Electromagnetic Theory

JEST-2012

Q1. A magnetic field $\vec{B} = B_0(\hat{i} + 2\hat{j} - 4\hat{k})$ exists at point. If a test charge moving with a velocity, $\vec{v} = v_0(3\hat{i} - \hat{j} + 2\hat{k})$ experiences no force at a certain point, the electric field at that point in SI units is

(a) $\vec{E} = -v_0 B_0(3\hat{i} - 2\hat{j} - 4\hat{k})$ (b) $\vec{E} = -v_0 B_0(\hat{i} + \hat{j} + 7\hat{k})$

(c) $\vec{E} = v_0 B_0(14\hat{j} + 7\hat{k})$ (d) $\vec{E} = -v_0 B_0(14\hat{j} + 7\hat{k})$

Ans. : (d)

Solution: $\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}] = 0 \Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$

$\Rightarrow \vec{E} = -v_0 B_0\{(4-4)\hat{i} + (2+12)\hat{j} + (6+1)\hat{k}\} \quad = -v_0 B_0(14\hat{j} + 7\hat{k})$

Q2. An observer in an inertial frame finds that at a point P the electric field vanishes but the magnetic field does not. This implies that in any other inertial frame the electric field $\vec{E}$ and the magnetic field $\vec{B}$ satisfy

(a) $|\vec{E}|^2 = |\vec{B}|^2$ (b) $\vec{E} \cdot \vec{B} = 0$ (c) $\vec{E} \times \vec{B} = 0$ (d) $\vec{E} = 0$

Ans.: (b)

Q3. A circular conducting ring of radius R rotates with constant angular velocity ω about its diameter placed along the x-axis. A uniform magnetic field B is applied along the y-axis. If at time $t = 0$ the ring is entirely in the xy-plane, the emf induced in the ring at time $t > 0$ is

(a) $B\omega^2 \pi R^2 t$ (b) $B\omega\pi R^2 \tan(\omega t)$

(c) $B\omega\pi R^2 \sin(\omega t)$ (d) $B\omega\pi R^2 \cos(\omega t)$

Ans. : (d)

Solution: $\phi_m = \vec{B} \cdot \vec{A} = BA\cos(90 - \theta) = BA\sin\omega t$

$$\varepsilon = -\frac{d\phi_m}{dt} = -\frac{d}{dt}(\vec{B} \cdot \vec{A}) = -\frac{d}{dt}[BA\sin\omega t] = -BA(\cos\omega t)\omega$$

$\Rightarrow \varepsilon = -B\pi R^2 \omega\cos\omega t \Rightarrow \varepsilon = B\omega\pi R^2 \cos\omega t$

Q4. An electric field in a region is given by $\vec{E}(x,y,z) = ax\hat{i} + cz\hat{j} + 6by\hat{k}$. For which values of a,b,c does this represent an electrostatic field?

(a) 13,1,12 (b) 17,6,1 (c) 13,1,6 (d) 45,6,1

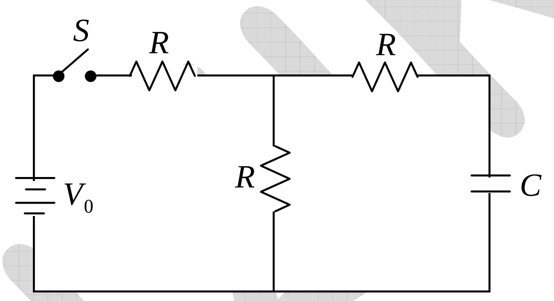
Ans.: (c)

Solution: For electrostatic field $\vec{\nabla} \times \vec{E} = 0$

$$\vec{\nabla} \times \vec{E} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax & cz & 6by \end{bmatrix} = 0 \Rightarrow \vec{\nabla} \times \vec{E} = (6b-c)\hat{i} + \hat{j}[0-0] + \hat{k}[0] = 0$$

$$\Rightarrow (6b-c)\hat{i} = 0 \Rightarrow c = 6b$$

Q5. A capacitor C is connected to a battery V_0 through three equal resistors R and a switch S as shown below:



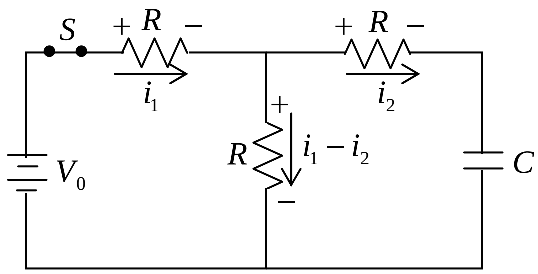
The capacitor is initially uncharged. At time $t = 0$, the switch S is closed. The voltage across the capacitor as a function of time t for $t > 0$ is given by

(a) $(V_0/2)(1-\exp(-t/2RC))$ (b) $(V_0/3)(1-\exp(-t/3RC))$

(c) $(V_0/3)(1-\exp(-3t/2RC))$ (d) $(V_0/2)(1-\exp(-2t/3RC))$

Ans.: (d)

Solution:



Apply KVL in loop 1: $-V_0 + i_1(t)R + (i_1 - i_2)R = 0 \Rightarrow 2i_1(t)R - i_2(t)R = V_0$ ….. (i)

Apply KVL in loop 2:

$$i_2 R + \frac{1}{C}\int_0^t i_2 dt - (i_1 - i_2)R = 0 \Rightarrow \frac{1}{C}\int_0^t i_2 dt + 2i_2 R - \left(\frac{V_0 + i_2 R}{2}\right) = 0$$

$$\Rightarrow \frac{1}{C}\int_0^t i_2 dt + \frac{3}{2}i_2 R = \frac{-V_0}{2} \quad \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

$$\Rightarrow \frac{1}{C}i_2 + \frac{3}{2}R\frac{di_2}{dt} = 0 \quad \Rightarrow \frac{3}{2}R\frac{di_2}{dt} = -\frac{1}{C}i_2 \Rightarrow \frac{di_2}{dt} = \frac{-2}{3RC}i_2 \Rightarrow i_2(t) = Ke^{\frac{-2}{3RC}t}$$

Initial Conditions

$$i_1(0^+) = \frac{V_0}{R + \dfrac{R\times R}{R+R}} = \frac{V_0}{R + \dfrac{R}{2}} = \frac{2}{3}\frac{V_0}{R} \text{ and } i_2(0^+) = \frac{R}{R+R}\times\frac{2V_0}{3R} = \frac{V_0}{3R},\ v_C(0^+) = 0$$

$$\because i_2(0^+) = \frac{V_0}{3R} \Rightarrow K = \frac{V_0}{3R} \Rightarrow i_2(t) = \frac{V_0}{3R}e^{\frac{-2}{3RC}t}$$

$$v_C(t) = \frac{1}{C}\int_0^t i_2 dt = \frac{V_0}{3RC}\int_0^t e^{\frac{-2t}{3RC}}dt = \frac{V_0}{3RC}\left[\frac{e^{\frac{-2t}{3RC}}}{\frac{-2}{3}RC}\right]_0^t$$

$$\Rightarrow v_C(t) = \frac{V_0}{3RC}\times\frac{-3RC}{2}\left[e^{\frac{-2t}{3RC}} - 1\right] \Rightarrow v_C(t) = \frac{V_0}{2}\left[1 - e^{\frac{-2t}{3RC}}\right]$$

Q6. A small magnet is dropped down a long vertical copper tube in a uniform gravitational field. After a long time, the magnet

(a) attains a constant velocity (b) moves with a constant acceleration

(c) moves with a constant deceleration (d) executes simple harmonic motion

Ans. : (a)

Q7. Consider a particle of electric charge e and mass m moving under the influence of a constant horizontal electric field E and constant vertical gravitational field described by acceleration due to gravity g. If the particle starts from rest, what will be its trajectory?

(a) parabolic (b) elliptic (c) straight line (d) circular

Ans.: (c)

Solution: Equation of motion $qE = \frac{md^2x}{dt^2} \Rightarrow \frac{dx}{dt} = \alpha_1 t + c_1$

at $t = 0,\quad v = 0 \Rightarrow c_1 = 0$, $\Rightarrow \frac{dx}{dt} = \alpha_1 t \Rightarrow x = \frac{\alpha_1 t^2}{2}\left(\text{where } \alpha_1 = \frac{qE}{m}\right)$

similarly, $mg = \frac{md^2y}{dt^2}$

$y = \frac{\alpha_2 t^2}{2} \Rightarrow y = \frac{\alpha_2 x}{\alpha_1}, \alpha_2 = g$

Q8. A point charge $+q$ is placed at $(0,0,d)$ above a grounded infinite conducting plane defined by $z = 0$. There are no charges present anywhere else. What is the magnitude of electric field at $(0,0,-d)$?

