Oscillations, Waves and Optics

IIT-JAM 2005

Q1. Consider a beam of light of wavelength λ incident on a system of a polarizer and an analyzer. The analyzer is oriented at 45^0 to the polarizer. When an optical component is introduced between them, the output intensity becomes zero. (Light is incident normally on all components). The optical component is

(a) a full-wave plate (b) a half-wave plate

(c) a quarter-wave plate (d) an ordinary glass plate

Ans.: (b)

Solution: Half wave plate introduce phase difference of π, if incidence wave is plane polarized than after passing through HWP the wave is also plane polarized. If electric field of the incidence wave makes angle 45^0 with optic axis of HWP than plane polarized at output will be at 45^0, as a result it will incidence on polarizer at 90^0. According to malus law intensity at output will be

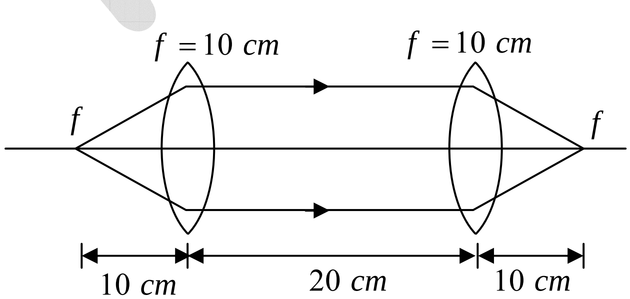
$$I = I_0 \cos^2\theta = I_0 \cos^2(\pi/2) = 0$$

Q2. A combination of two thin convex lenses of equal focal lengths, is kept separated along the optic axes by a distance of $20\,cm$ between them. The combination behaves as a lens system of infinite focal length. If an object is kept at $10\,cm$ from the first lens, its image will be formed on the other side at a distance x from the second lens. The value of x is

(a) $10\,cm$ (b) $20\,cm$ (c) $6.67\,cm$ (d) infinite

Ans.: (a)

Solution: As we see in the figure that the image is formed $10\,cm$ apart from the second lens.



IIT-JAM 2006

Q3. At a given point in space the total light wave is composed of three phasors $P_1 = a$, $P_2 = \frac{a}{2}e^{i\theta}$ and $P_3 = \frac{a}{2}e^{-i\theta}$. The intensity of light at this point is

(a) $4a^2\cos^2\left(\frac{\theta}{2}\right)$ (b) $4a^2\cos^4\left(\frac{\theta}{2}\right)$

(c) $a^2\cos^2(\theta)$ (d) $4a^2\cos^2(2\theta)$

Ans.: (b)

Solution: $P = P_1 + P_2 + P_3 = a + \frac{a}{2}e^{i\theta} + \frac{a}{2}e^{-i\theta} = \frac{a}{2}(2+\cos\theta + i\sin\theta + \cos\theta - i\sin\theta)$

$$= a(1+\cos\theta) = 2a\cos^2\frac{\theta}{2}$$

$$I = P^2 = 4a^2\cos^4\left(\frac{\theta}{2}\right)$$

Q4. A spring-mass system has undamped natural angular frequency $\omega_0 = 100\, rad\, s^{-1}$. The solution $x(t)$ at critical damping is given by $x(t) = x_0(1+\omega_0 t)\exp(-\omega_0 t)$, where x_0 is a constant. The system experiences the maximum damping force at time

(a) $0.01s$ (b) $0.1s$ (c) $0.01\pi s$ (d) $0.1\pi s$

Ans.: (a)

Solution: Damping force, $F_d = -b\frac{dx}{dt}$

For maximum damping force, $\frac{dF_d}{dt} = 0 \Rightarrow -b\frac{d^2x}{dt^2} = 0 \Rightarrow \frac{d^2x}{dt^2} = 0$

$$\frac{dx}{dt} = x_0(1+\omega_0 t)e^{-\omega_0 t}(-\omega_0) + x_0\omega_0 e^{-\omega_0 t} = (x_0\omega_0 + x_0\omega_0(1+\omega_0 t))e^{-\omega_0 t}$$

$$\frac{d^2x}{dt^2} = (x_0\omega_0 + x_0\omega_0(1+\omega_0 t))e^{-\omega_0 t}(-\omega_0) + x_0\omega_0\omega_0 e^{-\omega_0 t} = -x_0\omega_0^2(1+\omega_0 t)e^{-\omega_0 t} = 0$$

$$1+\omega_0 t = 0 \Rightarrow t = 1/\omega_0 \Rightarrow t = 0.01\sec$$

Q5. $\vec{E}(x,y,z,t)=A(3\hat{i}+4\hat{j})\exp[i(\omega t-kz)]$ represents an electromagnetic wave. Possible directions of the fast axis of a quarter wave plate which convert this wave into a circularly polarized wave are

(a) $\frac{1}{\sqrt{2}}[7\hat{i}+\hat{j}]$ *and* $\frac{1}{\sqrt{2}}[-\hat{i}+7\hat{j}]$ (b) $\frac{1}{\sqrt{2}}[3\hat{i}+4\hat{j}]$ *and* $\frac{1}{\sqrt{2}}[4\hat{i}-3\hat{j}]$

(c) $\frac{1}{\sqrt{2}}[3\hat{i}-4\hat{j}]$ *and* $\frac{1}{\sqrt{2}}[4\hat{i}+3\hat{j}]$ (d) $\frac{1}{\sqrt{2}}[7\hat{i}-\hat{j}]$ *and* $\frac{1}{\sqrt{2}}[\hat{i}+7\hat{j}]$

Ans.: (a)

Solution: The fast axis of the quarter wave plate must make angle of 45^0 with the direction of vibration of electric field so that amplitude of ordinary ray and extra-ordinary ray is equal to produce circularly polarized light.

$\vec{E}(x,y,z,t)=A(3\hat{i}+4\hat{j})\exp[i(\omega t-kz)]=\vec{E}_0 A\exp[i(\omega t-kz)]$

Where $\vec{E}_0=(3\hat{i}+4\hat{j})$

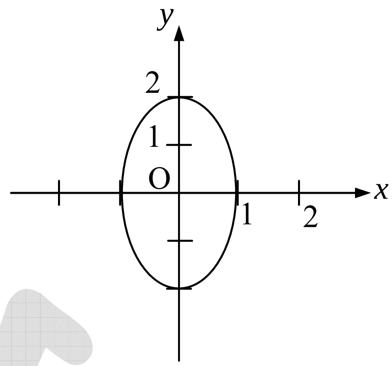
Let us calculate the angle between $\vec{E}_0$ and $\vec{A}=\frac{1}{\sqrt{2}}[7\hat{i}+\hat{j}]$ *and* $\vec{B}=\frac{1}{\sqrt{2}}[-\hat{i}+7\hat{j}]$

$$\cos\theta=\frac{\vec{E}_0\cdot\vec{A}}{|\vec{E}_0||\vec{A}|}=\frac{\frac{1}{\sqrt{2}}(21+4)}{\sqrt{25}\times\sqrt{50}/\sqrt{2}}=\frac{\frac{1}{\sqrt{2}}\times 25}{25}=\frac{1}{\sqrt{2}}\Rightarrow\theta=45^0 \quad \text{and}$$

$$\cos\theta=\frac{\vec{E}_0\cdot\vec{B}}{|\vec{E}_0||\vec{B}|}=\frac{\frac{1}{\sqrt{2}}(-3+28)}{\sqrt{25}\times\sqrt{50}/\sqrt{2}}=\frac{\frac{1}{\sqrt{2}}\times 25}{25}=\frac{1}{\sqrt{2}}\Rightarrow\theta=45^0$$

IIT-JAM 2007

Q6. When two simple harmonic oscillations represented by $x = A_0 \cos(\omega t + \alpha)$ and $y = B_0 \cos(\omega t + \beta)$ are superposed at right angles, the resultant is an ellipse with its major axis along the y-axis as shown in the figure. The conditions which correspond to this are



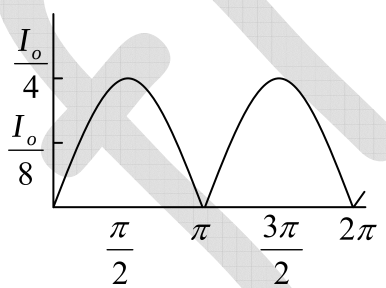
(a) $\beta = \alpha + \frac{\pi}{2}; A_0 = 2B_0$ (b) $\beta = \alpha - \frac{\pi}{4}; A_0 = B_0$

(c) $\beta = \alpha + \frac{\pi}{2}; 2A_0 = B_0$ (d) $\beta = \alpha + \frac{\pi}{4}; A_0 = B_0$

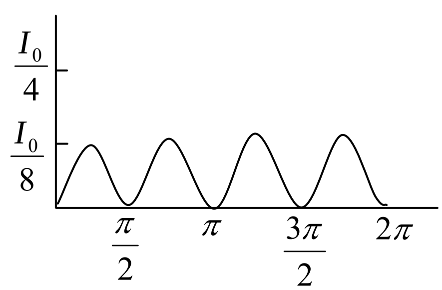
Ans.: (c)

Q7. Three polarizers P, Q and R are placed parallel to each other with their planes perpendicular to the z-axis. Q is placed between P and R. Initially the polarizing directions of P and Q are parallel, but that of R is perpendicular to them. In this arrangement when unpolaized light of intensity I_0 is incident on P, the intensity coming out of R is zero. The polarizer Q is now rotated about the z-axis. As a function of angle of rotation, the intensity of light coming out of R is best represented by

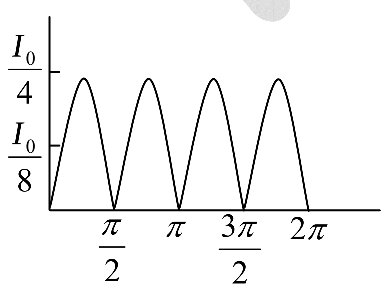
(a)



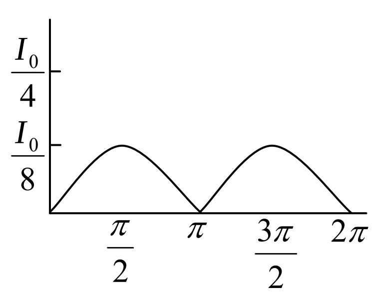
(b)



(c)

