = \frac{\alpha_m}{\alpha_c}$

Ans. 20: (c)

Q21. The 2's complement of 1111 1111 is

- (a) 00000001 (b) 00000000 (c) 1111 1111 (d) 1000 0000

Ans. 21: (a)

Solution: $(1111 1111)_2 \rightarrow 1^{st}$ compliment $(0000 0000)_2$

2^s compliment = 1^{st} comp. + 1 = $(0000 0001)_2$

Q22. Two tuning forks A and B are struck instantaneously to obtain Lissajous figures. The figures go through a complete cycle in 20 s . Fork A is loaded with wax, so that the cycle period changes to 10 s . If the frequency of fork B is 256.10 Hz , what is the frequency of fork A after loading?

- (a) 256.00 Hz (b) 256.05 Hz (c) 256.15 Hz (d) 256.20 Hz

Ans. 22: (a)

Solution: $n_1 \sim n_2 = \frac{1}{t_1}$

$$n_1 \sim 256.10 = \frac{1}{20} = 0.05$$

So, $n_1 = 256.05$ or 256.15

Since cycle period is decreasing on loading wax, So, $n_1 < n_2$; $n_1 = 256.05$.

Suppose on loading wax n_1 is decreased to n_1^{wax} , then

$$n_2 - n_1^{\text{wax}} = \frac{1}{t_2} - \frac{1}{10\text{ s}} = 0.1\text{ Hz} \Rightarrow n_1^{\text{wax}} = n_2 - 0.10 = 256.10 - 0.10$$

$$\Rightarrow n_1^{\text{wax}} = 256\text{ Hz}$$

Q23. Consider a classical harmonic oscillator in thermal equilibrium at a temperature T . If the spring constant is changed to twice its value isothermally, then the amount of work done on the system is

- (a) $k_B T \ln 2$ (b) $k_B T \frac{\ln 2}{2}$ (c) $2k_B T \ln 2$ (d) $-k_B T \ln 2$

Ans. 23: (b)

Solution: $Z = \frac{2\pi k_B T}{hw}$, $F = -k_B T \ln Z$, $F = -k_B T \ln \frac{2\pi k_B T}{h\sqrt{\frac{k}{m}}}$, $F = -k_B T \ln \left(\frac{2\pi k_B T}{h\sqrt{\frac{1}{m}}} k^{-1/2} \right)$

Using general definition of pressure, $P = -\frac{\partial F}{\partial \text{Parameter}} = -\frac{\partial F}{\partial k}$

$$P = k_B T \frac{1}{\frac{2\pi k_B T}{h\sqrt{\frac{1}{m}}} k^{-1/2}} \frac{2\pi k_B T}{h\sqrt{\frac{1}{m}}} \left(-\frac{1}{2}\right) k^{-3/2} = \frac{k_B T}{2k}$$

$$Work = \int_k^{2k} P dk = \int_k^{2k} \frac{k_B T}{2k} dk = \frac{k_B T}{2} \int_k^{2k} \frac{dk}{k} = k_B T \frac{\ln 2}{2}$$

Q24. The solution of the differential equation $y'' - 2y' - 3y = e^u$ is given as $C_1 e^{-t} + C_2 e^{2t} + C_3 e^{3t}$.

The values of the coefficients C_1, C_2 and C_3 are:

- (a) C_1, C_2 and C_3 are arbitrary (b) C_1, C_3 are arbitrary and $C_2 = \frac{-1}{3}$
 (c) C_2, C_3 are arbitrary and $C_1 = \frac{-1}{3}$ (d) C_1, C_2 are arbitrary and $C_3 = \frac{-1}{3}$

Ans. 24: (b)

Solution: $\frac{1}{D^2 - 2D - 3} e^{2t} = \frac{1}{4 - 2 \times 2 - 3} e^{2t}$

P.I. = $-\frac{1}{3} e^{2t} \Rightarrow C_2 = -\frac{1}{3}$

Q25. A free particle of energy E , characterized by a plane wave of wavelength λ enters a region of constant potential $-V$ (where $E > V > 0$). Within the region of the potential, the wavelength of the particle is $\frac{\lambda}{2}$. The ratio $\frac{V}{E}$ is:

- (a) $\frac{-1}{3}$ (b) -3 (c) 3 (d) $\frac{1}{3}$

Ans. 25: (c)

Solution: $E = \frac{P^2}{2m} = \frac{\hbar^2}{2m\lambda^2} \Rightarrow \lambda^2 = \frac{\hbar^2}{2mE}$

$$\therefore \frac{\lambda}{2} = \sqrt{\frac{\hbar^2}{2m(E+V)}} \Rightarrow \left(\frac{\lambda}{2}\right)^2 = \frac{\hbar^2}{2m(E+V)} \Rightarrow \frac{1}{4} \frac{\hbar^2}{2mE} = \frac{\hbar^2}{2m(E+V)}$$

$$\Rightarrow \frac{E+V}{E} = 4 \Rightarrow 1 + \frac{V}{E} = 4 \Rightarrow \frac{V}{E} = 3$$

PART B: 3-Mark Questions

Q1. Consider a quantum particle of mass m moving in a potential

$$V(x, y) = \begin{cases} \frac{1}{2} m \omega^2 (x^2 + y^2), & \text{for } x > 0, y > 0 \\ \infty, & \text{otherwise} \end{cases}$$

what is the degeneracy of the energy state $9\hbar\omega$, where $\omega > 0$ measures the strength of the potential?