(a) $q/(8\pi\epsilon_0 d^2)$ (b) $-\infty$ (c) 0 (d) $q/(16\pi\epsilon_0 d^2)$

Ans.: (d)

Solution: Electric field at Q

$$\vec{E} = \frac{-q}{4\pi\epsilon_0 (2d)^2}(\hat{z}) = \frac{-q}{16\pi\epsilon_0 d^2}\hat{z} \Rightarrow |E| = \frac{q}{16\pi\epsilon_0 d^2}$$

Q9. A time-dependent magnetic field $\vec{B}(t)$ is produced in a circular region of space, infinitely long and of radius R. The magnetic field is given as $\vec{B} = B_0 t\,\hat{z}$ and is zero for $r > R$, where B_0 is a positive constant. The electric field at point $r = 2R$ is

(a) $\frac{B_0 R}{2}\hat{r}$ (b) $-\frac{B_0 R}{4}\hat{\theta}$ (c) $-\frac{B_0 R}{2}\hat{r}$ (d) $\frac{B_0 R}{4}\hat{\theta}$

Ans.: (b)

Solution: Solution: $\oint_{line} \vec{E}\cdot dl = \int\left(-\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{a} \Rightarrow |E| \times 2\pi r = -B_0 \pi R^2$

$\Rightarrow |E| = -B_0 \frac{R^2}{2r} \Rightarrow \vec{E} = -\frac{B_0 R^2}{2r}\hat{\theta}$

The electric field at point $r = 2R$ is $\vec{E} = -\frac{B_0 R}{4}\hat{\theta}$

Q10. When unpolarised light is incident on a glass plate at a particular angle, it is observed that the reflected beam is linearly polarized. What is the angle of the refracted beam with respect to the surface normal?

(a) 56.7^0

(b) 33.4^0

(c) 23.3^0

(d) The light is completely reflected and there is no refracted beam.

Ans.: (b)

Solution: Since $n_1 = 1$, $n_2 = 1.52$

Brewster angle $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1}\right) = 56.7^0$

Now $\theta_R = 180 - 90 - 56.7 = 33.4^0$

Q11. A cube has a constant electric potential V on its surface. If there are no charges inside the cube, the potential at the center of the cube is

(a) V (b) $\frac{V}{8}$ (c) 0 (d) $\frac{V}{6}$

Ans.: (a)

JEST-2013

Q12. At equilibrium, there can not be any free charge inside a metal. However, if you forcibly put charge in the interior then it takes some finite time to 'disappear' i.e. move to the surface. If the conductivity σ of a metal is $10^6 (\Omega m)^{-1}$ and the dielectric constant $\varepsilon_0 = 8.85\times 10^{-12}$ Farad/m, this time will be approximately:

(a) 10^{-5} sec (b) 10^{-11} sec (c) 10^{-9} sec (d) 10^{-17} sec

Ans.: (d)

Solution: Characteristic time: $\tau = \frac{\epsilon_0}{\sigma} = \frac{8.85\times 10^{-12}}{10^6} = 8.85\times 10^{-18}$

Q13. The electric fields outside $(r > R)$ and inside $(r < R)$ a solid sphere with a uniform volume charge density are given by $\vec{E}_{r>R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ and $\vec{E}_{r<R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r\hat{r}$ respectively, while the electric field outside a spherical shell with a uniform surface charge density is given by $\vec{E}_{r>R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$, q being the total charge. The correct ratio of the electrostatic energies for the second case to the first case is

(a) $1:3$ (b) $9:16$ (c) $3:8$ (d) $5:6$

Ans.: (d)

Solution: Electrostatic energy in spherical shell $w_{sp} = \frac{\in_0}{2}\int_0^R \left|\vec{E}_1\right|^2 4\pi r^2 dr + \frac{\in_0}{2}\int_R^\infty \left|\vec{E}_2\right|^2 4\pi r^2 dr$

$$\Rightarrow \frac{\in_0}{2}\int_R^\infty \frac{q^2}{(4\pi\in_0)^2 r^4}4\pi r^2 dr = \frac{q^2}{8\pi\in_0}\left(-\frac{1}{r}\right)_R^\infty = \frac{q^2}{8\pi\in_0}\frac{1}{R}$$

Electrostatic energy in solid sphere $w_s = \frac{\in_0}{2}\int_0^R |E_1|^2 4\pi r^2 dr + \frac{\in_0}{2}\int_R^\infty |E_2|^2 4\pi r^2 dr$

$$\Rightarrow \frac{q^2}{8\pi\in_0}\times\frac{1}{R^6}\left[\frac{r^5}{5}\right]_0^R + \frac{q^2}{8\pi\in_0}\left[-\frac{1}{r}\right]_R^\infty$$

$$w_s = \frac{q^2}{5\times 8\pi\in_0}\cdot\frac{1}{R} + \frac{q^2}{8\pi\in_0 R} = \frac{6q^2}{40\pi\in_0 R}$$

Now $\frac{W_{spherical}}{W_{sphere}} = \frac{\frac{q^2}{8\pi\in_0}}{\frac{6q^2}{40\pi\in_0 R}} = \frac{5}{6}$

Q14. A thin uniform ring carrying charge Q and mass M rotates about its axis. What is the gyromagnetic ratio (defined as ratio of magnetic dipole moment to the angular momentum) of this ring?

(a) $\frac{Q}{2\pi M}$ (b) $\frac{Q}{M}$ (c) $\frac{Q}{2M}$ (d) $\frac{Q}{\pi M}$

Ans.: (c)

Solution: Magnetic dipole moment $M' = IA = \frac{Q}{T}\pi r^2 \Rightarrow \frac{Q}{2\pi T}\times 2\pi\times\pi r^2 = \frac{Q\omega r^2}{2}$

Angular momentum $J = Mr^2\omega \Rightarrow \frac{M'}{J} = \frac{Q}{2M}$

Q15. The electric and magnetic field caused by an accelerated charged particle are found to scale as $E \propto r^{-n}$ and $B \propto r^{-m}$ at large distances. What are the value of n and m?

(a) $n=1,\ m=2$ (b) $n=2,\ m=1$ (c) $n=1,\ m=1$ (d) $n=2,\ m=2$

Ans.: (c)

Solution: For large distance $F = \dfrac{qa\sin\theta}{r},\ B = \dfrac{qa\sin\theta}{r} \Rightarrow E \propto \dfrac{1}{r},\ \ B \propto \dfrac{1}{r}$

So $m = n = 1$

Q16. If $\vec{E}_1 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$ and $\vec{E}_2 = y^2\hat{i} + \left(2xy + z^2\right)\hat{j} + 2yz\hat{k}$ then

(a) Both are impossible electrostatic fields

(b) Both are possible electrostatic fields

(c) Only $\vec{E}_1$ is a possible electrostatic field

(d) Only $\vec{E}_2$ is a possible electrostatic field

Ans.: (d)

Solution: For electrostatic field $\vec{\nabla} \times \vec{E} = 0$

$$\vec{\nabla} \times \vec{E}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ y^2 & 2xy + z^2 & 2yz \end{vmatrix}$$

$$(2z - 2z)\hat{i} + 0 + (2y - 2y)\hat{z} = 0$$

$$\vec{\nabla} \times \vec{E}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix} = (0 - 2y)\hat{i} + 0 + x\hat{j} \neq 0$$

Q17. A charge q is placed at the centre of an otherwise neutral dielectric sphere of radius a and relative permittivity ε_r. We denote the expression $q/4\pi\varepsilon_0 r^2$ by $E(r)$. Which of the following statements is false?

(a) The electric field inside the sphere, $r < a$, is given by $E(r)/\varepsilon_r$

(b) The field outside the sphere, $r > a$, is given by $E(r)$

(c) The total charge inside a sphere of radius $r > a$ is given by q.

(d) The total charge inside a sphere of radius $r < a$ is given by q.

Ans.: (d)

Q18. An electromagnetic wave of frequency ω travels in the x- direction through vacuum. It is polarized in the y- direction and the amplitude of the electric field is E_0. With $k = \frac{\omega}{c}$ where c is the speed of light in vacuum, the electric and the magnetic fields are then conventionally given by

(a) $\vec{E} = E_0 \cos(ky - \omega t)\hat{x}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega t)\hat{z}$

(b) $\vec{E} = E_0 \cos(kx - \omega t)\hat{y}$ and $\vec{B} = \frac{E_0}{c}\cos(kx - \omega t)\hat{z}$

(c) $\vec{E} = E_0 \cos(kx - \omega t)\hat{z}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega t)\hat{y}$

(d) $\vec{E} = E_0 \cos(kx - \omega t)\hat{x}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega t)\hat{y}$

Ans.: (b)

Solution: $\vec{E} = E_0 \cos(kx - \omega t)\hat{y}$

$$\vec{B} = \frac{1}{c}\left(\hat{k} \times \vec{E}\right) \Rightarrow \vec{B} = \frac{1}{c}\left[\hat{x} \times E_0 \cos(kx - \omega t)\hat{y}\right]$$

$$\Rightarrow \vec{B} = \frac{E_0}{c}\cos(kx - \omega t)(\hat{x} \times \hat{y}) \Rightarrow \vec{B} = \frac{E_0}{c}\cos(kx - \omega t)(\hat{z})$$

JEST-2014

Q19. For an optical fiber with core and cladding index of $n_1 = 1.45$ and $n_2 = 1.44$, respectively, what is the approximate cut-off angle of incidence? Cut-off angle of incidence is defined as the incidence angle below which light will be guided.