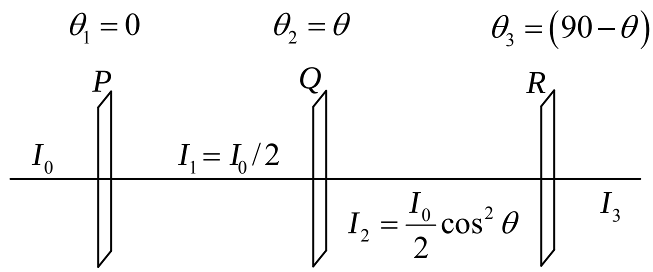


(d)



Ans.: (b)

Solution: $I_1 = \dfrac{I_0}{2}$



$$I_2 = \frac{I_0}{2}\cos^2\theta$$

$$I_3 = \frac{I_0}{2}\cos^2\theta\cos^2(90-\theta)$$

$$\theta = 0 \rightarrow I_3 = 0$$

$$\theta = 45 \rightarrow I_3 = \frac{I_0}{2}\cos^2 45\cos^2 45 = \frac{I_0}{2}\frac{1}{2}\frac{1}{2} = \frac{I_0}{8}$$

$$\theta = 90^0 \rightarrow I_3 = 0$$

<u>IIT-JAM 2008</u>

Q8. The instantaneous position $x(t)$ of a small block performing one-dimensional damped oscillations $x(t) = Ae^{-rt}\cos(\omega t + a)$. Here ω is the angular frequency, γ the damping coefficient, A the initial amplitude and α the initial phase. If $x\big|_{t=0} = 0 \; and \; \left.\dfrac{dx}{dt}\right|_{t=0} = v,$ the values of A and α (with $n = 0, 1, 2, ...$) are

(a) $A = \dfrac{v}{2\omega}, \alpha = \dfrac{(2n+1)}{2}\pi$ (b) $A = \dfrac{v}{\omega}, \alpha = n\pi$

(c) $A = \dfrac{v}{\omega}, \alpha = \dfrac{(2n+1)\pi}{2}$ (d) $A = \dfrac{2v}{\omega}, \alpha = \dfrac{(2n+1)\pi}{2}$

Ans.: (c)

Solution: $x\big|_{t=0} = A\cos\alpha = 0 \;\Rightarrow\; \cos\alpha = 0 \;\Rightarrow\; \alpha = \dfrac{(2n+1)}{2}$

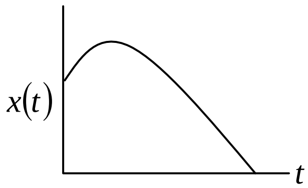
$$\left.\frac{dx}{dt}\right| = Ae^{-\gamma t}(-\gamma)\cos(\omega t + \alpha) + Ae^{-\gamma t}\sin(\omega t + \alpha)\omega$$

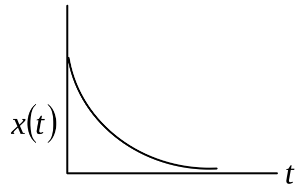
$$= Ae^{-\gamma t}\left(-\gamma\cos(\omega t + \alpha) + \sin(\omega t + \alpha)\omega\right)$$

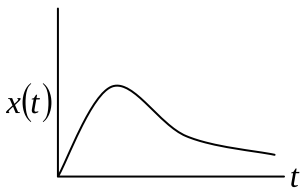
$$\left.\frac{dx}{dt}\right|_{t=0} = v \;\Rightarrow\; A\left(-\gamma\cos(\alpha) + \omega\sin(\alpha)\right) = v \;\Rightarrow\; A\omega = v \;\Rightarrow\; A = \frac{v}{\omega}$$

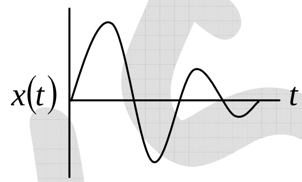
IIT-JAM 2009

Q9. Among the following displacement versus time plots, which ones may represent an over-damped oscillator?

(P)


(Q)


(R)


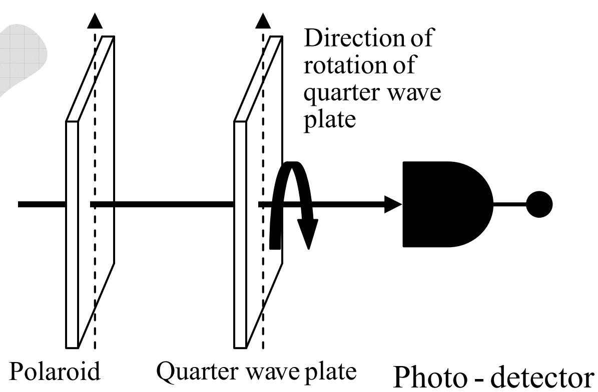
(S)


(a) only (P) and (Q)
(b) only (P) and (R)
(c) only (P) and (S)
(d) only (P), (R) and (S)

Ans.: (a)

IIT-JAM 2010

Q10. A quarter-wave plate is placed in between a polarizer and a photo-director. When the optic axis of the quarter-wave plate is kept initially parallel to the pass axis of the polarizer and perpendicular to the direction of light propagation. The intensity of light passing through the quarter-wave plate is measured to be I_0 (see figure). If the quarter wave plate is now rotated by 45^0 about an axis parallel to the light propagation, what would be the intensity of the emergent light measured by the photo-director?



(a) $\dfrac{I_o}{\sqrt{2}}$
(b) $\dfrac{I_0}{2}$
(c) $\dfrac{I_0}{2\sqrt{2}}$
(d) I_0

Ans.: (d)

Solution: After passing through QWP plane polarized light of intensity I_0 will convert into circularly polarized with intensity I_0.

IIT-JAM 2011

Q11. Six simple harmonic oscillations each of same frequency and equal amplitude are superposed. The phase difference between any two consecutive oscillations i.e., $\phi_n - \phi_{n-1} = \Delta\phi$ is constant, where ϕ_n is the phase of the n^{th} oscillation. If the resultant amplitude of the superposition is zero, what is the phase difference $\Delta\phi$?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 2π

Ans.: (d)

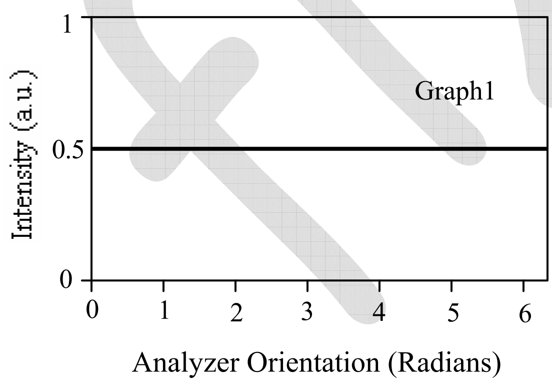
Solution: Resultant amplitude of the superposition of n SHM is

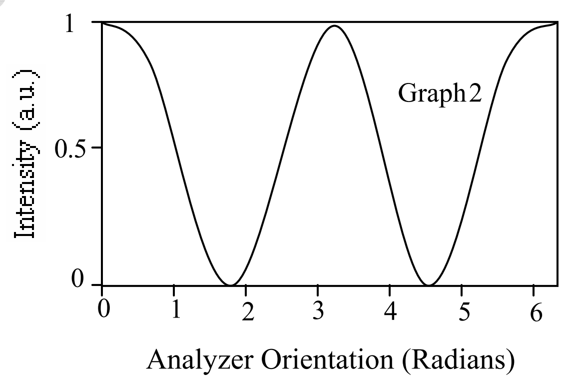
$$R = \frac{a\sin(n\delta/2)}{\sin(\delta/2)}, \quad \text{where, } \delta = \frac{\Delta\phi}{n} \quad \Rightarrow \quad n\delta = \Delta\phi$$

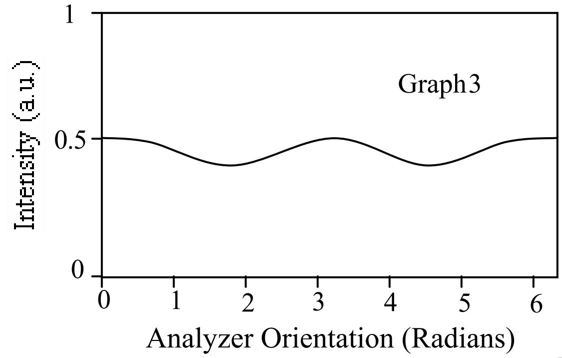
$$R = \frac{a\sin(n\delta/2)}{\sin(\delta/2)} = 0 \Rightarrow \sin(n\delta/2) = 0 \Rightarrow n\delta/2 = \pi$$

$$\frac{\Delta\phi}{2} = \pi \Rightarrow \Delta\phi = 2\pi$$

Q12. Intensity of three different light beams after passing through an analyzer is found to vary as shown in the following graphs. Identify the option giving the correct states of polarization of the incident beams from graphs.







(a) Graph1: Linear Polarization Graph2: Circular Polarization, Graph3: Elliptic Polarization

(b) Graph 1: Circular Polarization, Graph 2: Linear Polarization, Graph 3: Elliptic Polarization

(c) Graph 1: Unpolarized Graph 2: Circular Polarization, Graph 3: Linear Polarization

(d) Graph 1: Unpolarized Graph 2: Elliptic Polarization, Graph 3: Circular Polarization

Ans.: (b)

Solution: Graph I $\rightarrow$ UPL or CPL

Graph II $\rightarrow$ LPL

Graph III $\rightarrow$ EPL or UPL + LPL or CPL + LPL

Hence answer is (b)

IIT-JAM 2012

Q13. A lightly damped harmonic oscillator loses energy at the rate of 1% per minute. The decrease in amplitude of the oscillator per minute will be closest to

(a) 0.5% (b) 1% (c) 1.5% (d) 2%

Ans.: (d)

Solution: Decay of energy is governed by equation, $E = E_0 e^{-2\gamma t}$

Decay of amplitude is governed by equation, $A = ae^{-\gamma t}$

Q14. Group I contains *x*- and *y*- components of the electric field and Group II contains the type of polarization of light.

	Group I	Group II
P.	$E_x = \frac{E_0}{\sqrt{2}}\cos(\omega t + kz)$, $E_y = E_0 \sin(\omega t + kz)$	1. Linearly Polarized
Q.	$E_x = E_0 \sin(\omega t + kz)$, $E_y = E_0 \cos(\omega t + kz)$	2. Circularly Polarized
R.	$E_x = E_1 \sin(\omega t + kz)$, $E_y = E_2 \sin(\omega t + kz)$	3. Unpolarized
S.	$E_x = E_0 \sin(\omega t + kz)$, $E_y = E_0 \sin\left(\omega t + kz + \frac{\pi}{4}\right)$	4. Elliptically Polarized

The correct set of matches is

(a) $P \to 4; Q \to 2; R \to 4; S \to 1$ (b) $P \to 1; Q \to 3; R \to 1; S \to 4$

(c) $P \to 4; Q \to 2; R \to 1; S \to 4$ (d) $P \to 3; Q \to 1; R \to 3; S \to 2$

Ans.: (c)

Solution: P. $E_x = \frac{E_0}{\sqrt{2}}\cos(\omega t + kz)$ *and* $E_y = E_0 \sin(\omega t + kz)$

The phase difference between E_x and E_y is $\pi/2$ with different amplitude. Therefore the resultant will be elliptically polarized.