- (a) 4 (b) 2 (c) 10 (d) 5

Ans. 1: (a)

Solution: $V(x, y) = \begin{cases} \frac{1}{2} m \omega^2 (x^2 + y^2), & \text{for } x > 0, y > 0 \\ \infty, & \text{otherwise} \end{cases}$

This is semi-parabolic potential in x and y .

So only odd at x_x odd at x_y will give

$$(x_x, x_y) = (1, 7)(7, 1)(3, 5)(5, 3).$$

So option (a) is correct.

Q2. Consider a particle with total energy E is oscillating in a potential $U(x) = A|x|^n$ with $A > 0$ and $n > 0$ in one dimension. Which one of the following gives the relation between the time-period of oscillation T and the total energy E :

- (a) $T \propto E^{1/n-1/2}$ (b) $T \propto E^0$ (c) $T \propto E^n$ (d) $T \propto E^{1/n}$

Ans. 2: (a)

Solution: $J = \oint \sqrt{2m(E-V)} dx \Rightarrow J \propto \int_0^{(E/A)^{1/n}} \sqrt{2m(E - \hbar x^n)} dx$

$$J \propto \sqrt{2mE} \times \left(\frac{E}{A}\right)^{1/n} \int_0^1 \sqrt{1-t^n} dt \Rightarrow J \propto E^{\frac{1}{n} + \frac{1}{2}}$$

$$T \propto \frac{\partial J}{\partial E} \Rightarrow T \propto E^{\frac{1}{n} - \frac{1}{2}}$$

Q3. A classical gas of N particles is kept at a temperature T and is confined to move on a two-dimensional surface (xy -plane). If an external linear force field is applied along the x -axis, then the partition function of the system will be proportional to

- (a) T^N (b) T^{2N} (c) $T^{N/2}$ (d) $T^{3N/2}$

Ans. 3: (d)

Solution: $F = -kx = -\frac{dV}{dx} \Rightarrow V = +kx^2$

$$z_1 = \frac{1}{h^2} \iint_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\beta \left[\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + kx^2 \right]} dp_x dp_y dx dy \propto \left(\sqrt{2\pi x k_B T} \right)^2 \sqrt{\frac{\pi}{\beta k}} \Rightarrow z_1 \propto T^{\frac{1}{2}+1} \propto T^{\frac{3}{2}}$$

Thus for N particles $z_1 \propto T^{\frac{3}{2}N}$

Q4. An electromagnetic field is given by

$$\vec{E}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \theta(vt-r) \hat{r}, \quad \vec{B}(\vec{r}, t) = 0$$

$$\text{where } \theta(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

The corresponding charge density ρ and current density \vec{J} are given by

(a) $\rho = -q\delta^3(\vec{r})\theta(vt-r) + \frac{q}{4\pi r^2} \theta(vt-r); \vec{J} = 0$

(b) $\rho = -q\delta^3(\vec{r})\theta(vt-r); \vec{J} = 0$

(c) $\rho = \frac{q}{4\pi r^2} \delta(vt-r); \vec{J} = \frac{qv}{4\pi r^2} \delta(vt-r) \hat{r}$

(d) $\rho = -q\delta^3(\vec{r})\theta(vt-r) + \frac{q}{4\pi r^2} \delta(vt-r); \vec{J} = \frac{qv}{4\pi r^2} \delta(vt-r) \hat{r}$

Ans. 4: (d)

Solution: $\because \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow \rho = \epsilon_0 (\vec{\nabla} \cdot \vec{E}) \Rightarrow \rho = -\frac{q}{4\pi} \vec{\nabla} \cdot \left\{ \theta(vt-r) \frac{\hat{r}}{r^2} \right\}$

$$\Rightarrow \rho = -\frac{q}{4\pi} \left[\theta(vt-r) \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) + \frac{\hat{r}}{r^2} \cdot \vec{\nabla} \{ \theta(vt-r) \} \right]$$

$$\Rightarrow \rho = -q\delta^3(r)\theta(vt-r) + \frac{q}{4\pi r^2} \delta(vt-r) \quad \because \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = 4\pi\delta^3(r)$$

$$\text{and } \theta(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

$$\because \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{J} = -\epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \because \vec{B} = 0$$

$$\Rightarrow \vec{J} = -\epsilon_0 \frac{\partial \vec{E}}{\partial t} = -\epsilon_0 \times \frac{q}{4\pi\epsilon_0 r^2} \frac{\partial \theta}{\partial t} \times v \hat{r} \Rightarrow \vec{J} = \frac{qv}{4\pi r^2} \delta(vt - r) \hat{r}$$

Q5. A particle of mass m is placed in a potential well $U(x) = cx^n$, where c is a positive constant and n is an even positive integer. If the particle is in equilibrium at constant temperature, which one of the following relations between average kinetic energy $\langle K \rangle$ and average potential energy $\langle U \rangle$ is correct?

- (a) $\langle K \rangle = \frac{2}{n} \langle U \rangle$ (b) $\langle K \rangle = \langle U \rangle$ (c) $\langle K \rangle = \frac{n}{2} \langle U \rangle$ (d) $\langle K \rangle = 2 \langle U \rangle$

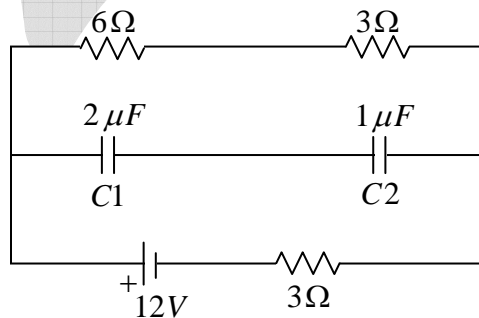
Ans. 5: (c)

Solution: If $V = ar^{n+1}$ then $\langle T \rangle = \frac{n+1}{2} \langle V \rangle$.

