Atomic and Molecular Physics

JEST 2012

Q1. The binding energy of the hydrogen atom (electron bound to proton) is 13.6 eV. The binding energy of positronium (electron bound to positron) is

(a) 13.6 / 2 eV (b) 13.6 / 1810 eV

(c) 13.6 × 1810 eV (d) 13.6 × 2 eV

Ans.: (a)

Solution: $E_n' = -\frac{13.6}{n^2}\frac{\mu}{m_e}$

$$\mu = \frac{m_e \cdot m_e}{m_e + m_e} = \frac{m}{2}$$

$E_n' = -\frac{13.6}{n^2}\cdot\frac{m_e}{2m_e} = -\frac{13.6}{2}\times\frac{1}{n^2}$. Thus binding energy will be $\frac{13.6}{2}eV$

JEST 2013

Q2. A sodium atom in the first excited $3P$ states has a lifetime of $16ns$ for decaying to the ground $3S$ state. The wavelength of the emitted photon is 589 nm. The corresponding line width of the transition (in frequency units) is about

(a) 1.7 x 10^6 Hz (b) 1 x 10^7 Hz

(c) 6.3 x 10^7 Hz (d) 5 x 10^{14} Hz

Ans.: (c)

Solution: $\Delta E \cdot \Delta t = \hbar$

$$\Delta f = \frac{1}{\Delta t} = \frac{1}{16\times 10^{-9}} Hz = 6.25\times 10^7 Hz = 6.3\times 10^7 Hz$$

Q3. If a proton were ten times, the ground state energy of the electron in a hydrogen atom would be

(a) less (b) more

(c) the same (d) less, more or equal depending on the electron mass

Ans.: (b)

Solution: $E_n = \frac{-13.6}{n^2}\times\frac{0.99995\ m_e}{m_e} \Rightarrow -13.59932 \qquad \because \mu = 0.99995 m_e$

JEST 2014

Q4. The value of elastic constant for copper is about $100\,Nm^{-1}$ and the atomic spacing is $0.256\,\text{nm}$. What is the amplitude of the vibration of the Cu atoms at 300 K as a percentage of the equilibrium separation?

(a) 4.55 % (b) 3.55 % (c) 2.55 % (d) 1.55 %

Ans.: (b)

Solution: We know that, $E=\frac{1}{2}\beta A^2$ and $a=0.256\times10^{-9}\,m$

For one dimension, $K.E=\frac{1}{2}k_BT$ and $P.E=\frac{1}{2}k_BT$

$$\langle E\rangle = k_BT=\frac{1}{2}\beta A^2 \Rightarrow A=\sqrt{\frac{2k_BT}{\beta}}=\sqrt{\frac{2\times1.38\times10^{-23}\times300}{100}}=\sqrt{8.28\times10^{-23}}$$

$$=9.09\times10^{-12}=0.0090\ nm$$

Now, x of $0.256nm=0.009nm$

$$x=\frac{0.00909nm}{0.256nm}=0.03551=3.5\%$$

Q5. Which functional form of potential best describes the interaction between a neutral atom and an ion at large distances (i.e. much larger than their diameters)

(a) $V\propto -1/r^2$ (b) $V\propto -1/r$ (c) $V\propto -e^{-r/a}/r$ (d) $V\propto -1/r^3$

Ans.: (a)

Q6. If a proton were ten times lighter, then the ground state energy of the electron in a hydrogen atom would have been

(a) Less (b) More

(c) The same (d) Depends on the electron mass

Ans.: (b)

Solution: $$E_n=\frac{-13.6}{n^2}\frac{\mu}{m_e}=\frac{-13.6}{n^2}\times\frac{1.00545m_e}{m_e}=-13.526$$

$$\because m_p=\frac{M_p}{10},\ \mu=\frac{m_e\cdot m_p}{m_e+m_p}=1.00545\,m_e$$

Q7. If hydrogen atom is bombarded by energetic electrons, it will emit

(a) K_α X - rays (b) β -rays

(c) Neutrons (d) none of the above

Ans.: (d)

Q8. A hydrogen atom in its ground state is collided with an electron of kinetic energy 13.377 eV. The maximum factor by which the radius of the atom would increase is

(a) 7 (b) 8 (c) 49 (d) 64

Ans.: (c)