Q. $E_x = E_0 \sin(\omega t + kz)$ *and* $E_y = E_0 \cos(\omega t + kz)$

The phase difference between E_x and E_y is $\pi/2$ with same amplitude. Therefore the resultant will be circularly polarized.

R. $E_x = E_1 \sin(\omega t + kz)$ *and* $E_y = E_2 \sin(\omega t + kz)$

The phase difference between E_x and E_y is 0 with different amplitude. Therefore the resultant will be linarly polarized.

S. $E_x = E_0 \sin(\omega t + kz)$ *and* $E_y = E_0 \sin\left(\omega t + kz + \frac{\pi}{4}\right)$

The phase difference between E_x and E_y is $\pi/4$ with same amplitude. Therefore the resultant will be elliptically polarized.

IIT-JAM 2013

Q15. A traveling pulse is given by $f(x,t) = A\exp\left(\dfrac{2abxt - a^2x^2 - b^2t^2}{c^2}\right)$ where A, a, b and c are positive constants of appropriate dimensions. The speed of the pulse is

(a) $\dfrac{b}{a}$ (b) $\dfrac{2b}{a}$ (c) $\dfrac{cb}{a}$ (d) $\dfrac{b}{2a}$

Ans.: (a)

Solution: $f(x,t) = A\exp\left(\dfrac{2abxt - a^2x^2 - b^2t^2}{c^2}\right) = A\exp\left[\dfrac{-(ax-bt)^2}{c^2}\right]$

Phase factor is constant

$$\frac{-(ax-bt)^2}{c^2} = const \quad \Rightarrow \quad -(ax-bt)^2 = const \times c^2$$

Taking differentiation, we get

$$(ax-bt)(adx-bdt) = 0 \quad \Rightarrow \quad adx - bdt = 0 \quad \Rightarrow dx/dt = b/a$$

Velocity of the pulse is *b/a*

IIT-JAM 2014

Q16. A collimated beam of light of diameter 1 mm is propagating along the *x*-axis. The beam is to be expanded to a collimated beam of diameter 10 mm using a combination of two convex lenses. A lens of focal length of 50 mm and another lens with focal length *F* are to be kept at a distance *d* between them. The values of *F* and *d* respectively, are

(a) 450 mm and 10 mm (b) 400 mm and 500 mm

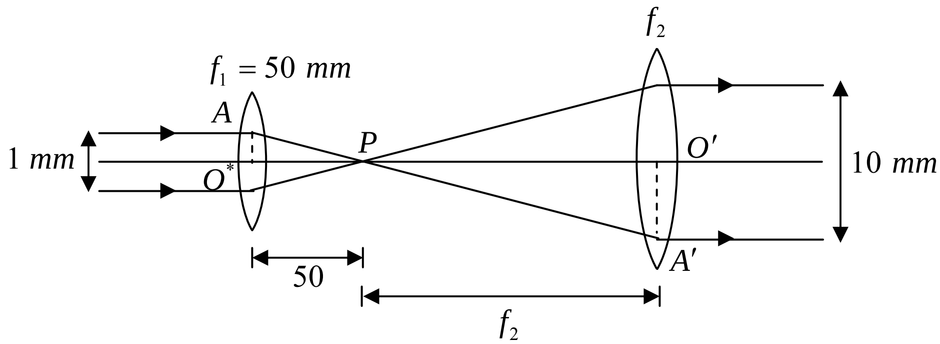
(c) 550 mm and 600 mm (d) 500 mm and 550 mm

Ans.: (d)

Solution: $\Delta\, AO^*P$ and $\Delta\, A'O'P$ (Refer to the figure)

$$\frac{AO^*}{A'O'} = \frac{OP}{OP'} \Rightarrow \frac{0.5\,mm}{5\,mm} = \frac{50\,mm}{f_2} \Rightarrow f_2 = 500\,mm$$

$$d = OO' = 50 + 500 = 550\ mm$$



Q17. The electric fields of two light sources with nearby frequencies ω_1 and ω_2, and wave vectors k_1 and k_2, are expressed as $\vec{E}_1 = E_{10}\hat{i}\, e^{-i(k_1 z - \omega_1 t)}$ and $\vec{E}_2 = E_{20}\hat{i}\, e^{-i(k_2 z - \omega_2 t)}$, respectively. The interference pattern on the screen is photographed at $t = t_0$; denote $(k_1 - k_2)z - (\omega_1 - \omega_2)t_0$ by θ. For this pattern

(a) a bright fringe will be obtained for $\cos\theta = -1$

(b) a bright fringe intensity is given by $(E_{10})^2 + (E_{20})^2$

(c) a dark fringe will be obtained for $\cos\theta = 1$

(d) a drak fringe intensity is given by $(E_{10} - E_{20})^2$

Ans.: (d)

Solution: A bright fringe will be obtained for $\cos\theta = +1$

A bright fringe intensity is given by $(E_{10} + E_{20})^2$

A dark fringe will be obtained for $\cos\theta = -1$

Q18. White light is incident on a grating G_1 with groove density 600 lines/mm and width 50 mm. A small portion of the diffracted light is incident on another grating G_2 with groove density 1800 lines/mm and width 15 mm. The resolving power of the combined system is

(a) 3×10^3 (b) 57×10^3 (c) 81×10^7 (d) 108×10^5

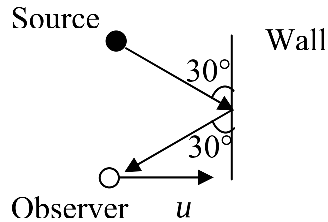
Ans.: (c)

Solution: $R_1 = n_1 N_1 = 1\times N_1 = N_1 = 600\times50$

$R_2 = n_2 N_2 = 1\times N_2 = N_2 = 1800\times15$

$R = R_1 R_2 = 81\times10^7$

Q19. A stationary source (see figure) emits sound waves of frequency f towards a wall. If an observer moving with speed u in a direction perpendicular to the wall, measures a frequency $f'=\frac{9}{8}f$ at the instant shown, then u is related to the speed of sound V_s as



(a) V_s (b) $V_s/2$ (c) $V_s/4$ (d) $V_s/8$

Ans.: (c)

Solution: Velocity component along sound wave

$$u\cos 60^0=\frac{u}{2}$$

$$\therefore f'=f\left(\frac{V_s+\frac{u}{2}}{V_s}\right)\Rightarrow \frac{9}{8}f=f\left(\frac{V_s+\frac{u}{2}}{V_s}\right)$$

$$9V_s=8V_s+4u$$

$$V_s=4u\Rightarrow u=\frac{V_s}{4}$$

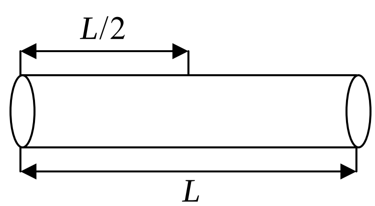
IIT-JAM 2015

Q20. At room temperature, the speed of sound in air is 340 m/sec. An organ pipe with both ends open has a length $L=29\ cm$. An extra hole is created at the position $L/2$. The lowest frequency of sound produced is

(a) $293\,Hz$ (b) $586\,Hz$ (c) $1172\,Hz$ (d) $2344\,Hz$

Ans.: (c)

Solution: The fundamental frequency in organ pipe with both end open is $f=\frac{v}{2L}$



with additional rate at $\frac{L}{2}$, the fundamental frequency becomes

$$f' = \frac{v}{2L'} = \frac{v}{2\frac{L}{2}} = \frac{v}{L} = \frac{340\,m/\sec}{29\times10^{-2}m} = 1172\ Hz$$

Q21. Vibrations of diatomic molecules can be represented as those of harmonic oscillators. Two halogen molecules X_2 and Y_2 have fundamental vibrational frequencies $v_X = 16.7\times10^{12}\ Hz$ and $v_Y = 26.8\times10^{12}\ Hz$, respectively. The respective force constants are $K_X = 325\,N/m$ and $K_Y = 446\,N/m$. The atomic masses of F, Cl and Br are 19.0, 35.5 and 79.9 atomic mass unit respectively. The halogen molecules X_2 and Y_2 are

(a) $X_2 = F_2$ and $Y_2 = Cl_2$

(b) $X_2 = Cl_2$ and $Y_2 = F_2$

(c) $X_2 = Br_2$ and $Y_2 = F_2$

(d) $X_2 = F_2$ and $Y_2 = Br_2$

Ans.: (b)

Solution: The oscillation frequency of diatomic molecule with reduce mass 'μ' is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \Rightarrow \mu = \frac{1}{4\pi^2}\frac{k}{f^2}$$ where k is force constant.

For X_2 molecule: $\mu = \frac{m_x m_x}{m_x + m_x} = \frac{m_x}{2}$

$$\Rightarrow m_x = \frac{1}{2\pi^2}\times\frac{k_x}{f_x^2} = \frac{1}{2\times(3.14)^2}\times\frac{325N/m}{\left(16.7\times10^{12}Hz\right)^2}$$

$$\Rightarrow m_x = 59.07\times10^{-27}kg = 35.5\times1.67\times10^{-27}kg = 35.5\,a.m.u.$$

This is the atomic mass of chlorine (Cl).

For Y_2 molecule: $\mu = \frac{m_y m_y}{m_y + m_y} = \frac{m_y}{2}$

$$\Rightarrow m_y = \frac{1}{2\pi^2}\times\frac{k_y}{\left(f_y\right)^2} = \frac{1}{2\times(3.14)^2}\times\frac{446N/m}{\left(26.8\times10^{12}Hz\right)^2}$$

$$\Rightarrow m_y = 31.73\times10^{-27}kg = 19\times1.67\times10^{-27}kg = 19\ a.m.u.$$

This is the atomic mass of F. Thus, correct answer is option (b)

Q22. Doppler effect can be used to measure the speed of blood through vessels. Sound of frequency $1.0522\ MHz$ is sent through the vessels along the direction of blood flow. The reflected sound generates a beat signal of frequency $100\ Hz$. The speed of sound in blood is $1545\ m/\sec$. The speed of blood through the vessel, in $m/\sec$, is

(a) 14.68 (b) 1.468 (c) 0.1468 (d) 0.01468

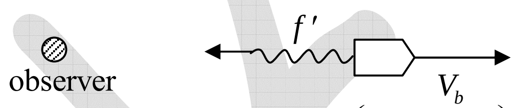
Ans.: (d)

Solution: Consider V_b, V_{sound} are velocities of blood cell and sound in blood. The sound of frequency (f_0) is traveling towards blood cell where blood cell is moving away with velocity V_b

Frequency of sound observed on blood cell is

$$f' = f_0\left(\frac{V_{sound} - V_b}{V_{sound}}\right) \qquad \text{(i)}$$

Sound from blood cell of frequency f' reflect back.