Thus if $U(x) = cx^n \Rightarrow \langle K \rangle = \frac{n}{2} \langle U \rangle$.

Q6. What is the charge stored on each capacitor C_1 and C_2 in the circuit shown in the given figure?

- (a) $6\mu C, 6\mu C$
 (b) $6\mu C, 3\mu C$
 (c) $3\mu C, 6\mu C$
 (d) $3\mu C, 3\mu C$



Ans. 6: (a)

Solution: $V_1 = \frac{q_1}{C_1}$, $V_2 = \frac{q_2}{C_2}$ and $V_1 + V_2 = 9V$.

$$\Rightarrow \frac{q_1}{C_1} + \frac{q_2}{C_2} = 9V \Rightarrow \frac{q_1}{2\mu F} + \frac{q_2}{1\mu F} = 9V$$

$$\Rightarrow q_1 + 2q_2 = 18 \text{ (Check that option (a) satisfies this condition)}$$

Q7. A particle moving in two dimensions satisfies the equations of motion

$$\dot{x}(t) = x(t) + y(t)$$

$$\dot{y}(t) = x(t) - y(t)$$

with $\dot{x}(0) = 0$. What is the ratio of $\frac{x(\infty)}{y(\infty)}$?

- (a) $1 - \frac{1}{\sqrt{2}}$ (b) $1 + \frac{1}{\sqrt{2}}$ (c) $\sqrt{2} - 1$ (d) $\sqrt{2} + 1$

Ans. 7: (d)

Solution: $\ddot{x} = \dot{x} + \dot{y} \Rightarrow \ddot{x} = \dot{x} + x - (\dot{x} - x) \Rightarrow \ddot{x} = \dot{x} + x - \dot{x} + x$

$$\Rightarrow \ddot{x} - 2x = 0 \Rightarrow m^2 - 2 = 0 \Rightarrow m = \pm\sqrt{2}$$

$$\Rightarrow x = C_1 e^{\sqrt{2}t} + C_2 e^{-\sqrt{2}t}$$

$$\dot{x}(t) = C_1 \sqrt{2} e^{\sqrt{2}t} - C_2 \sqrt{2} e^{-\sqrt{2}t}$$

$$\because \dot{x}(0) = 0 \Rightarrow C_1 - C_2 = 0 \Rightarrow C_2 = C_1$$

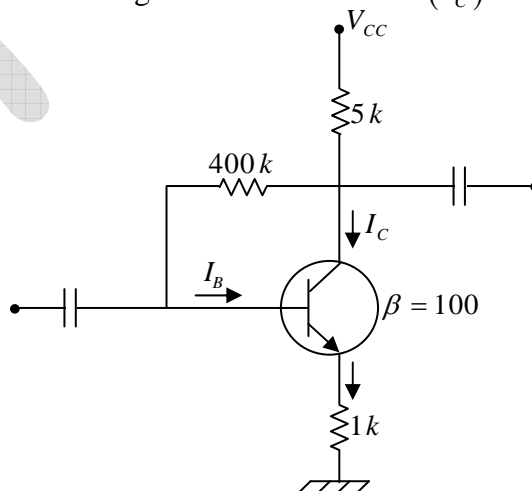
$$\Rightarrow x = 2C_1 \sinh \sqrt{2}t$$

$$\text{and } y = \dot{x} - x = \sqrt{2}C_1 \cosh \sqrt{2}t - 2C_1 \sinh \sqrt{2}t$$

$$\frac{x}{y} = \frac{2C_1 \sinh \sqrt{2}t}{\sqrt{2}C_1 \cosh \sqrt{2}t - 2C_1 \sinh \sqrt{2}t} \Rightarrow \frac{x}{y} = \frac{1}{\sqrt{2} \coth \sqrt{2}t - 1} = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1, \quad \text{at } t \rightarrow \infty$$

Q8. Analyse the common emitter transistor circuit given in the figure. If the current gain (β) increases by 50%, the relative change in collector current (I_C) is approximately

- (a) 5%
(b) 15%
(c) 20%
(d) 25%



Ans. 8: (b)

Solution: $I_C = \frac{V_{CC} - V_{BE}}{5k + 1k + \frac{400k}{100}} = \frac{V_{CC} - V_{BE}}{10k}$ and $I'_C = \frac{V_{CC} - V_{BE}}{5k + 1k + \frac{400k}{150}} = \frac{V_{CC} - V_{BE}}{8.66k}$.

Thus $\frac{I_C - I'_C}{I_C} \times 100\% \approx 15\%$.