(a) 7^o (b) 22^o (c) 5^o (d) 0^o

Ans.: (a)

Solution: $\theta = \sin^{-1}\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}$, where $n_2 = 1.44,\ n_1 = 1.45$

$$\theta = \sin^{-1}\left(1 - \frac{1.44 \times 1.44}{1.45 \times 1.45}\right)^{1/2} \Rightarrow \theta = \sin^{-1}(0.11726)^{1/2} \Rightarrow \theta = 6.67^0 \approx 7^0$$

Q20. Two large nonconducting sheets one with a fixed uniform positive charge and another with a fixed uniform negative charge are placed at a distance of 1 meter from each other. The magnitude of the surface charge densities are $\sigma_+ = 6.8\mu C/m^2$ for the positively charged sheet and $\sigma_- = 4.3\mu C/m^2$ for the negatively charged sheet. What is the electric field in the region between the sheets?

(a) $6.30 \times 10^5\ N/C$ (b) $3.84 \times 10^5\ N/C$

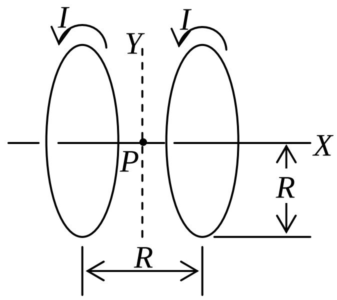
(c) $1.40 \times 10^5\ N/C$ (d) $1.16 \times 10^5\ N/C$

Ans.: (a)

Solution: Electric field between the sheet is $= \frac{\sigma_+}{2\in_0} + \frac{\sigma_-}{2\in_0} = \frac{6.8 \times 10^{-6}}{2\in_0} + \frac{4.3 \times 10^{-6}}{2\in_0}$

$$\Rightarrow \frac{11.2 \times 10^{-6}}{2 \times 8.86 \times 10^{-12}} = 0.626 \times 10^6 \Rightarrow 6.3 \times 10^5\ N/C$$

Q21. A system of two circular co-axial coils carrying equal currents I along same direction having equal radius R and separated by a distance R (as shown in the figure below). The magnitude of magnetic field at the midpoint P is given by



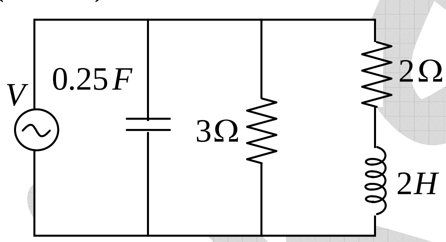
(a) $\frac{\mu_0 I}{2\sqrt{2}R}$ (b) $\frac{4\mu_0 I}{5\sqrt{5}R}$ (c) $\frac{8\mu_0 I}{5\sqrt{5}R}$ (d) 0

Ans.: (c)

Solution: $\because B = \dfrac{\mu_0 I R^2}{2\left(R^2+d^2\right)^{\frac{3}{2}}} \Rightarrow B_1 = \dfrac{\mu_0 I R^2}{2\left(R^2+\dfrac{R^2}{4}\right)^{\frac{3}{2}}}$, $B_2 = \dfrac{\mu_0 I R^2}{2\left(R^2+\dfrac{R^2}{4}\right)^{\frac{3}{2}}}$ $\because d = \dfrac{R}{2}$

$$B = B_1 + B_2 = \frac{\mu_0 I \times 2}{2R\left(\frac{5}{4}\right)^{\frac{3}{2}}} \Rightarrow B = \frac{\mu_0 I\, 4^{\frac{3}{2}}}{R\ \ 5^{\frac{3}{2}}} = \frac{8\mu_0 I}{5\sqrt{5}R}$$

Q22. Find the resonance frequency (rad/sec) of the circuit shown in the figure below



(a) 1.41 (b) 1.0 (c) 2.0 (d) 1.73

Ans.: (b)

Solution: $\omega = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}} = 1.0$ (where $R = 2\Omega,\ L = 2H,\ C = 0.25\ F$)

Q23. An electron is executing simple harmonic motion along the y-axis in right handed coordinate system. Which of the following statements is true for emitted radiation?

(a) The radiation will be most intense in xz plane

(b) The radiation will be most intense in xy plane

(c) The radiation will violate causality

(d) The electron's rest mass energy will reduce due to radiation loss

Ans.: (a)

Solution: Oscillating electron does not emit radiation in the direction of oscillation.

In the perpendicular direction of oscillation intensity is maximum.

So in this case the intensity will be maximum along x and z - axis or xz - plane (intensity is also en xy-plane but less).

Q24. A conducting sphere of radius r has charge Q on its surface. If the charge on the sphere is doubled and its radius is halved, the energy associated with the electric field will

(a) increase four times (b) increase eight times

(c) remain the same (d) decrease four times

Ans.: (b)

Solution: $E = \frac{Q}{4\pi \in_0 r^2}\hat{r}\; W = \frac{\in_0}{2}\int_0^R E_1^2 4\pi r^2 dr + \frac{\in_0}{2}\int_R^\infty E_2^2 4\pi r^2 dr \Rightarrow W = \frac{Q^2}{8\pi \in_0 R}$

$$\Rightarrow W' = \frac{(2Q)^2}{8\pi \in_0 \frac{R}{2}} = \frac{8Q^2}{8\pi \in_0 R} = 8W$$

JEST-2015

Q25. A circular loop of radius R, carries a uniform line charge density λ. The electric field, calculated at a distance z directly above the center of the loop, is maximum if z is equal to,

(a) $\frac{R}{\sqrt{3}}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{2}$ (d) $2R$

Ans.: (b)

Solution: $E = \frac{1}{4\pi\varepsilon_0}\frac{(\lambda \times 2\pi R)z}{(R^2+z^2)^{3/2}}$

For maximum E, $\frac{dE}{dz} = 0 \Rightarrow \frac{\lambda \times 2\pi R}{4\pi\varepsilon_0}\left[\frac{(R^2+z^2)^{3/2} - z\times 3/2\sqrt{R^2+z^2}\times 2z}{(R^2+z^2)^3}\right] = 0$

$$\Rightarrow (R^2+z^2)^{3/2} = 3z^2\sqrt{R^2+z^2} \Rightarrow R^2+z^2 = 3z^2 \Rightarrow R^2 = 2z^2 \Rightarrow z = \frac{R}{\sqrt{2}}$$

Q26. Consider two point charges q and λq located at the points, $x = a$ and $x = \mu a$, respectively. Assuming that the sum of the two charges is constant, what is the value of λ for which the magnitude of the electrostatic force is maximum?

(a) μ (b) 1 (c) $\frac{1}{\mu}$ (d) $1+\mu$

Ans.: (b)

Solution: $F=\frac{1}{4\pi\varepsilon_0}\frac{(\lambda q\times q)}{(\mu a-a)^2}=\frac{1}{4\pi\varepsilon_0}\frac{\lambda q^2}{a^2(\mu-1)^2}=\frac{1}{4\pi\varepsilon_0 a^2(\mu-1)^2}\frac{\lambda c^2}{(1+\lambda)^2}$ $\quad\because q+\lambda q=c$

For maximum F, $\frac{dF}{dz}=0\Rightarrow\frac{1}{4\pi\varepsilon_0 a^2(\mu-1)^2}\left[\frac{(1+\lambda)^2c^2-\lambda c^2\times 2(1+\lambda)}{(1+\lambda)^4}\right]=0$

$\Rightarrow(1+\lambda)^2c^2=\lambda c^2\times 2(1+\lambda)\Rightarrow 1+\lambda=2\lambda\Rightarrow\lambda=1$

Q27. A spherical shell of inner and outer radii a and b, respectively, is made of a dielectric material with frozen polarization $\vec{P}(r)=\frac{k}{r}\hat{r}$,where k is a constant and r is the distance from the its centre. The electric field in the region $a<r<b$ is,

(a) $\vec{E}=\frac{k}{\epsilon_0 r}\hat{r}$ (b) $\vec{E}=-\frac{k}{\epsilon_0 r}\hat{r}$ (c) $\vec{E}=0$ (d) $\vec{E}=\frac{k}{\epsilon_0 r^2}\hat{r}$

Ans.: (b)