The frequency observed by observer is $f = f'\left(\frac{V_{sound}}{V_{sound} + V_b}\right)$ (ii)

From equation (i) and (ii), we get $f = f_0\left(\frac{V_{sound} - V_b}{V_{sound}}\right)\left(\frac{V_{sound}}{V_{sound} + V_b}\right)$

$$\Rightarrow f = f_0 = \left(\frac{V_{sound} - V_b}{V_{sound} + V_b}\right) \qquad \text{(iii)}$$

Now, $\Delta f = f_0 - f = f_0 - f_0\left(\frac{V_{sound} - V_b}{V_{sound} + V_b}\right) \quad = f_0\left(\frac{2V_b}{V_{sound} + V_b}\right)$

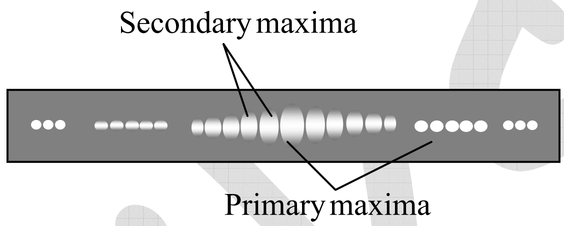
$$\Rightarrow \frac{2V_b}{V_{sound} + V_b} = \frac{\Delta f}{f_0} \Rightarrow \frac{V_{sound} + V_b}{V_b} = \frac{2f_0}{\Delta f} \Rightarrow V_b = \frac{V_{sound}}{\left(\frac{2f_0}{\Delta f} - 1\right)}$$

Given $V_{sound} = 1545\ m/\sec$, $f_0 = 1.0522\times10^6\ Hz$, $\Delta f = 100\ Hz$

$$\therefore V_b = \frac{1545}{\left(\frac{2\times1.0522\times10^6}{100}-1\right)} = \frac{1545}{21043} = 0.073 \Rightarrow V_b = 0.073\ m/\sec$$

Thus the best suitable answer is option (d).

Q23. The following figure shows a double slit Fraunhofer diffraction pattern produced by two slits, each of width a separated by a distance $b, a < b$.



Which of the following statements are correct?

(a) Reducing a increases the separation between consecutive primary maxima

(b) Reducing a increases the separation between consecutive secondary maxima

(c) Reducing b increases the separation between consecutive primary maxima

(d) Reducing b increases the separation between consecutive secondary maxima

Ans.: (a) and (d)

Solution: The minima condition for double slit Fraunhofer diffraction is

$$a\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{a}$$ where a is the width of slit.

Reducing 'a' increases the separation between diffraction minima i.e. increases the separation between consecutive primary maxima.

The condition of interference maxima is

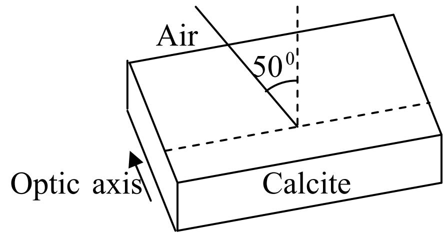
$$b\sin\theta = m\lambda \Rightarrow \sin\theta = \frac{m\lambda}{b}$$ where b is the separation between slits.

The position of interference maxima gives the separation between secondary maxima.

Reducing 'b' increases the separation between consecutive secondary maxima.

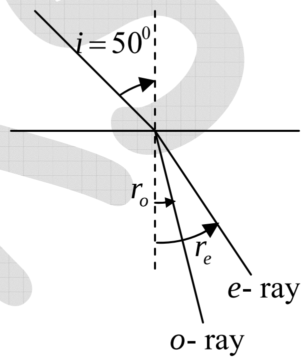
The correct answer is option (a) and (d).

Q24. Unpolarized light is incident on a calcite plate at an angle of incidence 50° as shown in the figure. Take $n_0 = 1.6584$ and $n_e = 1.4864$ for calcite. The angular separation (in degrees) between the two emerging rays within the plate is



Ans.: 3.51

Solution: Inside the crystal incident light split into two components, ordinary ray and extra-ordinary ray



According to Snell's law $\dfrac{\sin i}{\sin r} = n$

For ordinary ray $i = 50^0$, $n_o = 1.6584$

$$\therefore \sin r_o = \frac{\sin i}{n_o} \Rightarrow r_o = \sin^{-1}\left(\frac{\sin i}{n_o}\right)$$

$$\Rightarrow r_o = \sin^{-1}\left[\frac{\sin 50^0}{n_o}\right] = \sin^{-1}\left[\frac{0.766}{1.6584}\right] = \sin^{-1}[0.462] \Rightarrow r_0 = 27.51^0$$

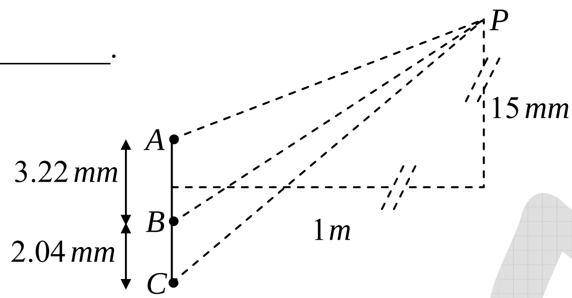
For extra-ordinary ray $i = 50^0$, $n_e = 1.4864$

$$\therefore \sin r_e = \frac{\sin i}{n_e} \Rightarrow r_e = \sin^{-1}\left(\frac{\sin i}{n_e}\right)$$

$$\Rightarrow r_e = \sin^{-1}\left[\frac{\sin 50^0}{n_e}\right] = \sin^{-1}\left[\frac{0.766}{1.4864}\right] = \sin^{-1}[0.515] \Rightarrow r_e = 31.02^0$$

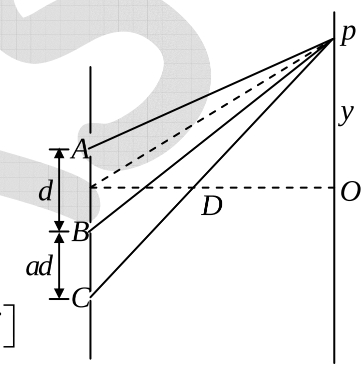
Thus, the angular separation between the o - ray and e - ray is $\theta = r_e - r_o = 3.51^0$

Q25. For the arrangement given in the following figure, the coherent light sources A, B and C have individual intensities of $2\,mW/m^2$, $2\,mW/m^2$ and $5\,mW/m^2$ respectively at point P. The wavelength of each of the sources is $600\,nm$. The resultant intensity at point P (in mW/m^2) is ___________.



Ans.: 9.23

Solution: The electric field on the screen is the sum of the fields produced by the slits individually.



$E = E_1 + E_2 + E_3 = A + Ae^{i\delta} + Be^{ia\delta}$, where $\delta = \frac{2\pi d}{\lambda}\sin\theta$

The total intensity at θ is

$$I = EE^* = 2A^2 + B^2 + 2A^2\cos\delta + 2AB\left[\cos(a\delta) + \cos(1-a)\delta\right]$$

where

$$\delta = \frac{2\pi d}{\lambda}\sin\theta \cong \frac{2\pi d}{\lambda}\theta = \frac{2\pi d}{\lambda}\times\frac{y}{D} = 2\pi\times\frac{3.22\times10^{-3}}{6\times10^{-7}}\times\frac{15\times10^{-3}}{1} = 505.7$$

$$\delta = 145.8^0$$

given, $A^2 = 2\ mw/m$, $B^2 = 5\ mw/m^2$, $d = 3.22\ mm$, $ad = 2.04\ mm$, $a = 0.6335\ mm$

$$\therefore I = 2\times2\times10^{-3} + 5\times10^{-3} + 2\times2\times10^{-3}\cos(\delta) + 2\sqrt{2}\sqrt{5}\times10^{-3}\left[\cos a\delta + \cos(1-a)\delta\right]$$

$$= 9.23\times10^{-3}\ w/m^2$$

$$I = 9.23\,mw/m^2$$

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Q26. Consider a particle of mass m following a trajectory given by $x = x_0 \cos \omega_1 t$ and $y = y_0 \sin \omega_2 t$, where x_0, y_0, ω_1 and ω_2 are constants of appropriate dimensions. The force on the particle is

(a) central only if $\omega_1 = \omega_2$ (b) central only if $x_0 = y_0$ and $\omega_1 = \omega_2$

(c) always central (d) central only if $x_0 = y_0$ and $\omega_1 \neq \omega_2$

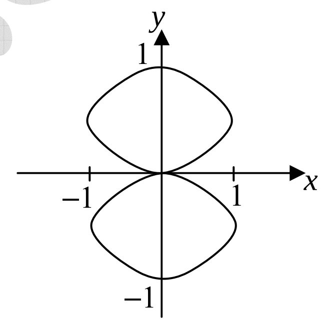
Ans.: (c)

Solution: $\ddot{x} = -x_0\omega_1^2 \cos \omega_1 t \;\; \ddot{y} = -y_0\omega_2^2 \cos \omega_2 t$

$$\vec{r} = x\hat{i} + y\hat{i} \Rightarrow \ddot{\vec{r}} = -\left(y_0\omega_2^2 \cos \omega_2 t\hat{i} + x_0\omega_1^2 \cos \omega_1 t\hat{j}\right)$$

If $\omega_1 = \omega_2$ then $\ddot{\vec{r}} = -\left(y_0\omega_2^2 \cos \omega_2 t\hat{i} + x_0\omega_1^2 \cos \omega_1 t\hat{j}\right) = -\omega^2 \vec{r}$

Q27. Two sinusoidal signals of frequency ω_x and ω_y having same amplitude are applied to x - and y - channels of a cathode ray oscilloscope (CRO), respectively. The following stationary figure will be observed when



(a) $\omega_y = \omega_x$ (b) phase difference is 0

(c) $\omega_y = 2\omega_x$ (d) phase difference is $\pi/2$

Ans.: (b)

Solution: $\dfrac{\omega_x}{\omega_y} = \dfrac{\text{number of cuts on } y\text{-axis}}{\text{number of cuts on } x\text{-axis}} = \dfrac{4}{2} \Rightarrow \omega_x = 2\omega_y$

and this lissajous figure appears when phase difference is 0. Thus correct option is (b)

Q28. Light traveling between two points takes a path for which

(a) time of flight is always minimum (b) distance is always minimum

(c) time of flight is extremum (d) distance is extremum

Ans.: (c)

Solution: According to Fermat's principle, the ray will correspond to that path for which the time taken is an extremum in comparison to nearby paths i.e. it is either a minimum or a maximum or stationary. Thus correction option is (c).