Q9. What is the value of the following contour integral I taken counterclockwise around the circle $|z| = 2$?

$$I = \oint_C \frac{dz}{z^3(z+4)}$$

(a) $\frac{\pi i}{2}$

(b) $\frac{\pi i}{32}$

(c) $\frac{\pi i}{16}$

(d) $\frac{\pi i}{4}$

Ans. 9: (b)

Solution: $f(z) = \frac{1}{4z^3} \left[1 + \frac{z}{4} \right]^{-1} = \frac{1}{4z^3} \left[1 - \frac{z}{4} + \frac{(-1)(-2)}{2!} \frac{z^2}{16} + \dots \right]$

$\Rightarrow b_1 = \frac{1}{64}, \quad I = 2\pi i \times \frac{1}{64} = \frac{\pi i}{32}$

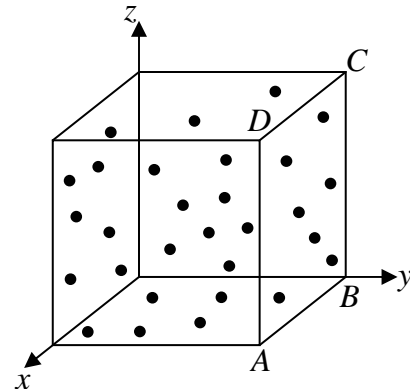
Q10. Consider a cube (see figure) of volume V containing N molecules each of mass m with uniform density $n = \frac{N}{V}$. Suppose this system is equivalent to a system of M non-interacting gases such that molecules of the i th gas are $N_i = n_i V$ in number, each with an identical y -component of velocity v_i . What is the pressure P on the surface $\square ABCD$ of area A ?

(a) $P = m \sum_{i=1}^M n_i v_i^2$

(b) $P = \frac{m \sum_{i=1}^M n_i v_i^2}{\sum_{i=1}^M n_i}$

(c) $P = \frac{m \sum_{i=1}^M n_i v_i^2}{2}$

(d) $P = 2m \sum_{i=1}^M n_i v_i^2$



Ans. 10: (a)

Solution: We will use kinetic theory of Gas model. Let the momentum of the i^{th} molecule before collision with the walls be $p = mv_i$. After the collision with the walls it will return in opposite direction. If the collision is elastic the momentum will be $p = -mv_i$.

$$\therefore \text{Change in momentum after collision } \Delta p = mv_i - (-mv_i) = 2mv_i$$

$$\therefore \text{Force applied by the wall on a molecule will be, } F_i = \frac{\Delta p}{\Delta t} = \frac{2mv_i}{l/v_i} = \frac{2mv_i^2}{l}$$

\Rightarrow Total force acting on the wall is the sum of all the forces applied by all the molecules is

$$\text{given by } F = \sum_{i=1}^M F_i N_i$$

\therefore Pressure acting on the wall will be,

$$P = \frac{F}{A} = \frac{\sum_{i=1}^M F_i N_i}{l^2} = \frac{\sum_{i=1}^M 2mv_i^2 N_i}{l^3} = \frac{1}{l^3} \sum_{i=1}^M 2mv_i^2 N_i = \frac{2m}{l^3} \sum_{i=1}^M v_i^2 n_i V = 2m \sum_{i=1}^M n_i v_i^2$$

Q11. The wave function of a particle subjected to a spherically symmetric potential $V(r)$ is given by $\psi(\vec{r}) = (x - y + 2z)f(r)$. Which one of the following statements is true about $\psi(\vec{r})$?

- (a) It is an eigenfunction of \vec{L}^2 with $l = 0$
- (b) It is an eigenfunction of \vec{L}^2 with $l = 1$
- (c) It is an eigenfunction of \vec{L}^2 with $l = 2$
- (d) It is not an eigenfunction of \vec{L}^2

Ans. 11: (b)

Solution: $\psi(r) = (x - y + 2z)f(r)$

$$x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta$$

$$\psi(r, \theta, \phi) = rf(r) [\sin \theta (\cos \phi - \sin \phi) + 2 \cos \theta] \propto rf(r) P_1(\cos \theta) f(\phi) \Rightarrow l = 1$$

Eigen state with $l = 1$

Q12. The Hamiltonian for a particle of mass m is given by $H = \frac{(p - \alpha q)^2}{(2m)}$, where α is a non-zero constant. Which one of the following equations is correct?

(a) $p = m\dot{q}$

(b) $\alpha \dot{p} = \dot{q}$

(c) $\ddot{q} = 0$

(d) $L = \frac{1}{2}m\dot{q}^2 - \alpha q\dot{q}$

Ans. 12: (c)

Solution: $H = \frac{(p - \alpha q)^2}{2m}$

$$\frac{\partial H}{\partial p} = \dot{q} \Rightarrow \frac{p - \alpha q}{m} = \dot{q} \Rightarrow (p - \alpha q) = m\dot{q}$$

$$\frac{\partial H}{\partial q} = -\dot{p} \Rightarrow \frac{-2\alpha(p - \alpha q)}{2m} = -\dot{p} \Rightarrow \dot{p} = \frac{\alpha(p - \alpha q)}{m} \Rightarrow \dot{p} = \frac{\alpha \times m\dot{q}}{m} = \alpha\dot{q}$$

$$\therefore (p - \alpha q) = m\dot{q} \Rightarrow p = \alpha q + m\dot{q} \Rightarrow \dot{p} = \alpha\dot{q} + m\ddot{q}$$

$$\text{Thus } \dot{p} = \alpha\dot{q} + m\ddot{q} = \alpha\dot{q} \Rightarrow m\ddot{q} = 0 \Rightarrow \ddot{q} = 0$$

Q13. A continuous He-Ne laser beam ($\lambda = 632.8 \text{ nm}$) is 'chopped', using a spinning aperture into $1 \mu\text{s}$ square pulses. The order-of-magnitude estimate of the spectral width $\Delta\lambda$ of the emerging 'pulsed' light is