Solution: $E_n = \frac{-13.6}{n^2}\ eV$

$\Rightarrow E_1 = -13.6\ eV$, $E_2 = -3.4\ eV$, $E_3 = 1.5\ eV$, $E_4 = 0.85\ eV$, $E_5 = 0.54\ eV$

$E_6 = 0.3777\ eV$, $E_7 = 0.2775\ eV$

Since Electron have kinetic energy 13.377 $eV = -13.6 + 0.2775\ eV \Rightarrow n = 7$

$\because r_n = a_0 n^2 \Rightarrow r_n = 49 a_0$

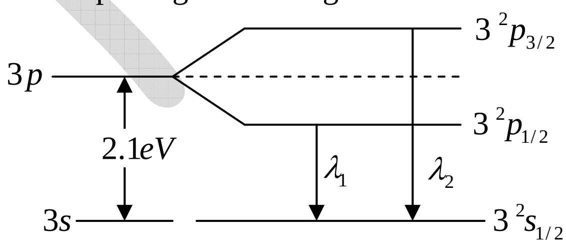
JEST 2015

Q9. The energy difference between the $3p$ and $3s$ levels in Na is $2.1\ eV$. Spin-orbit coupling splits the $3p$ level, resulting in two emission lines differing by $6\,\overset{o}{A}$. The splitting of the 3p level is approximately,

(a) $2\ eV$ (b) $0.2\,eV$ (c) $0.02\,eV$ (d) $2\,meV$

Ans: (d)

Solution: The fine structure splitting of Na in ground and excited state is



The transition $3\,{}^2p_{3/2} \to 3\,{}^2s_{1/2}$ produces photon of wavelength λ_2 and corresponding photon energy is $E_2 = \frac{12400}{\lambda_2\left(\overset{0}{\text{A}}\right)} eV$

The transition $3\,{}^2p_{1/2} \rightarrow 3\,{}^2s_{1/2}$ produces photon of wavelength λ_1 and corresponding photon energy is $E_1 = \frac{12400}{\lambda_1\left(\Lambda^0\right)} eV$ .The separation between ${}^2p_{3/2}$ and ${}^2p_{1/2}$ is

$$\Delta E = E_2 - E_1 = \frac{12400}{\lambda_2} - \frac{12400}{\lambda_1} = 12400\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = 12400\left(\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right)$$

Given $\Delta\lambda = \lambda_1 - \lambda_2 = 6A^0$ also fine structure splitting is of the order of $10^{-3} eV$.

Thus λ_1 and λ_2 are approximately same as λ corresponds to $3p \rightarrow 3s$, whereas wavelength (λ) corresponding to $3p \rightarrow 3s$ transition is $\lambda = \frac{12400}{2.1} A^0$

Thus, $\lambda_1\lambda_2 \cong \lambda^2 = \left(\frac{12400}{2.1}\right)^2$

$$\therefore \quad \Delta E = 12400 \times \frac{6}{\left(\frac{12400}{2.1}\right)^2} = \frac{2.1 \times 2.1 \times 6}{12400} \cong 2 \times 10^{-3}\, eV \Rightarrow \Delta E = 2\, meV$$

Q10. Which of the following excited states of a hydrogen atom has the highest lifetime?

(a) $2p$ (b) $2s$ (c) $3s$ (d) $3p$

Ans.: (b)

Solution: $2p$ and $3p$ are normal states. Electron from $2p$ and $3p$ make transition to ground state within 10^{-9} sec. Electron in $3s$ state although can not come directly to ground state but it can come to $1s$ through $2p$ as $3s \rightarrow 2p \rightarrow 1s$, while $2s$ is metastable state and electron in $2s$ state can make transition to $1s$ slowly. Thus $2s$ has long life time.

Q11. Which of the following statements is true for the energies of the terms of the carbon atom in the ground state electronic configuration $1s^2 2s^2 2p^2$?