Solution: $\rho_b=-\vec{\nabla}.\vec{P}=-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{k}{r}\right)=\frac{-k}{r^2}$ and $\sigma_b=\vec{P}.\hat{n}=\begin{cases}+\vec{P}.\hat{r}=\frac{k}{b}\ (\text{at } r=b)\\ -\vec{P}.\hat{r}=\frac{-k}{a}(\text{at } r=a)\end{cases}$

For $a<r<b$; $Q_{enc}=\left(\frac{-k}{a}\right)\times 4\pi a^2+\int_a^r\left(\frac{-k}{r^2}\right)4\pi r^2dr=-4\pi ka-4\pi k(r-a)=-4\pi kr$

$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}\Rightarrow\vec{E}=\frac{-k}{\varepsilon_0 r}\hat{r}$$

Q28. The electrostatic potential due to a charge distribution is given by $V(r)=A\frac{e^{-\lambda r}}{r}$ where A and λ are constants The total charge enclosed within a sphere of radius $\frac{1}{\lambda}$, with its origin at $r=0$ is given by,

(a) $\frac{8\pi\epsilon_0 A}{e}$ (b) $\frac{4\pi\epsilon_0 A}{e}$ (c) $\frac{\pi\epsilon_0 A}{e}$ (d) 0

Ans.: (a)

Solution: $\because V(r)=A\frac{e^{-\lambda r}}{r}$

$$\vec{E} = -\vec{\nabla}V = -A\left[\frac{re^{-\lambda r}\times(-\lambda) - e^{-\lambda r}}{r^2}\right]\hat{r} = \frac{Ae^{-\lambda r}}{r^2}(1+\lambda r)\hat{r}$$

$$Q_{enc} = \varepsilon_0 \oint \vec{E}.d\vec{a} = \varepsilon_0 \int_0^{\pi}\int_0^{2\pi} \frac{Ae^{-\lambda r}}{r^2}(1+\lambda r)\hat{r}.r^2 \sin\theta d\theta d\phi \hat{r} = 4\pi\varepsilon_0 Ae^{-\lambda r}(1+\lambda r)$$

Thus total charge enclosed within a sphere of radius $r = \frac{1}{\lambda}$ is

$$Q_{enc} = 4\pi\varepsilon_0 Ae^{-\lambda\frac{1}{\lambda}}\left(1+\lambda\frac{1}{\lambda}\right) = \frac{8\pi\varepsilon_0 A}{e}$$

Q29. The skin depth of a metal is dependent on the conductivity (σ) of the metal and the angular frequency ω of the incident field. For a metal of high conductivity, which of the following relations is correct? (Assume that $\sigma >> \in \omega$, where $\in$ is the electrical permittivity of the medium.)

(a) $d \propto \sqrt{\frac{\sigma}{\omega}}$

(b) $d \propto \sqrt{\frac{1}{\sigma\omega}}$

(c) $d \propto \sqrt{\sigma\omega}$

(d) $d \propto \sqrt{\frac{\omega}{\sigma}}$

Ans.: (b)

Solution: Skin depth $d = \sqrt{\frac{2}{\sigma\mu\omega}}$

Q30. The wavelength of red helium-neon laser in air is $6328\,\overset{o}{A}$. What happens to its frequency in glass that has a refractive index of 1.50?

(a) Increases by a factor of 3

(b) Decreases by a factor of 1.5

(c) Remains the same

(d) Decreases by a factor of 0.5

Ans.: (c)

Solution: Frequency of electromagnetic wave does not change when it enter in medium of any refractive index.

Q31. The approximate force exerted on a perfectly reflecting mirror by an incident laser beam of power $10\,mW$ at normal incidence is

(a) $10^{-13}\,N$ (b) $10^{-11}\,N$ (c) $10^{-9}\,N$ (d) $10^{-15}\,N$

Ans.: (b)

Solution: When electromagnetic wave is reflected by mirror the momentum transferred to the mirror per unit area per second is twice the momentum of the light striking the mirror per unit area per second

i.e. $$\frac{dp}{dt}=\frac{2\times Power}{c}=2\times\frac{10\times10^{-3}}{3\times10^{-8}}=6.6\times10^{-11}\ kg\,m/s^2$$

The force exerted on the reflecting mirror is $F=\frac{dp}{dt}=6.6\times10^{-11}N$

Thus best suitable answer is option (b).

Q32. Which of the following expressions represents an electric field due to a time varying magnetic field?

(a) $K(x\hat{x}+y\hat{y}+z\hat{z})$ (b) $K(x\hat{x}+y\hat{y}-z\hat{z})$

(c) $K(x\hat{x}-y\hat{y})$ (d) $K(y\hat{y}-x\hat{y}+2z\hat{z})$

Ans.: (d)

Solution: For time varying fields $\vec{\nabla}\times\vec{E}\neq0$

$$(a)\ \vec{\nabla}\times\vec{E}=K\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ x&y&z\end{vmatrix}=\hat{x}\left(\frac{\partial z}{\partial y}-\frac{\partial y}{\partial z}\right)+\hat{y}\left(\frac{\partial x}{\partial z}-\frac{\partial z}{\partial x}\right)+\hat{z}\left(\frac{\partial y}{\partial x}-\frac{\partial x}{\partial y}\right)=0$$

$$(b)\ \vec{\nabla}\times\vec{E}=K\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ x&y&-z\end{vmatrix}=\hat{x}\left(-\frac{\partial z}{\partial y}-\frac{\partial y}{\partial z}\right)+\hat{y}\left(\frac{\partial x}{\partial z}+\frac{\partial z}{\partial x}\right)+\hat{z}\left(\frac{\partial y}{\partial x}-\frac{\partial x}{\partial y}\right)=0$$

$$(c)\ \vec{\nabla}\times\vec{E}=K\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ x&-y&0\end{vmatrix}=\hat{x}\left(0+\frac{\partial y}{\partial z}\right)+\hat{y}\left(\frac{\partial x}{\partial z}-0\right)+\hat{z}\left(-\frac{\partial y}{\partial x}-\frac{\partial x}{\partial y}\right)=0$$

$$(d)\ \vec{\nabla}\times\vec{E}=K\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ y&-x&2z\end{vmatrix}=\hat{x}\left(\frac{\partial(2z)}{\partial y}+\frac{\partial x}{\partial z}\right)+\hat{y}\left(-\frac{\partial x}{\partial z}-\frac{\partial(2z)}{\partial x}\right)+\hat{z}\left(\frac{\partial y}{\partial x}-\frac{\partial y}{\partial y}\right)$$

$=-\hat{z}\neq0$

Q33. A charged particle is released at time $t=0$, from the origin in the presence of uniform static electric and magnetic fields given by $E = E_0\hat{y}$ and $B = B_0\hat{z}$ respectively. Which of the following statements is true for $t > 0$?

(a) The particle moves along the x-axis.

(b) The particle moves in a circular orbit.

(c) The particle moves in the (x, y) plane.

(d) Particle moves in the (y, z) plane

Ans.: (c)

Solution: In a cycloid charged particle will be always confined in a plane perpendicular to B.

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Q34. The maximum relativistic kinetic energy of β particles from a radioactive nucleus is equal to the rest mass energy of the particle. A magnetic field is applied perpendicular to the beam of β particles, which bends it to a circle of radius R. The field is given by:

(a) $\frac{3m_0c}{eR}$ (b) $\frac{\sqrt{2}m_0c}{eR}$ (c) $\frac{\sqrt{3}m_0c}{eR}$ (d) $\frac{\sqrt{3}m_0c}{2eR}$

Ans.: (c)

Solution: $KE_{\max} = mc^2 - m_0c^2 = m_0c^2 \Rightarrow m = 2m_0$

$$\because m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 2m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow v = \frac{\sqrt{3}}{2}c$$

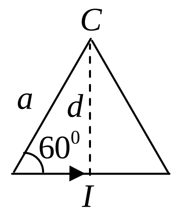
$$\because R = \frac{mv}{eB} \Rightarrow B = \frac{mv}{eR} = \frac{2m_0}{eR}\frac{\sqrt{3}}{2}c = \frac{\sqrt{3}m_0c}{eR}$$

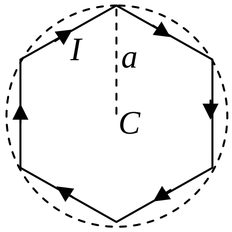
Q35. The strength of magnetic field at the center of a regular hexagon with sides of length a carrying a steady current I is:

(a) $\frac{\mu_0 I}{\sqrt{3}\pi a}$ (b) $\frac{\sqrt{6}\mu_0 I}{\pi a}$ (c) $\frac{3\mu_0 I}{\pi a}$ (d) $\frac{\sqrt{3}\mu_0 I}{\pi a}$

Ans.: (d)

$d = a\cos 30^0 = \frac{\sqrt{3}}{2}a$





$\because B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)$

$\Rightarrow B_1 = \frac{\mu_0 I}{4\pi d} 2\sin 30^0 = \frac{\mu_0 I}{4\pi \frac{\sqrt{3}}{2}a} 2\sin 30^0 = \frac{\mu_0 I}{2\sqrt{3}\pi a}$