Q29. A train passes through a station with a constant speed. A stationary observer at the station platform measures the tone of the train whistle as $484\,Hz$ when it approaches the station and $442\,Hz$ when it leaves the station. If the sound velocity in air is $330\,m/s$, then the tone of the whistle and the speed of the train are

(a) $462\,Hz,\ 54\,km/h$ (b) $463\,Hz,\ 52\,km/h$

(c) $463\,Hz,\ 56\,km/h$ (d) $464\,Hz,\ 52\,km/h$

Ans.: (a)

Solution: Let f_o = original frequency of the whistle

f_a = observed frequency when train approaches platform

f_r = observed frequency when train recedes platform

v_t = velocity of train

v = velocity of sound in air

$$\therefore\ f_a = f_o\left(\frac{v}{v-v_t}\right) \text{ and } f_r = f_o\left(\frac{v}{v+v_t}\right)$$

Now, $\dfrac{f_a}{f_r} = \dfrac{v+v_t}{v-v_t} \Rightarrow v_t = \left(\dfrac{f_a + f_r}{f_a + f_r}\right)v \Rightarrow v_t = \dfrac{484-442}{484+442}\times 330 = 15\,m/\sec = 54\,km/hr$

and $f_o = \dfrac{f_a\left(v-v_t\right)}{v} = \dfrac{330-15}{330}\times 484 = 462\,Hz$

Q30. The minimum length of a plane mirror to see the entire full-length image of an object is half of the object's height. Suppose δ is the distance between eye and top of the head of a person of height h. The person will be able to see his entire full-length image with a mirror of height $h/2$ fixed on the wall

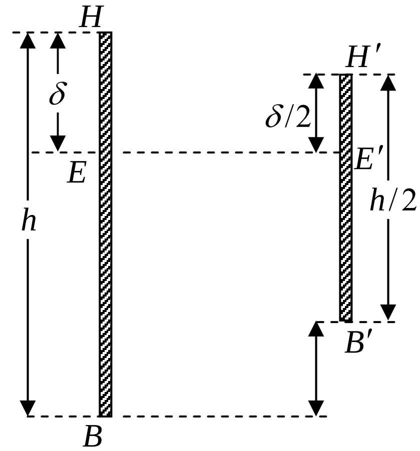
(a) when the bottom edge of mirror is kept $h/2$ above the floor

(b) when the bottom edge of mirror is kept $\left(h+\delta\right)/2$ above the floor

(c) when the bottom edge of mirror is kept $\left(h-\delta\right)/2$ above the floor

(d) when the centre of the mirror is at the same height as centre of the person

Ans.: (c)



Solution: Let $BH = h$ is the height of person

$HE = \delta$, where H represents top of head and E represents eye.

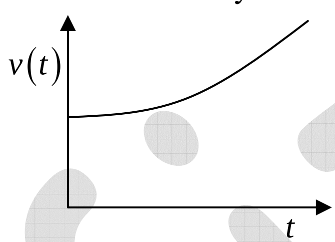
In the mirror, distance between eye and top of head will be $\frac{\delta}{2}$.

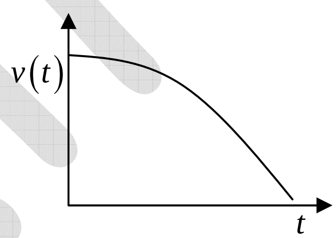
Since total height of mirror is $\frac{h}{2}$,

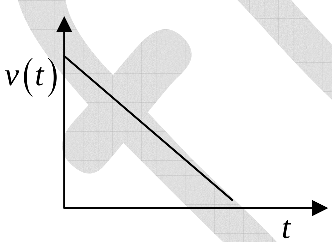
therefore, for diagram $B'E' = \frac{h}{2} - \frac{\delta}{2}$ and $BE' = h - \delta$

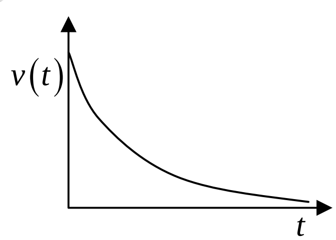
$\therefore BH' = h - \delta + \frac{\delta}{2}$ and $BB' = BH' - B'H' = h - \frac{\delta}{2} - \frac{\delta}{2} = \frac{h}{2} - \frac{\delta}{2} = \frac{h-\delta}{2}$

Q31. A particle travels in a medium along a horizontal linear path. The initial velocity of the particle is v_0 and the viscous force acting on it is proportional to its instantaneous velocity. In the absence of any other forces, which one of the following figures correctly represents the velocity of the particle as a function of time?

(a) 

(b) 

(c) 

(d) 

Ans.: (d)

Solution: Viscous force $\propto$ instantaneous velocity

$$F = -bv(t) \Rightarrow \frac{mdv(t)}{dt} = -bv(t) \Rightarrow \frac{dv(t)}{v(t)} = -\frac{b}{m}dt$$

Integrating on both sides

$$\int \frac{dv(t)}{v(t)} = -\int \frac{b}{m}dt \Rightarrow \ln v(t) = -\frac{b}{m}t + c$$

where $t=0,\ v(t)=v_0$ $\qquad\qquad \therefore c=\ln v_0$

$$\Rightarrow \qquad \ln v(t) = -\frac{b}{m}t + \ln v_0 \Rightarrow \ln\left(\frac{v(t)}{v_0}\right) = -\frac{b}{m}t \Rightarrow \frac{v(t)}{v_0} = e^{-\frac{b}{m}t}$$

Thus graph (d) correctly represent the variation of $v(t)$ w.r.t. time.

Q32. A lightly damped harmonic oscillator with natural frequency ω_0 is driven by a periodic force of frequency ω. The amplitude of oscillation is maximum when

(a) ω is slightly lower than ω_0

(b) $\omega = \omega_0$

(c) ω is slightly higher than ω_0

(d) The force is in phase with the displacement

Ans.: (a)

Solution: Amplitude in driven oscillator is

$$A = \frac{F_0/m}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+4b^2\omega^2}}$$

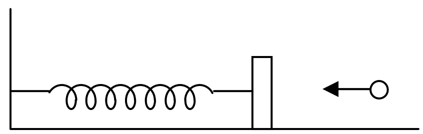
To find the condition for maximum amplitude, differentiate above equation w.r.t. ω and put $\frac{dA}{d\omega}=0$

i.e. $$\frac{dA}{d\omega} = \left(\frac{F_0}{m}\right)\left(-\frac{1}{2}\right)\frac{2\left(\omega_0^2-\omega^2\right)(-2\omega)+8b^2\omega}{\left[\left(\omega_0^2-\omega^2\right)^2+4b^2\omega^2\right]^{1/2}} = 0$$

$$\Rightarrow 2b^2-\omega_0^2+\omega^2 = 0 \Rightarrow \omega = \sqrt{\omega_0^2-2b^2}$$

Thus ω is slightly lower than ω_0. Correct option is (a).

Q33. A block of mass $0.38\,kg$ is kept at rest on a frictionless surface and attached to a wall with a spring of negligible mass. A bullet weighing $0.02\,kg$ moving with a speed of $200\,m/s$ hits the block at time $t=0$ and gets stuck to it. The displacement of the block (in metre) with respect to the equilibrium position is given by



(Spring constant $=640\,N/m$)

(a) $2\sin 5t$ (b) $\cos 10t$ (c) $0.4\cos 25t$ (d) $0.25\sin 40t$

Ans.: (d)

Solution: $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{k}{m+m'}} = \sqrt{\dfrac{640}{0.38+0.02}} = \sqrt{1600}$

$\omega = 40$ rad/sec

Let v' be the velocity acquired by the block m where bullet m strikes it and comes to rest in it.

By conservation of momentum

$$(m+m')v' = m'v \Rightarrow v' = \frac{m'}{m+m'}v = \frac{0.02}{0.38+0.02}\times 200 = \frac{0.02}{0.4}\times 200 = 10\,m/\text{sec}$$

The block is set in oscillation about it mean position with maximum amplitude A

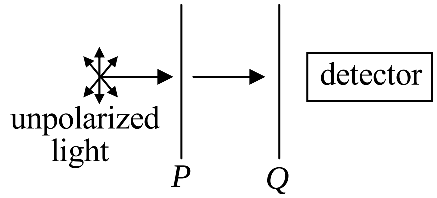
$\therefore\ x = A\sin\omega t \Rightarrow \dfrac{dx}{dt} = A\omega\cos\omega t$

In the mean position, the velocity is maximum

$\therefore\ A\omega = 10 \Rightarrow A = \dfrac{10}{\omega} = \dfrac{10}{40} = 0.25$

$\therefore\ x = 0.25\sin 40t$

Q34. In the optical arrangement as shown below, the axes of two polarizing sheets P and Q are oriented such that no light is detected. Now when a third polarizing sheet (R) is placed in between P and Q, then light is detected. Which of the following statement (s) is (are) true?



(a) Polarization axes of P and Q are perpendicular to each other.

(b) Polarization axis of R is not parallel to P

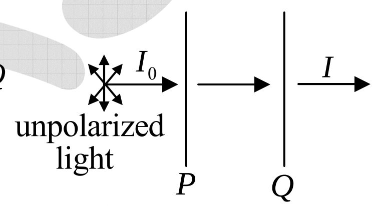
(c) Polarization axis of R is not parallel to Q

(d) Polarization axes of P and Q are parallel to each other.

Ans.: (a), (b) and (c)

Solution: According to Malu's law

$I = \frac{I_0}{2}\cos^2\theta$ where θ is angle between pass axis of P and Q



where $I = 0$, $\Rightarrow \theta = 90^0$

i.e. P and Q are perpendicular to each other. Thus option (a) is correct.

If third polarizer R is introduced between P and Q making angle θ_1 w.r.t. pass axis of P and $90^0 - \theta_1$ w.r.t. θ.

$$\therefore I = \frac{I_0}{2}\cos^2\theta_1 \cos^2(90-\theta_1)$$

If $\theta_1 = 0$, then $I = 0$ thus R can't be parallel to P. Now, If $\theta_1 = 90^0$, then again $I = 0$.