(a) 10^{-9} m

(b) 10^{-12} m

(c) 10^{-15} m

(d) 10^{-18} m

Ans. 13: (c)

Solution: $\Delta\nu \sim \frac{1}{\tau} \Rightarrow \frac{c}{\lambda^2} \Delta\lambda \sim \frac{1}{\tau}$

$$\Delta\lambda \sim \frac{\lambda^2}{c\tau} = \frac{(632.8 \times 10^{-9})^2}{(3 \times 10^8)(1 \times 10^{-6})} = 1.33 \times 10^{-15} \text{ m}$$

Q14. The Hamiltonian of a classical particle is given by $H(p, q) = \frac{p^2}{2m} + \frac{kq^2}{2}$. Given

$F(p, q, t) = \ln(p + im\omega q) - i\alpha\omega t$ is a constant of motion (where $\omega = \sqrt{\frac{k}{m}}$). What is the value of α ?

- (a) 2π (b) 0 (c) 1 (d) π

Ans. 14: (c)

Solution: $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$

$$F = \ln(p + im\omega q) - i\alpha\omega t$$

$$\frac{dF}{dt} = (F, H) + \frac{\partial F}{\partial t} = 0$$

$$\Rightarrow \frac{\partial F}{\partial q} \cdot \frac{\partial H}{\partial p} - \frac{\partial F}{\partial p} \cdot \frac{\partial H}{\partial q} + \frac{\partial F}{\partial t} = 0$$

$$\Rightarrow \frac{im\omega}{(p + im\omega q)} \cdot \frac{p}{m} - \frac{m\omega^2 q}{(p + im\omega q)} - i\alpha\omega = 0$$

$$\Rightarrow \frac{(i\omega p - m\omega^2 q)}{(p + im\omega q)} - i\alpha\omega = 0 \Rightarrow \frac{i\omega(p + i\omega m q)}{(p + i\omega m q)} - i\alpha\omega = 0$$

$$\Rightarrow i\omega - i\alpha\omega = 0 \Rightarrow \alpha = 1$$

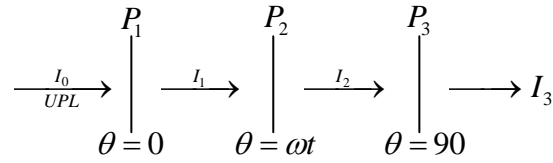
Q15. Three polarizers are stacked, normal to a central axis, along which is incident a beam of unpolarized light of intensity I_0 . The first and the third polarizers are perpendicular to each other and the middle polarizer is rotated at an angular frequency ω about the central axis (light beam). The time dependent intensity of light emerging after the third polarizer will be given by

- (a) $I(t) = \left(\frac{I_0}{16}\right)\{1 - \cos(4\omega t)\}$ (b) $I(t) = \left(\frac{I_0}{8}\right)\{1 - \cos(2\omega t)\}$
 (c) $I(t) = \left(\frac{I_0}{4}\right)\{1 - 2\cos\omega t\}$ (d) $I(t) = \left(\frac{I_0}{2}\right)\cos^2\omega t$

Ans. 15: (a)