$\Rightarrow B = 6B_1 = 6\times\frac{\mu_0 I}{2\sqrt{3}\pi a} = \frac{3\mu_0 I}{\sqrt{3}\pi a} = \frac{\sqrt{3}\mu_0 I}{\pi a}$

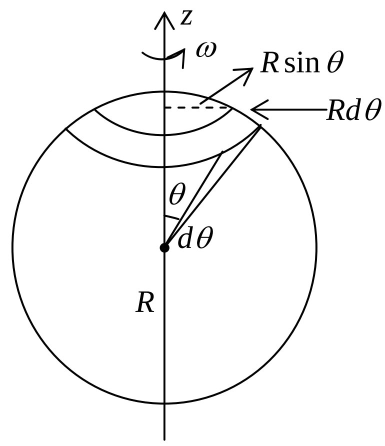
Q36. A spherical shell of radius R carries a constant surface charge density σ and is rotating about one of its diameters with an angular velocity ω. The magnitude of the magnetic moment of the shell is:

(a) $4\pi\sigma\omega R^4$ (b) $\frac{4\pi\sigma\omega R^4}{3}$ (c) $\frac{4\pi\sigma\omega R^4}{15}$ (d) $\frac{4\pi\sigma\omega R^4}{9}$

Ans. : (b)

Solution: The total charge on the shaded ring is

$$dq = \sigma(2\pi R\sin\theta)Rd\theta$$



Time for one revolution is $dt = \frac{2\pi}{\omega}$

$\Rightarrow$Current in the ring $I = \frac{dq}{dt} = \sigma\omega R^2 \sin\theta d\theta$

Area of the ring $= \pi(R\sin\theta)^2$, so the magnetic moment of the ring is

$$dm = (\sigma\omega R^2 \sin\theta d\theta)\times \pi R^2 \sin^2\theta$$

$$m = \sigma\omega R^4 \int_0^{\pi} \sin^3\theta d\theta = \frac{4}{3}\pi\times\sigma\omega R^4 \;\Rightarrow \vec{m} = \frac{4\pi}{3}\sigma\omega R^4 \hat{z}$$

Q37. The electric field $\vec{E} = E_0 \sin(\omega t - kz)\hat{x} + 2E_0 \sin\left(\omega t - kz + \frac{\pi}{2}\right)\hat{y}$ represents:

(a) a linearly polarized wave

(b) a right-hand circularly polarized wave

(c) a left-hand circularly polarized wave

(d) an elliptically polarized wave

Ans.: (d)

Q38. Suppose yz plane forms the boundary between two linear dielectric media I and II with dielectric constant $\in_I = 3$ and $\in_{II} = 4$, respectively. If the electric field in region I at the interface is given by $\vec{E}_I = 4\hat{x} + 3\hat{y} + 5\hat{z}$, then the electric field $\vec{E}_{II}$ at the interface in region II is:

(a) $4\hat{x} + 3\hat{y} + 5\hat{z}$ (b) $4\hat{x} + 0.75\hat{y} - 1.25\hat{z}$

(c) $-3\hat{x} + 3\hat{y} + 5\hat{z}$ (d) $3\hat{x} + 3\hat{y} + 5\hat{z}$

Ans.: (d)

Solution: $\because E^{\parallel}{}_I = E^{\parallel}{}_{II} \Rightarrow E^{\parallel}{}_{II} = 3\hat{y} + 5\hat{z}$ and $\frac{E^{\perp}{}_{II}}{E^{\perp}{}_I} = \frac{\varepsilon_I}{\varepsilon_{II}} \Rightarrow E^{\perp}{}_{II} = \frac{\varepsilon_I}{\varepsilon_{II}} E^{\perp}{}_I = \frac{3}{4} \cdot 4\hat{x} = 3\hat{x}$

$\Rightarrow \vec{E}_{II} = 3\hat{x} + 3\hat{y} + 5\hat{z}$

Q39. How much force does light from a 1.8 W laser exert when it is totally absorbed by an object?

(a) 6.0×10^{-9} N (b) 0.6×10^{-9} N (c) 0.6×10^{-8} N (d) 4.8×10^{-9} N

Ans: (a)

Solution: Radiation Pressure $\frac{F}{A} = \frac{I}{c} = \frac{P}{Ac} \Rightarrow F = \frac{P}{c} \Rightarrow F = \frac{1.8}{3 \times 10^8} = 6.0 \times 10^{-9}$ N

Q40. Self inductance per unit length of a long solenoid of radius R with n turns per unit length is:

(a) $\mu_0 \pi R^2 n^2$ (b) $2\mu_0 \pi R^2 n$ (c) $2\mu_0 \pi R^2 n^2$ (d) $\mu_0 \pi R^2 n$

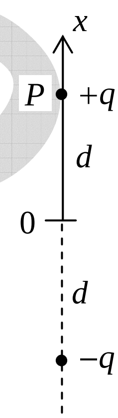
Ans.: (a)

Q41. A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane (ignore gravity). How long does it take for the charge to hit the plane?

(a) $\frac{\sqrt{2\pi^3\varepsilon_0 md^3}}{q}$ (b) $\frac{\sqrt{2\pi^3\varepsilon_0 md}}{q}$

(c) $\frac{\sqrt{\pi^3\varepsilon_0 md^3}}{q}$ (d) $\frac{\sqrt{\pi^3\varepsilon_0 md}}{q}$

Ans.: (a)



Solution: $F = ma = m\frac{d^2x}{dt^2} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d^2} \Rightarrow \frac{d^2x}{dt^2} = -\frac{A}{x^2}$ where $A = \frac{q^2}{16\pi m\varepsilon_0}$.

$$\Rightarrow \frac{dv}{dt} = -\frac{A}{x^2} \Rightarrow v\frac{dv}{dt} = -\frac{A}{x^2}\frac{dx}{dt} \Rightarrow \frac{1}{2}\frac{d}{dt}\left(v^2\right) = \frac{d}{dt}\left(\frac{A}{x}\right)$$

$$\Rightarrow \frac{v^2}{2} = \frac{A}{x} + C\text{, at } x = d,\ v = 0 \Rightarrow C = -\frac{A}{d} \Rightarrow v = \sqrt{2A}\sqrt{\left(\frac{1}{x} - \frac{1}{d}\right)}.$$

$$\Rightarrow -\frac{dx}{dt} = \sqrt{2A}\sqrt{\left(\frac{1}{x} - \frac{1}{d}\right)} \Rightarrow \int_d^0 \sqrt{\frac{xd}{d-x}}dx = -\sqrt{2A}\int_0^t dt$$

Put $x = d\sin^2\theta \Rightarrow dx = x = 2d\sin\theta\cos\theta d\theta$

$$\Rightarrow \int_{\pi/2}^{0}\sqrt{\frac{\left(d\sin^2\theta\right)d}{d\cos^2\theta}}2d\sin\theta\cos\theta d\theta = -\sqrt{2A}t$$

$$\Rightarrow -\sqrt{2A}t = \int_{\pi/2}^{0}\sqrt{d}\frac{\sin\theta}{\cos\theta}2d\sin\theta\cos\theta d\theta = 2d^{3/2}\int_{\pi/2}^{0}\sin^2\theta d\theta$$

$$\Rightarrow -\sqrt{2A}t = 2d^{3/2}\int_{\pi/2}^{0}\frac{(1-\cos 2\theta)}{2}d\theta = d^{3/2}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/2}^{0} = -d^{3/2}\frac{\pi}{2}$$

$$\Rightarrow -\sqrt{2A}t = -d^{3/2}\frac{\pi}{2} \Rightarrow -\sqrt{2\frac{q^2}{16\pi m\varepsilon_0}} \times t = -d^{3/2}\frac{\pi}{2}$$

$$\Rightarrow t = d^{3/2}\frac{\pi}{2} \times \sqrt{\frac{8\pi m\varepsilon_0}{q^2}} = \frac{\sqrt{2\pi^3\varepsilon_0 md^3}}{q}$$

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Q42. A plane electromagnetic wave propagating in air with $\vec{E}=\left(8\hat{i}+6\hat{j}+5\hat{k}\right)e^{i(\omega t+3x-4y)}$ is incident on a perfectly conducting slab positioned at $x=0$. $\vec{E}$ field of the reflected wave is

(a) $\left(-8\hat{i}-6\hat{j}-5\hat{k}\right)e^{i(\omega t+3x+4y)}$ (b) $\left(-8\hat{i}+6\hat{j}-5\hat{k}\right)e^{i(\omega t+3x+4y)}$

(c) $\left(-8\hat{i}+6\hat{j}-5\hat{k}\right)e^{i(\omega t-3x-4y)}$ (d) $\left(-8\hat{i}-6\hat{j}-5\hat{k}\right)e^{i(\omega t-3x-4y)}$