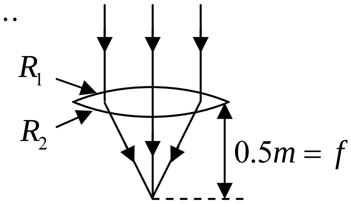
Thus R can't be parallel to θ also.

Thus options (a), (b) and (c) are correct.

Q35. When sunlight is focused on a paper using a bi-convex lens, it starts to burn in the shortest time if the lens is kept $0.5\,m$ above it. If the radius of curvature of the lens is $0.75\,m$ then, the refractive index of the material is…………….

Ans.: 1.75

Solution: For bi-convex lens



$$(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}$$

$$(\mu-1)=\frac{1}{f}\cdot\frac{1}{\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}=\frac{1}{0.5}\frac{1}{\left(\frac{1}{0.75}+\frac{1}{0.75}\right)}=0.75\Rightarrow\mu=1.75$$

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Q36. The dispersion relation for electromagnetic waves travelling in a plasma is given as $\omega^2=c^2k^2+\omega_p^2$, where c and ω_p are constants. In this plasma, the group velocity is:

(a) proportional to but not equal to the phase velocity

(b) inversely proportional to the phase velocity

(c) equal to the phase velocity

(d) a constant

Ans. : (b)

Solution: $\omega^2=c^2k^2+\omega_p^2\Rightarrow\omega=\sqrt{c^2k^2+\omega_p^2}\Rightarrow v_g=\frac{d\omega}{dk}=\frac{c^2k}{\sqrt{c^2k^2+\omega_p^2}}$ and $v_p=\frac{\omega}{k}=\frac{\sqrt{c^2k^2+\omega_p^2}}{k}$

$$\Rightarrow v_g=\frac{c^2}{v_p}$$

Q37. Which of the following is due to inhomogeneous refractive index of earth's atmosphere?

(a) Red colour of the evening Sun

(b) Blue colour of the sky

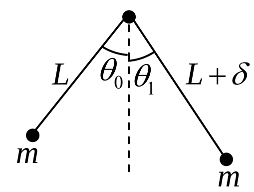
(c) Oval shape of the evening Sun

(d) Large apparent size of the evening Sun

Ans. : (c)

Solution: At sunrise and sunset, the Sun is near the horizon. Due to inhomogenous refractive index of atmosphere, the rays from the upper and lower part of the periphery of the Sun bend unequally on traveling through earth's atmosphere. That is why the Sun appears oval at the time of sunrise and sunset.

Q38. A pendulum is made of a massless string of length L and a small bob of negligible size and mass m. It is released making an angle $\theta_0\,(<<1\,\text{rad})$ from the vertical. When passing through the vertical, the string slips a bit from the pivot so that its length increases by a small amount $\delta\,(\delta << L)$ in negligible time. If it swings up to angle θ_1 on the other side before starting to swing back, then to a good approximation which of the following expressions is correct?



(a) $\theta_1 = \theta_0$ (b) $\theta_1 = \theta_0\left(1-\frac{\delta}{2L}\right)$

(c) $\theta_1 = \theta_0\left(1-\frac{\delta}{L}\right)$ (d) $\theta_1 = \theta_0\left(1-\frac{3\delta}{2L}\right)$

Ans. : (b)

Solution: $mgl\left(1-\cos\theta_i\right) = mg\left(l+\delta\right)\left(1-\cos\theta_f\right)$

$$l2\sin^2\left(\frac{\theta_i}{2}\right) = \left(l+\delta\right)2\sin^2\left(\frac{\theta_f}{2}\right)$$

$$\theta_i = \theta_f\left(1+\frac{\delta}{l}\right)^{1/2} \Rightarrow \theta_f = \theta_i\left(1+\frac{\delta}{l}\right)^{-1/2}$$

$$\theta_f = \theta_i\left(1-\frac{\delta}{2l}\right), \theta_i = \theta_0 \Rightarrow \theta_f = \theta_0\left(1-\frac{\delta}{2l}\right)$$

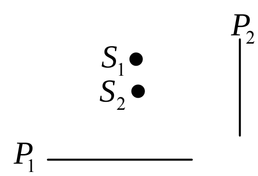
Q39. Consider two coherent point sources (S_1 and S_2) separated by a small distance along a vertical line and two screens P_1 and P_2 as shown in Figure. Which one of the choices represents the shapes of the interference fringes at the central regions on the screens?

(a) Circular on P_1 and straight line on P_2

(b) Circular on P_1 and circular on P_2

(c) Straight lines on P_1 and straight lines on P_2

(d) Straight lines on P_1 and circular on P_2



Ans. : (a)

Solution: The locus of constant path difference on plate P_2 is straight line. Therefore on plate P_2 interference fringes are straight line in nature. Whereas on plate P_1 the locus of constant path difference is circular, therefore fringes are circular

Q40. Unpolarized light is incident on a combination of polarizer, a $\frac{\lambda}{2}$ plate and a $\frac{\lambda}{2}$ and a $\frac{\lambda}{4}$ plate kept one after the other. What will be the output polarization for the following configurations?

Configuration 1: Axes of the polarizer, the $\frac{\lambda}{2}$ plate and the $\frac{\lambda}{4}$ plate are all parallel to each other

Configuration 2: The $\frac{\lambda}{2}$ plate is rotated by 45^0 with respect to configuration 1.

Configuration 3: The $\frac{\lambda}{4}$ plate is rotated by 45^0 with respect to configuration 1.

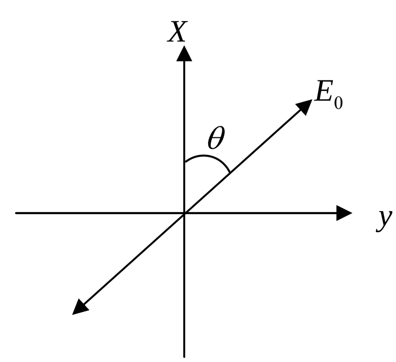
(a) Linear for configuration 1 linear for configuration 2, circular for configuration 3.

(b) Linear for configuration 1 circular for configuration 2, circular for configuration 3.

(c) Circular for configuration 1 circular for configuration 2, circular for configuration 3.

(d) Circular for configuration 1 linear for configuration 2, circular for configuration 3.

Ans. : (b)

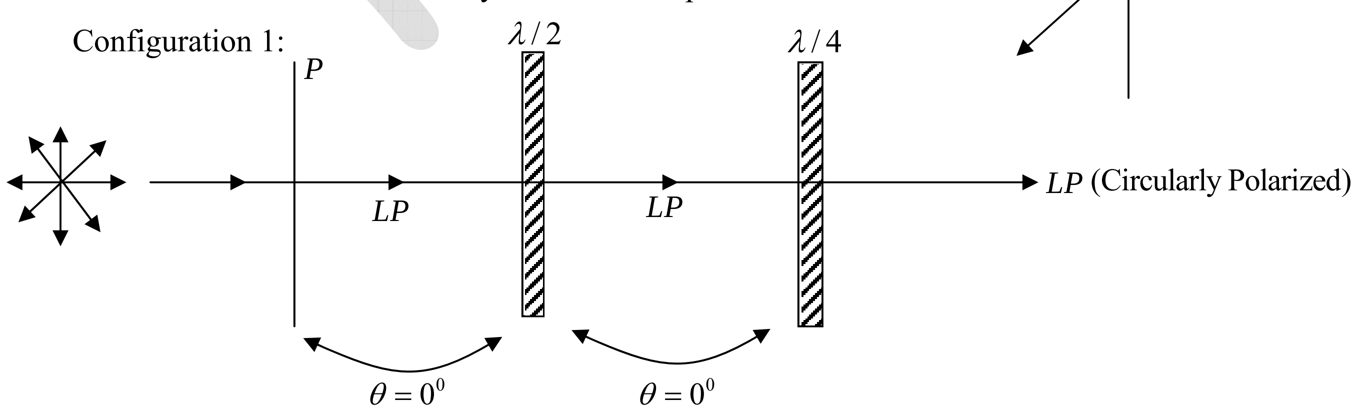


Solution: The output of polarization is $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\delta=\sin^2\delta$

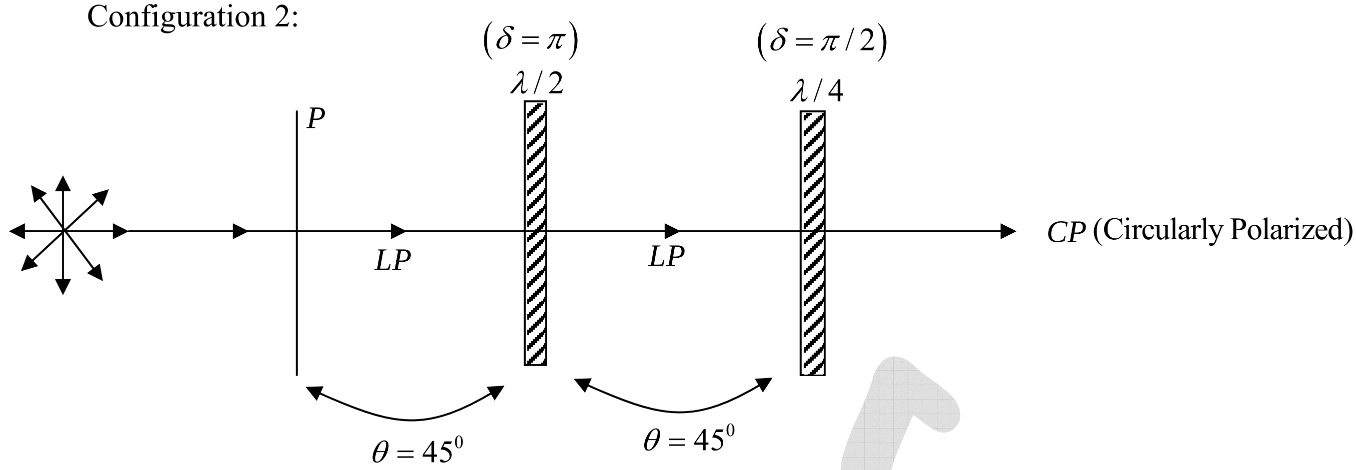
where $a=E_0\cos\theta, b=E_0\sin\theta$ and δ is phase