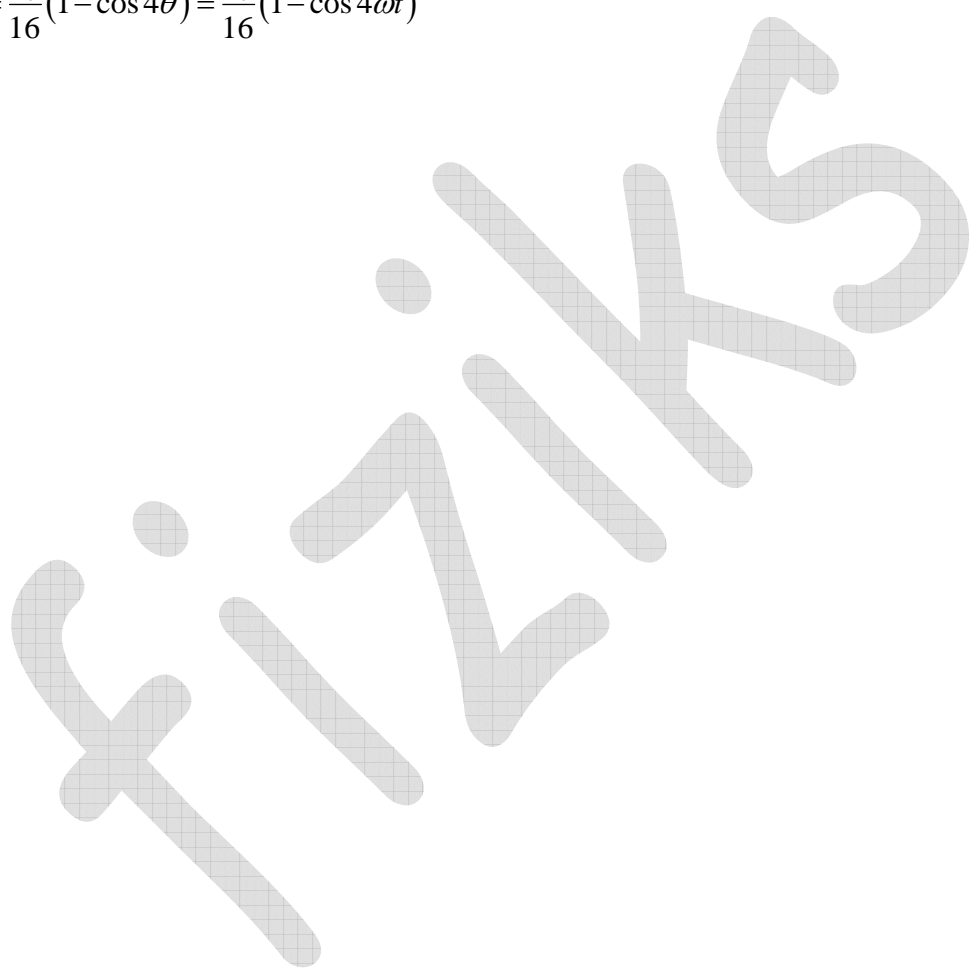
Solution: $I_1 = I_0 / 2$

$$I_2 = I_1 \cos^2 \theta = \frac{I_0}{2} \cos^2 \theta$$



$$I_3 = I_2 \cos^2 (90^\circ - \theta) = \frac{I_0}{2} \cos^2 \theta \sin^2 \theta = \frac{I_0}{8} \sin^2 2\theta$$

$$I_3 = \frac{I_0}{16} (1 - \cos 4\theta) = \frac{I_0}{16} (1 - \cos 4\omega t)$$



PART C: 3- Mark Numerical Questions

- Q1. A thin film of water having refractive index $n = 1.333$ floats on the surface of a beaker of silicone oil having refractive index $n_s = 1.40$. The arrangement is illuminated by 600nm light incident normally from top and a large region of the film appears bright red. What is the minimum possible thickness of the film (in nm)?

Ans. 1: 0225

Solution: $2\mu_f t \cos r = n\lambda$

$r = 0^\circ$, $n = 1$ for minimum thickness

$$t_{\min} = \frac{\lambda}{2\mu_f} = \frac{600\text{nm}}{2 \times 1.333} = 225\text{nm}$$

- Q2. What is the value of the following integral?

$$I = \frac{100\sqrt{2}}{\pi} \int_0^{\pi/2} x \delta(2 \sin x - \sqrt{2}) dx$$

Ans. 2: 0025

Solution: For the location of the delta function set $2 \sin x - \sqrt{2} = 0 \Rightarrow \sin x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}$

only this pole exists between 0 to $\pi/2$. Others will not be considered.

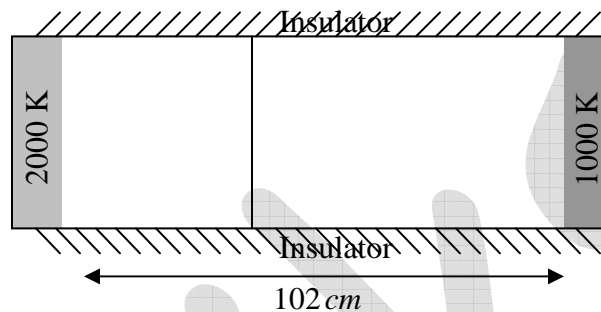
$$\Rightarrow \delta(2 \sin x - \sqrt{2}) = \delta\left(x - \frac{\pi}{4}\right) = \begin{cases} \infty & x = \frac{\pi}{4} \\ 0 & x \neq \frac{\pi}{4} \end{cases}$$

The general formula to calculate is given by: $\delta[g(x)] dx = \frac{1}{g'(x_0)} \delta(x - x_0)$

$$I = \frac{100\sqrt{2}}{\pi} \int_0^{\pi/2} x \frac{1}{2 \cos \frac{\pi}{4}} \delta\left(x - \frac{\pi}{4}\right) dx = \frac{100\sqrt{2}}{\pi} \frac{1}{2 \cos \frac{\pi}{4}} \int_0^{\pi/2} x \delta\left(x - \frac{\pi}{4}\right) dx$$

$$\Rightarrow I = \frac{100}{\pi} \int_0^{\pi/2} x \delta\left(x - \frac{\pi}{4}\right) dx = \frac{100}{\pi} \times \frac{\pi}{4} = 25$$

- Q3. Two compartments in a cylinder with uniform cross section and total length 102 cm are separated by a sliding partition which can move but does not allow heat to pass across it. No molecules are present in either of the compartments. The radiation inside each compartment is in thermal equilibrium with its walls. The walls at the two ends of the cylinder are maintained at temperatures 2000 K and 4000 K , respectively. The sides are perfectly insulated. Find the location of the partition, measured from the left end of the container.



Ans. 3: 0006

Solution: Balancing Radiation Pressure on both side of the wall, $\frac{P_1}{x} = \frac{P_2}{102-x}$

$$\Rightarrow \frac{(2000)^4}{x} = \frac{(4000)^4}{102-x} = \frac{(2)^4}{x} = \frac{(4)^4}{102-x} = \frac{1}{x} = \frac{16}{102-x} \Rightarrow x = 6\text{ cm}$$

- Q4. A laser has output power of 150 mW with beam diameter of 2 mm at a wavelength 630 nm . What is the value of the electric field in units of V/m is? Use Coulomb's constant,

$$\frac{1}{(4\pi\epsilon_0)} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