Ans. : (c)

Solution: For given $\vec{E}=\left(8\hat{i}+6\hat{j}+5\hat{k}\right)e^{i(\omega t+3x-4y)}$; $\hat{n}=\left(8\hat{i}+6\hat{j}+5\hat{k}\right)$ and $\vec{k}=3\hat{i}-4\hat{j}$

$\Rightarrow \vec{k}.\hat{n}=\left(3\hat{i}-4\hat{j}\right).\left(8\hat{i}+6\hat{j}+5\hat{k}\right)=24-24=0$

On a perfectly conducting slab wave is reflected so possible answer is (c) and (d)

(c) $\vec{E}_r=\left(-8\hat{i}+6\hat{j}-5\hat{k}\right)e^{i(\omega t-3x-4y)}$; $\hat{n}=\left(-8\hat{i}+6\hat{j}-5\hat{k}\right)$ and $\vec{k}=-3\hat{i}-4\hat{j}$

$\Rightarrow \vec{k}.\hat{n}=\left(-3\hat{i}-4\hat{j}\right).\left(-8\hat{i}+6\hat{j}-5\hat{k}\right)=24-24=0$

(d) $\vec{E}_r=\left(-8\hat{i}-6\hat{j}-5\hat{k}\right)e^{i(\omega t-3x-4y)}$; $\hat{n}=\left(-8\hat{i}-6\hat{j}-5\hat{k}\right)$ and $\vec{k}=-3\hat{i}-4\hat{j}$

$\Rightarrow \vec{k}.\hat{n}=\left(-3\hat{i}-4\hat{j}\right).\left(-8\hat{i}-6\hat{j}-5\hat{k}\right)=24+24=48\neq 0$

Correct answer is (c).

Q43. Consider magnetic vector potential $\vec{A}$ and scalar potential Φ which define the magnetic field $\vec{B}$ and electric field $\vec{E}$. If one adds $-\vec{\nabla}\lambda$ to $\vec{A}$ for a well-defined λ, then what should be added to Φ so that $\vec{E}$ remains unchanged up to an arbitrary function of time, $f(t)$?

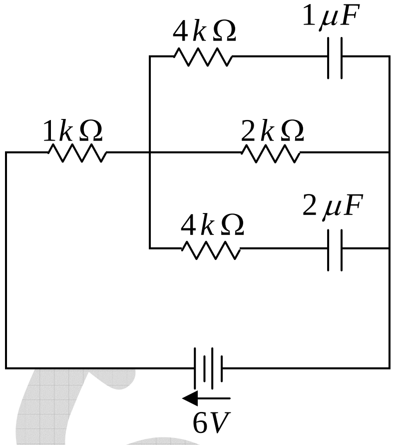
(a) $\frac{\partial\lambda}{\partial t}$ (b) $-\frac{\partial\lambda}{\partial t}$ (c) $\frac{1}{2}\frac{\partial\lambda}{\partial t}$ (d) $-\frac{1}{2}\frac{\partial\lambda}{\partial t}$

Ans. : (a)

Solution: Consider Gauge Transformation

$$\vec{A}'=\vec{A}-\vec{\nabla}\lambda=\vec{A}+\vec{\nabla}(-\lambda) \quad \text{and} \quad \Phi'=\Phi-\frac{\partial(-\lambda)}{\partial t}=\Phi+\frac{\partial\lambda}{\partial t}$$

Q44. Consider the following circuit in steady state condition. Calculate the amount of charge stored in $1\mu F$ and $2\mu F$ capacitors respectively.



(a) $4\mu C$ and $8\mu C$

(b) $8\mu C$ and $4\mu C$

(c) $3\mu C$ and $6\mu C$

(d) $6\mu C$ and $3\mu C$

Ans. : (a)

Solution: For DC voltage capacitors are open circuited.

Voltage across $2k\Omega$ resistance $V_{2k\Omega} = \frac{2}{(1+2)} \times 6V = 4V$

Amount of charge stored in $1\mu F$ is $Q_{1\mu F} = CV = 1\mu F \times 4V = 4\,\mu C$

Amount of charge stored in $2\mu F$ is $Q_{2\mu F} = CV = 2\mu F \times 4V = 8\,\mu C$

Q45. Two equal positive charges of magnitude $+q$ separated by a distance d are surrounded by a uniformly charged thin spherical shell of radius $2d$ bearing a total charge $-2q$ and centred at the midpoint between the two positive charges. The net electric field at distance r from the midpoint $(>> d)$ is

(a) zero

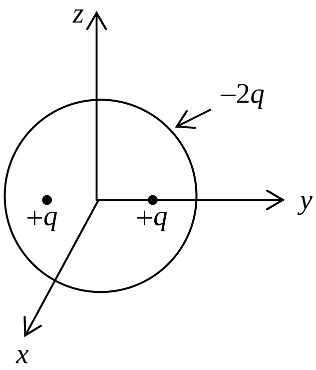
(b) proportional to d

(c) proportional to $1/r^3$

(d) proportional to $1/r^4$

Ans. : (d)

Solution: $Q_{mono} = q + q - 2q = 0$



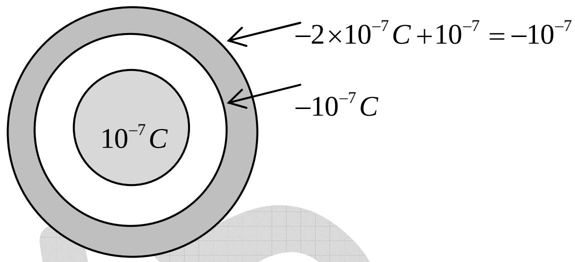
Since, the surface is symmetrical, so the net contribution by the $-2q$ charge in dipole moment vanishes, so

$\vec{p} = qd\hat{y} + q(-d\hat{y}) - 0 = 0$

$\Rightarrow V \propto \frac{1}{r^3}$ and $E \propto \frac{1}{r^4}$

Q46. A solid, insulating sphere of radius $1cm$ has charge $10^{-7}C$ distributed uniformly over its volume. It is surrounded concentrically by a conducting thick spherical shell of inner radius $2cm$, outer radius $2.5cm$ and is charged with $-2\times10^{-7}C$. What is the electrostatic potential in Volts on the surface of the sphere?

Ans. : 9000

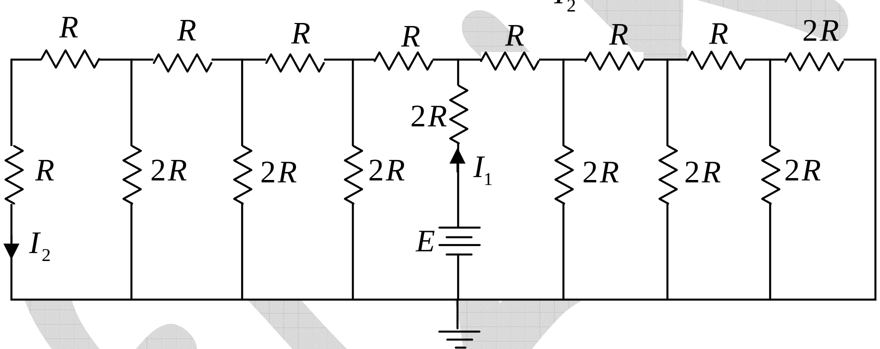


Solution: $V=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R_1}+\frac{q_2}{R_2}+\frac{q_3}{R_3}\right]$

$$\Rightarrow V=9\times10^9\left[\frac{10^{-7}}{1\times10^{-2}}+\frac{\left(-10^{-7}\right)}{2\times10^{-2}}+\frac{\left(-10^{-7}\right)}{2.5\times10^{-2}}\right]$$

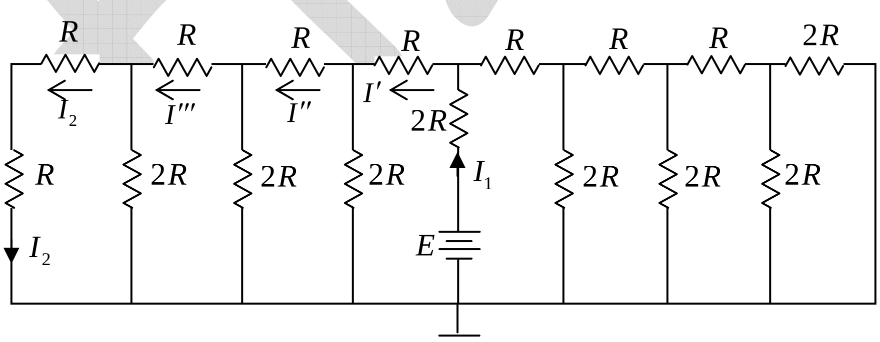
$$\Rightarrow V=9\times10^9\times10^{-5}\left[1-\frac{1}{2}-\frac{1}{2.5}\right]=9\times10^4\times0.1=9000\,Volts$$

Q47. For the circuit shown below, what is the ratio $\frac{I_1}{I_2}$?