Difference between o and e-rays at the exit of plate

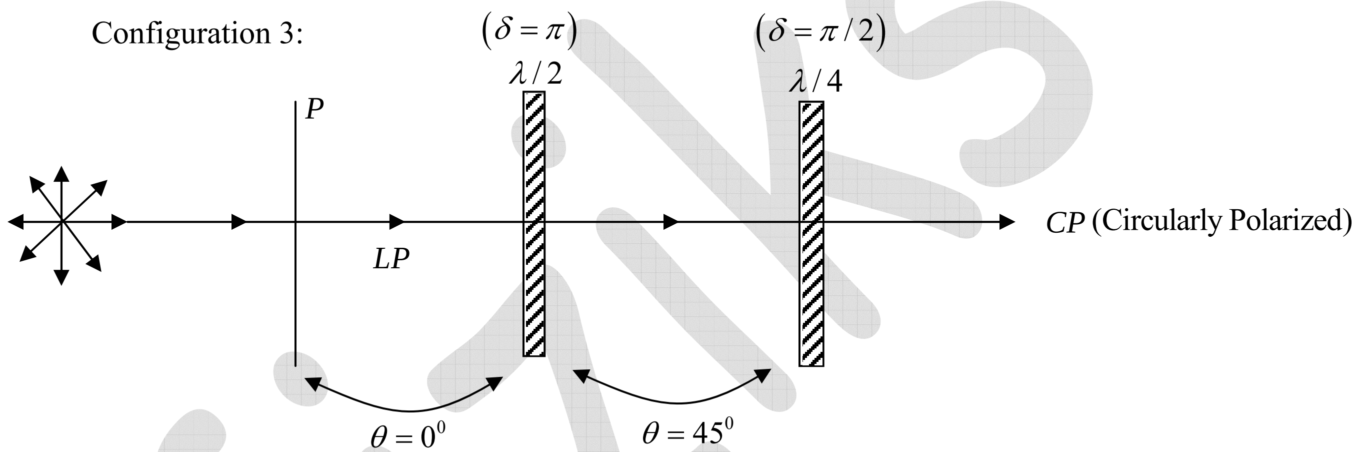
Configuration 1:



Configuration 2:



Configuration 3:



Thus correct option is (b)

Q41. Unpolarized light of intensity I_0 passes through a polarizer P_1. The light coming out of the polarizer falls on a quarter-wave plate with its optical axis at 45^0 with respect to the polarization axis of P_1 and then passes through another polarizer P_2 with its polarization axis perpendicular to that of P_1. The intensity of the light coming out of P_2 is I. The ratio I_0/I is............

(Specify your answer to two digits after the decimal point)

Ans. : 4.00

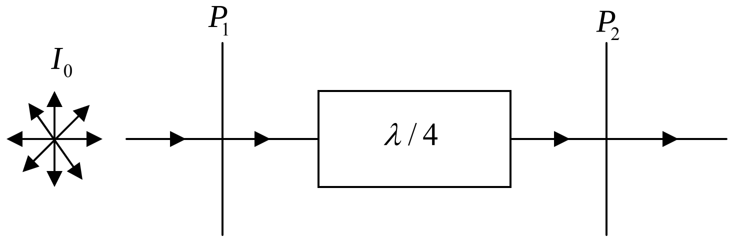
Solution The intensity at the emerged beam at the exit of second polaroid P_2 is

$$I=\frac{I_0}{4}\sin^2(2\theta)$$

when $\theta = 45^0$

$$I = \frac{I_0}{4}\sin^2\left(90^0\right) = \frac{I_0}{4}$$

$$\therefore \frac{I_0}{I} = 4$$



Q42. An anti-reflection film coating of thickness $0.1\,\mu m$ is to be deposited on a glass plate for normal incidence of light of wavelength $0.5\,\mu m$. What should be the refractive index of the film?

(Specify your answer to two digits after the decimal point)

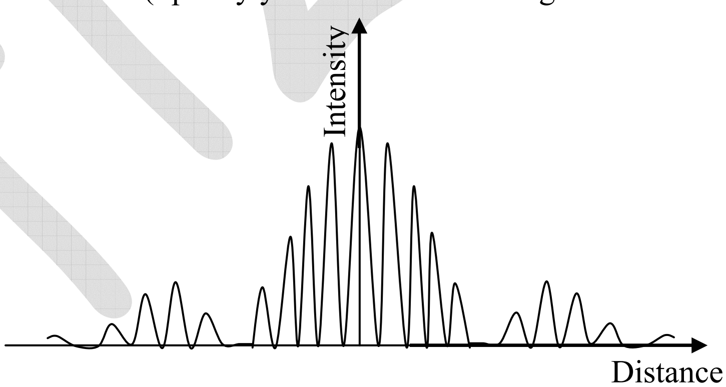
Ans. : 1.25

Solution The condition for constructive interference is $2\mu t = \left(m + \frac{1}{2}\right)\lambda$, where $m = 0,1,2$

$\therefore$ for $m = 0$

$$\mu = \frac{\lambda/2}{2t} = \frac{\lambda}{4t} = \frac{0.5\times 10^{-6}\,m}{4\times 0.1\times 10^{-6}\,m} = \frac{5}{4} = 1.25$$

Q43. Intensity versus distance curve for a double slit diffraction experiment is shown in the figure below. If the width of each of the slits is $0.7\,\mu m$, what is the separation between the two slits in micrometers? (Specify your answer to two digits after the decimal point)



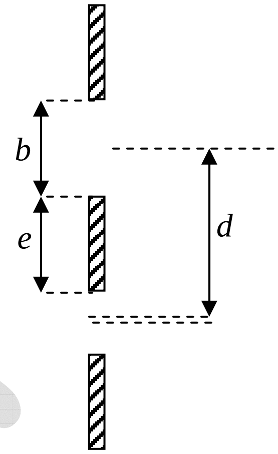
Ans. : 3.50

Solution: The condition for the absent order is $\frac{d}{b}=\frac{n}{m}$

where, for $m=1$, 5^{th} interference maxima is absent (from figure)

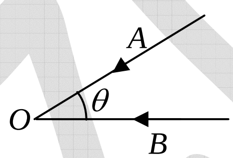
$\therefore m=1, n=5$

$\Rightarrow \frac{d}{b}=5 \Rightarrow d=5b=5\times 0.7\mu m=3.5\mu m$



IIT-JAM 2018

Q44. Two vehicles A and B are approaching an observer O at rest with equal speed as shown in the figure. Both vehicles have identical sirens blowing at a frequency f_s. The observer hears these sirens at frequency f_A and f_B, respectively from the two vehicles. Which one of the following is correct?



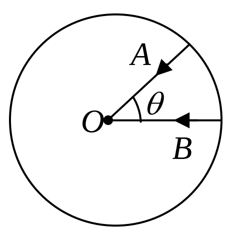
(a) $f_A = f_B < f_s$ (b) $f_A = f_B > f_s$

(c) $f_A > f_B > f_s$ (d) $f_A < f_B < f_s$

Ans.: (b)

Solution: Doppler shift

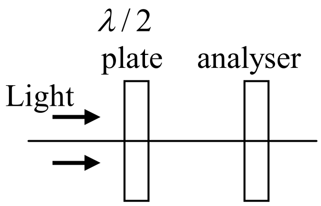
$$f_A = f_s\left(\frac{v_s}{v_s - v_A}\right), \qquad f_B = f_s\left(\frac{v_s}{v_s - v_B}\right)$$

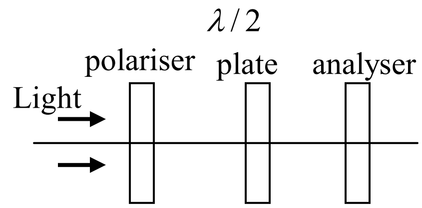


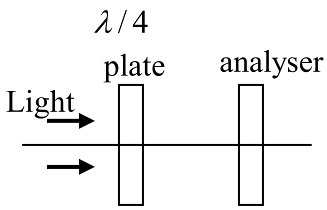
$V_A = V_B < V_s$

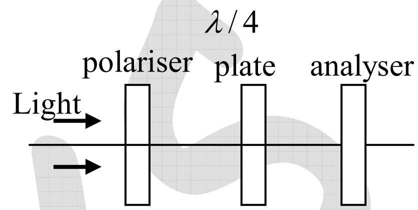
$\therefore f_A = f_B > f_s$

Q45. Which of the following arrangements of optical components can be used to distinguish between an unpolarised light and a circularly polarised light?

(a) 

(b) 

(c) 

(d) 

Ans.: (c)

Solution: (i) In configuration (A), output will be linearly polarized for both

(ii) In configuration (B), output will be linearly polarized for both

(iii) In configuration (C), output will be linearly polarized of constant intensity if input is unpolarised whereas it is linearly polarized with intensity varying from zero to maximum if input is circularly polarized.

(iv) In configuration (D) output will be linearly polarized for both.

Q46. The plane of polarisation of a plane polarized light rotates by 60^0 after passing through a wave plate. The pass-axis of the wave plate is at an angle α with respect to the plane of polarization of the incident light. The wave plate and α are

(a) $\frac{\lambda}{4}, 60^0$ (b) $\frac{\lambda}{2}, 30^0$ (c) $\frac{\lambda}{2}, 120^0$ (d) $\frac{\lambda}{4}, 30^0$

Ans.: (b)

Solution: When plane polarized light is incident on the $\pi/4$ plate, it converts it into circularly polarized light, whereas $\pi/2$ plate rotates is by angle 2α, where α is angle between fast axis and polarization direction.

Given, $2\alpha = 60^0 \Rightarrow \alpha = 30^0$.

Q47. Consider two waves $y_1 = a\cos(\omega t - kz)$ and $y_2 = a\cos[(\omega + \Delta\omega)t - (k + \Delta k)z]$. The group velocity of the superposed wave will be ($\Delta\omega \ll \omega$ and $\Delta k \ll k$)

(a) $\frac{(\omega - \Delta\omega)}{(k - \Delta k)}$ (b) $\frac{(2\omega - \Delta\omega)}{(2k + \Delta k)}$ (c) $\frac{\Delta\omega}{\Delta k}$ (d) $\frac{(\omega + \Delta\omega)}{(k + \Delta k)}$

Ans. : (c)

Solution: $y_1 = a\cos(\omega t - kz)$, $y_2 = a\cos[(\omega + \Delta\omega)t - (k + \Delta k)z]$

$$\Rightarrow y = y_1 - y_2 = 2a\cos\left[\frac{\Delta\omega t - \Delta kz}{2}\right] \times \cos\left[\frac{2\omega + \Delta\omega}{2}t - \frac{2k + \Delta k}{2}z\right]$$

$$v_g = \frac{\Delta\omega/2}{\Delta k/2} = \frac{\Delta\omega}{\Delta k}$$

Q48. Consider a convex lens of focal length f. A point object moves towards the lens along its axis between $2f$ and f. If the speed of the object is V_0, then its image would move with speed V_I. Which of the following is correct?