Ans. 4: 6000

$$\text{Solution: } I = \frac{P}{A} = \frac{1}{2} \epsilon_0 E_0^2 c = \frac{P}{\pi r^2} \Rightarrow E_0^2 = \frac{2P}{\pi \epsilon_0 c r^2} \Rightarrow E_0 = \sqrt{\frac{2P}{\pi \epsilon_0 c r^2}} = \sqrt{\frac{1}{4\pi \epsilon_0} \frac{8P}{c r^2}}$$

$$\Rightarrow E_0 = \sqrt{9 \times 10^9 \times \frac{8 \times 150 \times 10^{-3}}{3 \times 10^8 \times (10^{-3})^2}} \Rightarrow E_0 = \sqrt{\frac{1200 \times 3 \times 10^6}{10^2}} = 6000 \text{ V/m}$$

- Q5. A two-state quantum system has energy eigenvalues $\pm \epsilon$ corresponding to normalised states ψ_{\pm} . At time $t=0$ the system is in the quantum state $\frac{[\psi_{+} + \psi_{-}]}{\sqrt{2}}$. Find the $10000 \times$ probability that the system will be in the same state at time $t = \frac{h}{(6\epsilon)}$, where h is the

Planck's constant.

Ans. 5: 2500

Solution: $t = 0$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\psi_{+} + \psi_{-})$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[\psi_{+}e^{\frac{i\epsilon t}{h}} + \psi_{-}e^{\frac{i\epsilon t}{h}}\right]$$

$$t = \frac{h}{6\epsilon}$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[\psi_{+}e^{\frac{i2\pi\epsilon h}{h \cdot 6\epsilon}} + \psi_{-}e^{\frac{i2\pi\epsilon h}{h \cdot 6\epsilon}}\right] = \frac{1}{\sqrt{2}}\left[\psi_{+}e^{\frac{\pi i}{3}} + \psi_{-}e^{\frac{\pi i}{3}}\right]$$

$$\langle\psi|\psi(t)\rangle^2 = \frac{1}{4} \left| \frac{\left(e^{\frac{\pi i}{3}} + e^{\frac{\pi i}{3}}\right)}{2} \right|^2 = \frac{|2\cos\frac{\pi}{3}|^2}{4} = \frac{1}{4}$$

$$\frac{1}{4} \times 10000 = 2500$$

- Q6. A small insect of mass m is sitting on the rim of a uniform circular horizontal disk of radius R and mass M . The system is rotating at a constant angular velocity ω_i about a frictionless vertical axis passing through the center of the disk. The insect started to crawl towards the center of the disk. Assume $\frac{M}{m} = 10$, and let the final angular velocity of the system, when the insect reaches the centre of the disk be ω_f . What is the value of $\frac{100\omega_f}{\omega_i}$?

Ans. 6: 0120

Solution: $F_\theta = 0$, so angular momentum is constant.

$$I_{in} = I_f$$

$$\left(\frac{MR^2}{2} + mR^2\right)\omega_i = \left(\frac{MR^2}{2}\right)\omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{\frac{MR^2}{2} + mR^2}{\frac{MR^2}{2}} = 1 + \left(\frac{m}{M}\right) \frac{\omega_f}{\omega_i} = \left(1 + \frac{2}{10}\right) = \frac{12}{10}$$

$$\left(\frac{\omega_f}{\omega_i}\right) \times 100 = \frac{12}{10} \times 100 = 120$$

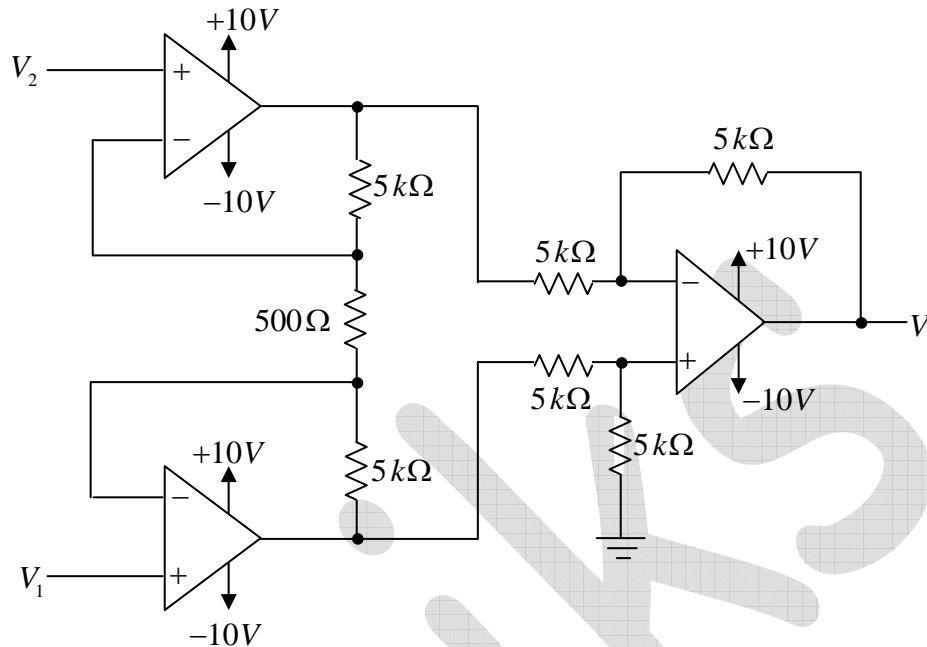
- Q7. A particle is moving on a one-dimensional discrete lattice with lattice spacing unity. It can move from a site to its nearest neighbour site every $1/5$ seconds with p being the probability to move right and $q = (1 - p)$ being the probability to move left. Consider that the particle starts at origin, $x = 0$ at time $t = 0$. Taking $p = \frac{3}{4}$, calculate the variance $\langle (x - \langle x \rangle)^2 \rangle$ at time $t = 5 \times 10^4$ seconds, where $\langle x \rangle$ is the average position.