Ans. : 16

Solution:



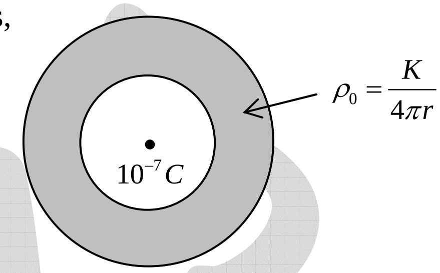
From voltage divider rule $I'=\frac{I_1}{2}$, $I''=\frac{I'}{2}=\frac{I_1}{4}$, $I'''=\frac{I''}{2}=\frac{I_1}{8}$ and $I_2=\frac{I'''}{2}=\frac{I_1}{16}$

$$\Rightarrow \frac{I_1}{I_2}=16$$

Q48. A sphere of inner radius $1cm$ and outer radius $2cm$, centered at origin has a volume charge density $\rho_0 = \dfrac{K}{4\pi r}$, where K is a non-zero constant and r is the radial distance. A point charge of magnitude $10^{-3}C$ is placed at the origin. For what value of K in units of C/m^2, the electric field inside shell is constant?

Ans. : 20

Solution: Electric field inside the sphere as a function of r is,



$$|E| \times 4\pi r^2 = \frac{1}{\varepsilon_0}\left[10^{-3} + \int_{10^{-2}}^{r} \frac{K}{4\pi r} 4\pi r^2 dr\right]$$

$$|E| = \frac{1}{4\pi r^2 \varepsilon_0}\left[10^{-3} + \frac{K}{2}\left(r^2 - 10^{-4}\right)\right]$$

Lets equate, $|E|_{r=1.5\times10^{-2}} = |E|_{r=2\times10^{-2}}$

$$\Rightarrow \frac{1}{4\pi \times 2.25\times10^{-4}\varepsilon_0}\left[10^{-3} + \frac{K}{2}\left(2.25\times10^{-4} - 10^{-4}\right)\right]$$

$$= \frac{1}{4\pi \times 4\times10^{-4}\varepsilon_0}\left[10^{-3} + \frac{K}{2}\left(4\times10^{-4} - 10^{-4}\right)\right]$$

$$\Rightarrow \frac{1}{2.25}\left[10^{-3} + \frac{K}{2}\left(1.25\times10^{-4}\right)\right] = \frac{1}{4}\left[10^{-3} + \frac{K}{2}\left(3\times10^{-4}\right)\right] \Rightarrow K = 20$$

Q49. Consider a grounded conducting plane which is infinitely extended perpendicular to the y-axis at $y = 0$. If an infinite line of charge per unit length λ runs parallel to x-axis at $y = d$, then surface charge density on the conducting plane is

(a) $\dfrac{-\lambda d}{\left(x^2 + d^2 + z^2\right)}$ (b) $\dfrac{-\lambda d}{\left(x^2 + d^2 + z^2\right)}$

(c) $\dfrac{-\lambda d}{\pi\left(x^2 + d^2 + z^2\right)}$ (d) $\dfrac{-\lambda d}{2\pi\left(x^2 + d^2 + z^2\right)}$

Ans. : (c)

Solution: Lets say the wire runs parallel to x- axis and directly above it, and the conducting plane is the x-z plane.

Potential of $+\lambda$ is $V_+ = -\dfrac{\lambda}{2\pi\varepsilon_0}\ln\left(\dfrac{r_+}{a}\right)$ and Potential of $-\lambda$ is $V_- = \dfrac{\lambda}{2\pi\varepsilon_0}\ln\left(\dfrac{r_-}{a}\right)$

Total potential $V(d, z) = \dfrac{2\lambda}{4\pi\varepsilon_0}\ln\left(\dfrac{r_-}{r_+}\right) = \dfrac{\lambda}{4\pi\varepsilon_0}\ln\left(\dfrac{r_-^2}{r_+^2}\right)$

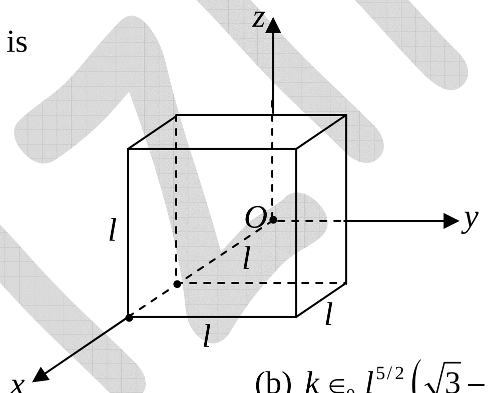
$$V(y,z) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{(y+d)^2+z^2}{(y-d)^2+z^2}\right\}$$

$\because \sigma = -\varepsilon_0 \dfrac{\partial V}{\partial n}$. Here, $\dfrac{\partial V}{\partial n} = \dfrac{\partial V}{\partial y}$, evaluated at y = 0

$$\sigma(z) = -\varepsilon_0 \frac{\lambda}{4\pi\varepsilon_0}\left\{\frac{1}{(y+d)^2+z^2}\times 2(y+d) - \frac{1}{(y-d)^2+z^2}\times 2(y-d)\right\}\Bigg|_{y=0}$$

$$\sigma(z) = \frac{-\lambda d}{\pi(z^2+d^2)}$$

Q50. For an electric field $\vec{E} = k\sqrt{x}\hat{x}$ where k is a non-zero constant, total charge enclosed by the cube as shown below is



(a) 0

(b) $k \in_0 l^{5/2}\left(\sqrt{3}-1\right)$

(c) $k \in_0 l^{5/2}\left(\sqrt{5}-1\right)$

(d) $k \in_0 l^{5/2}\left(\sqrt{2}-1\right)$

Ans. : (d)

Solution: $Q_{enc} = \varepsilon_0 \oint \vec{E}.d\vec{a}$

At $x = 2l;\ Q_{enc} = \varepsilon_0\left(k\sqrt{2l}\hat{x}\right).\left(l^2\hat{x}\right) = k\varepsilon_0 l^{5/2}\sqrt{2}$

At $x = l;\ Q_{enc} = \varepsilon_0\left(k\sqrt{l}\hat{x}\right).\left(-l^2\hat{x}\right) = -k\varepsilon_0 l^{5/2}$

At all other surface it will be zero. So, $Q_T = k\varepsilon_0 l^{5/2}\sqrt{2} - k\varepsilon_0 l^{5/2} = k\varepsilon_0 l^{5/2}\left(\sqrt{2}-1\right)$

JEST-2018

Q51. Two dielectric spheres of radius R are separated by a distance a such that $a >> R$. One of the spheres (sphere1) has a charge q and the other is neutral. If the linear dimensions of the systems are scaled up by a factor two, by what factor should be charge on the sphere 1 be changed so that the force between the two spheres remain unchanged?

(a) 2 (b) $4\sqrt{2}$ (c) 4 (d) $2\sqrt{2}$

Ans. : (c)

Solution: $F = \frac{kq \times q_1}{a^2} = k\frac{q' \times q_1}{(2a)^2} \Rightarrow q' = 4q$

Q52. An electric charge distribution produces an electric field

$$\vec{E} = \left(1 - e^{-\alpha r}\right)\frac{\vec{r}}{r^3}$$

where δ and α are constants. The net charge within a sphere of radius α^{-1} centered at the origin is

(a) $4\pi \in_0 \left(1 - e^{-1}\right)$ (b) $4\pi \in_0 \left(1 + e^{-1}\right)$

(c) $-4\pi \in_0 \frac{1}{\alpha e}$ (d) $4\pi \in_0 \frac{1}{\alpha e}$

Ans. : (a)

Solution: $Q_{enc} = \varepsilon_0 \oint \vec{E}.d\vec{a} = \varepsilon_0 \int_0^{\pi}\int_0^{2\pi} \left(1 - e^{-\alpha r}\right)\frac{\hat{r}}{r^2}.\left(r^2 \sin\theta d\theta d\phi \hat{r}\right) = \varepsilon_0 \left(1 - e^{-\alpha \times \frac{1}{\alpha}}\right) 4\pi$

$\Rightarrow Q_{enc} = 4\pi\varepsilon_0 \left(1 - e^{-1}\right)$

Q53. The charge density as a function of the radial distance r is given by

$$\rho(r) = \rho_0 \frac{R^2 - r^2}{R^2} \text{ for } r < R \text{ and zero otherwise.}$$

The electric flux over the surface of an ellipsoid with axes $3R, 4R$ and $5R$ centered at the origin is

(a) $\frac{4}{3 \in_0}\pi\rho_0 R^3$ (b) $\frac{8}{9 \in_0}\pi\rho_0 R^3$ (c) $\frac{8}{15 \in_0}\pi\rho_0 R^3$ (d) zero