(a) $V_I = V_o$; the image moves away from the lens.

(b) $V_I = -V_o$; the image moves away from the lens.

(c) $V_I > V_o$; the image moves away from the lens.

(d) $V_I < V_o$; the image moves away from the lens.

Ans. : (c)

Solution: $V_I = \left(\frac{f}{f - u}\right)^2 V_0$ and $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For, $u > f$, $v_i > V_0$ and u decreases than v increases.

$\therefore V_I > V_0$ and image moves away from the lens.

Q49. Two beams of light in the visible range $(400\ nm - 700\ nm)$ interfere with each other at a point. The optical path difference between them is $5000\ nm$. Which of the following wavelengths will interfere constructively at the given point?

(a) $416.67 nm$ (b) $555.55\ nm$ (c) $625\ nm$ (d) $666.66\ nm$

Ans.: (a),(b) and (c)

Solution: $\delta = \frac{2\pi}{\lambda}(p.d)$

For constructive interference $\delta = 2n\pi$ where n is integer

$\therefore\ 2n\pi = \frac{2\pi}{\lambda}(p.d) \Rightarrow \lambda = \frac{p.d}{n} = \frac{5000nm}{n}$

for, $n = 8, \lambda = 625\,nm$

$n = 9, \lambda = 555\cdot 55\,nm$

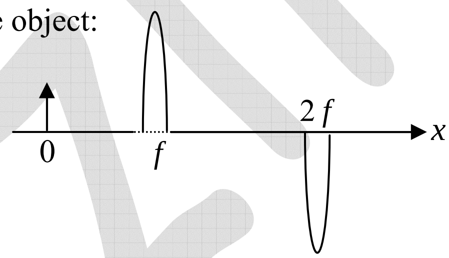
$n = 10, \lambda = 500\,nm$

$n = 11,\ \lambda = 454\cdot 5\,nm$

$n = 12,\ \lambda = 416\cdot 67\,nm$

Thus, correct options are (a),(b) and (c)

Q50. Consider a convex lens of focal length f. The lens is cut along a diameter in two parts. The two lens parts and an object are kept as shown in the figure. The images are formed at following distances from the object:



(a) $2f$ (b) $3f$ (c) $4f$ (d) ∞

Ans.: (b), (c) and (d)

Solution: For first lens $\frac{1}{v'} - \frac{1}{u} = \frac{1}{f}$

For second lens $\frac{1}{v} - \frac{1}{u'} = \frac{1}{f}$

(i) if $u = \infty, v' = f, v = \infty$ (ii) if $u > 2f, v' < 2f, v < 2f$

(iii) if $u = 2f, v' = 2f$ No image (iv) if $u < 2f, v' > 2f, v > 2f$

(v) if $u = f, v' = \infty, v = \infty$ (vi) $u < f, v' = -ve$, No image

Thus, V cannot be $2f$. The correct options are (b),(c) and (d)

Q51. In a grating with grating constant $d = a + b$, where a is the slit width and b is the separation between the slits, the diffraction pattern has the fourth order missing. The value of $\frac{b}{a}$ is______. (Specific your answer as an integer.)

Ans. : 3

Solution: $n = m\frac{d}{a} = m\left(\frac{a+b}{a}\right) = m\left(1+\frac{b}{a}\right)$

For, $m = 1, n = 4$ $\qquad \therefore 4 = 1 + \frac{b}{a} \Rightarrow \frac{b}{a} = 3$

Q52. Consider a slit of width $18\,\mu m$ which is being illuminated simultaneously with light of orange color (wavelength $600\,nm$) and of blue color (wavelength $450\,nm$). The diffraction pattern is observed on a screen kept at a distance in front of the slit. The smallest angle at which only the orange color is observed is θ_1 and the smallest angle at which only the blue color is observed is θ_2. The angular difference $\theta_2 - \theta_1$ (in degrees) is___________

(Specify your answer upto two digits after the decimal point)

Ans.: 0.48

Solution: $d \sin\theta = \lambda$

$$\theta_1 = \sin^{-1}\left(\frac{\lambda_1}{d}\right) = \sin^{-1}\left(\frac{600\times10^{-9}}{18\times10^{-6}}\right) = 1.91^0$$

$$\theta_2 = \sin^{-1}\left(\frac{\lambda_2}{d}\right) = \sin^{-1}\left(\frac{450\times10^{-9}}{18\times10^{-6}}\right) = 1.43^0$$

$\therefore \theta_2 - \theta_1 = 0.48^0$.

IIT-JAM 2019

Q53. A thin lens of refractive index $\frac{3}{2}$ is kept inside a liquid of refractive index $\frac{4}{3}$. If the focal length of the lens in air is 10 cm, then the focal length inside the liquid is

(a) 10 cm (b) 30 cm (c) 40 cm (d) 50 cm

Ans. : (c)

Solution: $\frac{1}{f_a}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{f_l}=\left(\frac{3/2}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{f_l}{f_a}=\frac{\left(\frac{3}{2}-1\right)}{\left(\frac{9}{8}-1\right)}=4$$

$$f_l=4f_a=4\times10=40\,cm$$

Q54. Light of wavelength λ (in free space) propagates through a dispersive medium with refractive index $n(\lambda)=1.5+0.6\lambda$. The group velocity of a wave travelling inside this medium in units of 10^8 m/s is

(a) 1.5 (b) 2.0 (c) 3.0 (d) 4.0

Ans. : (b)

Solution: $v_g=\frac{d\omega}{dk}=\frac{d\omega}{d\lambda}\frac{d\lambda}{dk}\qquad\because k=\frac{2\pi}{\lambda}$

$$=-\frac{\lambda^2}{2\pi}\frac{d\omega}{d\lambda}\qquad\qquad\frac{dk}{d\lambda}=-\frac{2\pi}{\lambda^2}$$

$$=-\frac{\lambda^2}{2\pi}\frac{d}{d\lambda}\left(\frac{c\,2\pi}{n\lambda}\right)\qquad\because\ n=\frac{c}{v_p}=\frac{ck}{\omega}$$

$$=-c\lambda^2\frac{d}{d\lambda}\left(\frac{1}{n\lambda}\right)$$

$$=-c\lambda^2\frac{(n\lambda)\cdot0-1\cdot\frac{d}{d\lambda}(n\lambda)}{n^2\lambda^2}\quad=-c\lambda^2\frac{-\left[1\cdot n+\lambda\frac{d}{d\lambda}n\right]}{n^2\lambda^2}$$

$$= c\frac{n+\lambda(0.6)}{n^2} = c\frac{1.5+1.2\lambda}{(1.5+2.6\lambda)^2} \simeq \frac{c}{1.5} \qquad \because \lambda \sim 10^{-7}\, m$$

$$\simeq \frac{2}{3}c \simeq 2\times 10^8$$

Q55. The maximum number of intensity minima that can be observed I the Fraunhofer diffraction pattern of a single slit (width $10\,\mu m$) illuminated by a laser bean (wavelength $0.630\,\mu m$) will be

(a) 4 (b) 7 (c) 12 (d) 15

Ans. : (d)

Solution: $e\sin\theta = n\lambda$

$$n_{\max} = \frac{e}{\lambda} = \frac{10\,\mu m}{0.63\,\mu m} = 15.87 \approx 15$$

Q56. For an under damped harmonic oscillator with velocity $v(t)$

(a) Rate of energy dissipation varies linearly with $v(t)$

(b) Rate of energy dissipation varies as square of $v(t)$

(c) The reduction in the oscillator frequency, compared to the undamped case, is independent of $v(t)$

(d) For weak damping, the amplitude decays exponentially to zero

Ans. : (b), (c), (d)

Solution: Displacement $x = Ae^{-rt}\sin(\omega t+\phi)$

Velocity $v = \frac{dx}{dt} \cong A\omega e^{-rt}\cos(\omega t+\phi)$

Energy $E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2},\omega^2 A^2 e^{-2rt}$

Power dissipation, $P = \frac{dE}{dt} = \frac{1}{2}m\omega^2 A^2 e^{-2rt}(-2r)$

$$P \propto v^2$$

Power dissipation is proportional to v^2, thus option (a) is wrong and option (b) is correct.

Also, displacement $x = Ae^{-rt}\sin(\omega t + \phi)$ decays exponentially to zero, thus option (d) is also correct.

The damped oscillation frequency is

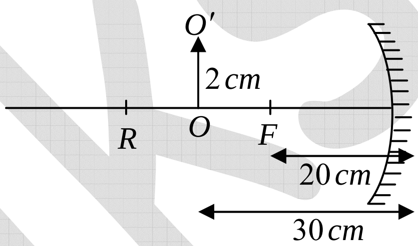
$$\omega = \sqrt{\omega_0^2 - r^2}$$

It is independent of $v(t)$. Thus option (c) is also correct.

Q57. An object of $2\,\text{cm}$ height is placed at a distance of $30\,\text{cm}$ in front of a concave mirror with radius of curvature $40\,\text{cm}$. The height of the image is ________cm.

Ans. : 4

Solution: $u = -30\,cm$



$$f = -20\,cm$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-20} - \frac{1}{-30}$$

$$\frac{1}{v} = -\frac{1}{60} \Rightarrow v = -60\,cm$$

$$m = \frac{I}{O} = -\frac{v}{u}$$

$$I = -\frac{(-60)}{(-30)} \times 2\,cm = -4\,cm$$

Q58. The 7^{th} bright fringe in the Young's double slit experiment using a light of wavelength $550\,\text{nm}$ shifts to the central maxima after covering the two slits with two sheets of different refractive indices n_1 and n_2 but having same thickness $6\,\mu m$. The value of $|n_1 - n_2|$ is _______.

(Round off to 2 decimal places)

Ans. : 0.64

Solution: $(n_1 - 1)t - (n_2 - 1)t = 7\lambda$

$$(n_1 - n_2) = \frac{7\lambda}{t} = \frac{7 \times 550 \times 10^{-9}}{6 \times 10^{-6}} = 0.64$$

Q59. Light of wavelength 680 nm is incident normally on a diffraction grating having 4000 lines/cm. The diffraction angle (in degrees) corresponding to the third-order maximum is ______

(Round off to 2 decimal places)

Ans. : 55^0

Solution: $(e+d)\sin\theta = n\lambda$

$$\frac{10^{-2}}{4000}\times\sin\theta = 3\times 680\times 10^{-9}$$

$$\theta = \sin^{-1}(0.82) \approx 55^0$$