Ans. 7: 46875

Solution: Step length $l = 1$, Number of steps $N = \frac{5 \times 10^4}{1/5} = 250000$

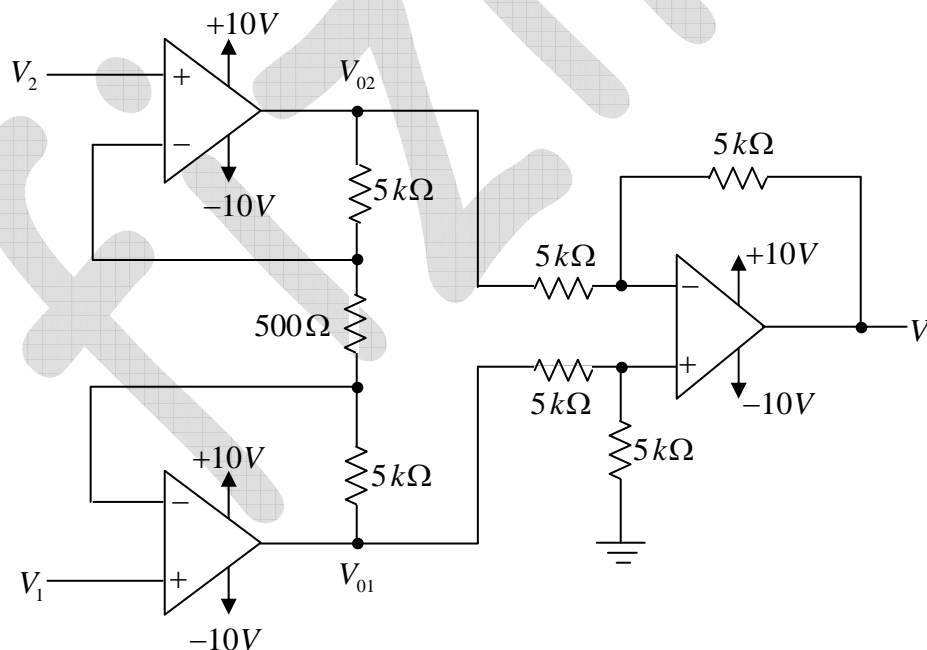
$$\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle = Npq = 250000 \times \frac{3}{4} \times \frac{1}{4} = 46875$$

Q8. Analyse the op-amp circuit shown in the figure below. What is the output voltage (V_0) in millivolts if $V_1 = 2.5V$ and $V_2 = 2.25V$?



Ans. 8: 5305

Solution:



From Superposition Theorem

$$V_{02} = V_2' + V_{01}' = \left(1 + \frac{5}{5.5}\right)V_2 - \frac{5}{5.5}V_{01} = 1.91V_2 - 0.91V_{01}$$

$$V_{01} = V_1' + V_{02}' = \left(1 + \frac{5}{5.5}\right)V_1 - \frac{5}{5.5}V_{02} = 1.91V_1 - 0.91V_{02}$$

$$\text{Thus } V_{01} - V_{02} = (1.91V_1 - 0.91V_{02}) - (1.91V_2 - 0.91V_{01})$$

$$\Rightarrow V_{01} - 0.91V_{01} - V_{02} + 0.91V_{02} = 1.91V_1 - 1.91V_2$$

$$\Rightarrow 0.09V_{01} - 0.09V_{02} = 1.91V_1 - 1.91V_2 \Rightarrow V_{01} - V_{02} = \frac{1.91}{0.09}(V_1 - V_2)$$

$$\Rightarrow V_{01} - V_{02} = \frac{1.91}{0.09}(2.5V - 2.25V) = 5.305V$$

$$\text{Now, } V_0 = V_{02}'' + V_{01}'' = -\frac{5}{5}V_{02} + \left(1 + \frac{5}{5}\right)\frac{5}{5+5}V_{01} = -V_{02} + V_{01} = V_{01} - V_{02}$$

$$\Rightarrow V_0 = V_{01} - V_{02} = 5.305V = 5305 \text{ mV}$$

- Q9. Some bacteria are added to a bucket at time 10 am. The number of bacteria doubles every minute and reaches a number 16×10^{15} at 10:18 am. How many seconds after 10 am were there 25×10^{13} bacteria?

Ans. 9: 0720

- Q10. A cleaning machine presses a circular mop of radius $R = 30 \text{ cm}$ vertically down on a floor with a total force $F = 25 \text{ N}$ and rotates it with a constant angular speed about the vertical axis passing through the centre of mop. If the force is distributed uniformly over the mop and if the coefficient of friction between the mop and the floor is $\mu = 0.25$, what is the value of torque in N-cm applied by the machine on the mop?

Ans. 10: 0125

Solution: dN is normal force in area $rdrd\theta$; $dN = \frac{F}{\pi R^2}rdrd\theta$

$$\tau = r \times dF = r\mu dN = r\mu \frac{F}{\pi R^2}rdrd\theta = \frac{F\mu}{\pi R^2} \int_0^R r^2 dr \int_0^{2\pi} d\theta = \frac{F\mu}{\pi R^2} \frac{R^3}{3} 2\pi = \frac{2F\mu R}{3}$$

$$\Rightarrow \tau = \frac{2 \times 25 \times 0.25 \times 0.30}{3} = 125 \text{ N-cm}$$