Ans. : (a)

Solution: Electric flux $\oint \vec{E}.d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_0^R\int_0^\pi\int_0^{2\pi} \rho_0\left(\frac{R^2-r^2}{R^2}\right)\left(r^2\sin\theta drd\theta d\phi\right) = \frac{8}{15\varepsilon_0}\pi\rho_0 R^3$

Q54. An electromagnetic wave of wavelength λ is incident normally on a dielectric slab of thickness t. If K is the dielectric constant of the slab, the change in phase of the emergent wave compared with the case of propagation in the absence of the dielectric slab is

(a) $\sqrt{K}-1$ (b) 2π (c) $\frac{2\pi t}{\lambda}$ (d) $\frac{2\pi t}{\lambda}\left(\sqrt{K}-1\right)$

Ans. : (d)

Solution: Assume wave is propagating is positive z-direction:

In absence of the dielectric slab, phase $\phi_1 = kz - \omega t = \frac{\omega}{c}d - \omega t$ (let thickness is d)

In presence of the dielectric slab, phase $\phi_2 = kz - \omega t = \frac{\omega}{c/n}d - \omega t = \frac{\omega\sqrt{K}}{c}d - \omega t$

$\because n \approx \sqrt{K}$

Thus change is phase is $\phi_2 - \phi_1 = \frac{\omega}{c}d\left(\sqrt{K}-1\right) = \frac{2\pi}{\lambda}d\left(\sqrt{K}-1\right)$

Q55. Two parallel rails of a railroad track are insulated from each other and from the ground. The distance between the rails is 1 meter. A voltmeter is electrically connected between the rails. Assume the vertical component of the earth's magnetic field to the 0.2 gauss. What is the voltage developed between the rails when a train travels at a speed of 180 km/h along the track? Give the answer in milli-volts.

Ans. : 1.0

Solution: Induced emf $\varepsilon = Blv = \left(0.2\times10^{-4}\right)\times 1m\times 180\times\frac{10}{60\times60} = 10^{-3}volts = 1mV$

Q56. Two conductors are embedded in a material of conductivity 10^{-4} $ohm-m$ and dielectric constant $\in = 80\in_0$ The resistance between the two conductors is 10^6 ohm. What is the capacitance (in pF) between the two conductors? Ignore the decimal part of the answer.

Ans. : **Data insufficient**

Q57. An apparatus is made from two concentric conducting cylinders of radii a and b respectively, where $a < b$. The inner cylinder is grounded and the outer cylinder is at a positive potential V. The space between the cylinders has a uniform magnetic field H directed along the axis of the cylinders. Electrons leave the inner cylinder with zero speed and travel towards the outer cylinder. What is the threshold value of V below which the electrons cannot reach the outer cylinder?

(a) $\dfrac{eH^2\left(b^2-a^2\right)}{8mc^2}$ (b) $\dfrac{eH^2\left(b^2-a^2\right)^2}{8mc^2b^2}$

(c) $\dfrac{eH^2\left(b^2-a^2\right)^2}{8mc^2a^2}$ (d) $\dfrac{eH^2b\sqrt{\left(b^2-a^2\right)}}{8mc^2}$

Ans. : (b)

JEST-2019

Q58. A dc voltage of 80 Volt is switched on across a circuit containing a resistance of 5Ω in series with an inductance of $20H$. What is the rate of change of current at the instant when the current is $12A$?

(a) $0\,A/s$ (b) $1\,A/s$ (c) $5\,A/s$ (d) $80\,A/s$

Ans. : (b)

Solution: $i(t)=\dfrac{V}{R}\left(1-e^{-\frac{R}{L}t}\right)\Rightarrow \dfrac{di}{dt}=\dfrac{V}{L}e^{-\frac{R}{L}t}\Rightarrow \dfrac{di}{dt}=\dfrac{V}{L}\left(1-\dfrac{R}{V}i\right)$

$$\Rightarrow \frac{di}{dt}=\frac{80}{20}\left(1-\frac{5}{80}\times 12\right)=\frac{80}{20}\left(\frac{20}{80}\right)=1A/s$$

Q59. A very long solenoid (axis along z direction) of n turns per unit length carries a current which increases linearly with time, $i=Kt$. What is the magnetic field inside the solenoid at a given time t?

(a) $B=\mu_0 nKt\hat{z}$ (b) $B=\mu_0 nK\hat{z}$

(c) $B=\mu_0 nKt\left(\hat{x}+\hat{y}\right)$ (d) $B=\mu_0 cnKt\hat{z}$

Ans. : (a)

Q60. The magnetic field (Gaussian units) in an empty space is described by

$$B = B_0 \exp(ax)\sin(ky - \omega t)\hat{z}$$

What is the y - component of the electric field?

(a) $-\frac{ac}{\omega}B_0 \sin(ky - \omega t)$ (b) $-\frac{ac}{\omega}B_0 \exp(ax)\cos(ky - \omega t)$

(c) $-B_0 \sin(ky - \omega t)$ (d) 0

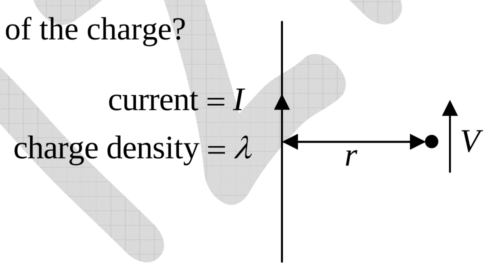
Ans. : (d)

Q61. Consider two concentric spherical metal shells of radii r_1 and $r_2\left(r_2 > r_1\right)$. The outer shell has a charge q and the inner shell is grounded. What is the charge on the inner shell?

(a) $\frac{r_1}{r_2}q$ (b) $\frac{r_1}{r_2}q$ (c) 0 (d) $\frac{r_2}{r_1}q$

Ans. : (a)

Q62. A wire with uniform line charge density λ per unit length carries a current I as shown in the figure. Take the permittivity and permeability of the medium to be $\varepsilon_0 = \mu_0 = 1$. A particle of charge q is at a distance r and is travelling along a trajectory parallel to the wire. What is the speed of the charge?



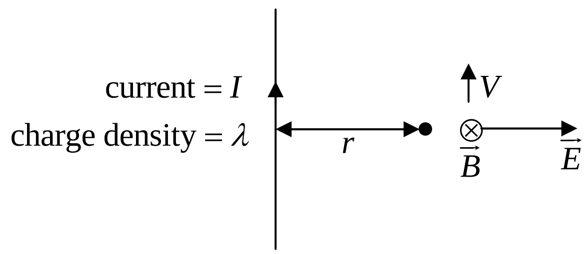
(a) $\frac{\lambda}{I}$ (b) $\frac{\lambda}{2I}$ (c) $\frac{\lambda}{3I}$ (d) $\frac{4\lambda}{I}$

Ans.: (a)

Solution: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ and $B = \frac{\mu_0 I}{2\pi r}$

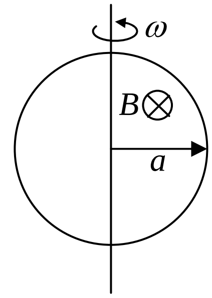
Directions are shown in the figure.

Net force on charge q is zero i.e. $\vec{F} = 0$.



$$\Rightarrow q\left[\vec{E} + \left(\vec{v} \times \vec{B}\right)\right] = 0 \Rightarrow E = vB \Rightarrow \frac{\lambda}{2\pi\varepsilon_0 r} = v\frac{\mu_0 I}{2\pi r} \Rightarrow v = \frac{\lambda}{I} \qquad \because \varepsilon_0 = \mu_0 = 1$$

Q63. A circular metal loop of radius $a = 1m$ spins with a constant angular velocity $\omega = 20\pi$ rad/s in a magnetic field $B = 3$ Tesla, as shown in the figure. The resistance of the loop is 10 ohms. Let P be the power dissipated in one complete cycle. What is the value of $\frac{P}{\pi^4}$ in Watts?



Ans. : 18

Solution: Magnetic flux through the loop is $\phi_m = \int_S \vec{B}.d\vec{a} = B \times \pi a^2 \times \cos\omega t$

Induced e.m.f $\varepsilon = -\frac{d\phi_m}{dt} = \omega B \times \pi a^2 \times \sin\omega t$.

Power dissipated $p = \frac{\varepsilon^2}{R} = \frac{\omega^2 B^2 \pi^2 a^4 \sin^2\omega t}{R}$

Power dissipated in one complete cycle $P = \langle p \rangle = \frac{\omega^2 B^2 \pi^2 a^4}{2R}$ $\quad \because \langle \sin^2\omega t \rangle = \frac{1}{2}$

$$\frac{P}{\pi^4} = \frac{\omega^2 B^2 a^4}{2\pi^2 R} \Rightarrow P = \frac{(20\pi)^2 (3)^2 (1)^4}{2(10)(10)} = 18$$