(a) ${}^3P <{}^1D <{}^1S$ (b) ${}^3P <{}^1S <{}^1D$

(c) ${}^3P <{}^1F <{}^1S$ (d) ${}^3P <{}^1F <{}^1D$

Ans.: (a)

Solution: The spectroscopy terms for p^2 are ${}^1S_0, {}^1D_2, {}^3P_2$. According to Hund's rule, state with highest multiplicity lies lowest. Then, out of same multiplicity, state with highest L lies lowest. Thus these terms can be arranged as ${}^3P < {}^1D < {}^1S$

JEST 2016

Q12. The H_2 molecule has a reduced mass $M = 8.35\times10^{-28}\ kg$ and an equilibrium internuclear distance $R = 0.742\times10^{-10}\ m$. The rotational energy in terms of the rotational quantum number J is

(a) $E_{rot}(J) = 7J(J-1)\ meV$ (b) $E_{rot}(J) = \frac{5}{2}J(J+1)\ meV$

(c) $E_{rot}(J) = 7J(J+1)\ meV$ (d) $E_{rot}(J) = \frac{5}{2}J(J-1)\ meV$

Ans. : (c)

Solution: $E = \frac{\hbar^2}{2I}J(J+1)$,

where, $I = \mu r^2 = 8.35\times10^{-28}\,kg\times\left(0.742\times10^{-10}\,m\right)^2 = 4.597\times10^{-48}\,kgm^2$

$$\therefore \frac{\hbar^2}{2I} = \frac{\left(1.05\times10^{-34}\,J\cdot s\right)^2}{2\times\left(4.597\times10^{-48}\,kgm^2\right)} = \frac{1.112\times10^{-68}}{9.18\times10^{-48}}$$

$$= 1.21\times10^{-21}\,J = 1.21\times10^{-21}\times\frac{1}{1.6\times10^{-19}}eV = 7.57\times10^{-3}eV = 7.57\,meV$$

$$\therefore E \cong 7J(J+1)\,meV$$

Q13. If the Rydberg constant of an atom of finite nuclear mass is αR_∞, where R_∞ the Rydberg constant corresponding to an infinite nuclear mass, the ratio of the electronic to nuclear mass of the atom is:-

(a) $\frac{(1-\alpha)}{\alpha}$ (b) $\frac{(\alpha-1)}{\alpha}$ (c) $(1-\alpha)$ (d) $\frac{1}{\alpha}$

Ans. : (a)

Solution: $R_M = \frac{R_\infty}{1+\frac{m_c}{M}} \Rightarrow \alpha = \frac{1}{1+\frac{m_c}{M}} \Rightarrow 1+\frac{m_c}{M} = \frac{1}{\alpha} \Rightarrow \frac{m_c}{M} = \frac{1}{\alpha} - 1 = \frac{1-\alpha}{\alpha}$

JEST 2018

Q14. What is the difference between the maximum and the minimum eigenvalues of a system of two electrons whose Hamiltonian is $H = J\vec{S}_1.\vec{S}_2$, where $\vec{S}_1$ and $\vec{S}_2$ are the corresponding spin angular momentum operators of the two electrons?

(a) $\frac{J}{4}$ (b) $\frac{J}{2}$ (c) $\frac{3J}{4}$ (d) J

Ans. : (d)

Solution: $H = J\vec{S}_1.\vec{S}_2 = J\left(\frac{s^2 - s_1^2 - s_2^2}{2}\right) \Rightarrow J\left(\frac{s(s+1) - s_1(s_1+1) - s_2(s_2+1)}{2}\right)$

$s_1 = \frac{1}{2}, s_2 = \frac{1}{2} \quad s = 1, 0$

For $s = 1$ $E_f = J.\left(\frac{2\hbar^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2}\right) = \frac{J\hbar^2}{4}$

For $s = 0$ $E_g = J.\left(\frac{0\hbar^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2}\right) = -J\frac{3\hbar^2}{4}$

$E_f - E_g = \frac{J\hbar^2}{4} - \left(-\frac{3J\hbar^2}{4}\right) = \frac{4}{4}J\hbar^2 = J\hbar^2$

JEST 2019

Q15. Consider a hypothetical world in which the electron has spin $\frac{3}{2}$ instead of $\frac{1}{2}$. What will be the electronic configuration for an element with atomic number $Z = 5$?

(a) $1s^4, 2s^1$ (b) $1s^4, 2s^2, 2p^1$ (c) $1s^5$ (d) $1s^3, 2s^1, 2p^1$

Ans. : (a)

Solution: The degeneracy of level j is $d = 2j + 1$

For s - orbit, $d = 2s + 1 = 2\times\frac{3}{2} + 1 = 4$

$\therefore$ The electronic configuration for $z = 5$ is

$1s^4, 2s^1$. Thus correct option is (a)