D.U. M.Sc. 2017

Q1. The normalized wave functions ψ_1 and ψ_2 correspond to the ground state and the first excited states of a particle in a potential. The operators $\hat{A}$ act on the wave functions as:

$$\hat{A}\psi_1 = \psi_2 \text{ and } \hat{A}\psi_2 = \psi_1$$

The expectation value of the operator $\hat{A}$ for the state $\hat{A} = \frac{(3\psi_1 + 4\psi_2)}{5}$ is

(a) 0 (b) -0.32 (c) 0.75 (d) 0.96

Q2. The de-Broglie wavelength of a particle of mass m and charge q after being accelerated through a potential difference of V volts is:

(a) $\lambda = \frac{\hbar}{\sqrt{2mqV}}$ (b) $\lambda = \frac{\hbar}{\sqrt{mqV}}$ (c) $\lambda = \frac{\hbar m}{\sqrt{qV}}$ (d) $\lambda = \frac{\hbar}{qmV}$

Q3. The state of a hydrogen atom is described by the wave function

$$\psi = Ar^2 \exp\left(-\frac{r}{3a_0}\right)\left(3\cos^2\theta - 1\right)$$

Where A and a_0 are the normalization constant and Bohr radius, respectively. If one measures the z-component of the orbital angular momentum for this atom, the outcome will be:

(a) $3\hbar$ since ψ is proportional to $\exp\left(-\frac{r}{3a_0}\right)$

(b) $2\hbar$ since ψ depends on $\cos^2\theta$

(c) $1\hbar$ since ψ depends on $\cos^2\theta$

(d) $0\hbar$ since ψ does not depends on ϕ

Q4. An electron is confined within a region of size $< 10^{-15}\, m$ by an external potential. Then, the average kinetic energy of this electron must be:

(a) much greater than $0.511\, MeV$

(b) approximately equal to $0.511\, MeV$

(c) approximately equal to $0.511 \times 10^{-13}\, eV$

(d) approximately equal to $1\, eV$

Q5. An amplifier with initial open gain of 400 is used as negative feedback amplifier. The feedback factor is 0.05. If the gain of the amplifier charges by 10% due to temperature, then the closed loop gain changes approximately by:

(a) 0.05% (b) 0.1% (c) 0.5% (d) 1%

Q6. What is the value of the least significant bit of an 8-bit Digital to Analog Converter (DAC) for $0-12.8\,V$ range?

(a) $0.050\ V$ (b) $0.625\,V$ (c) $1.28\,V$ (d) $1.6\,V$

Q7. If the distance between two adjacent peaks on the x-axis is $2\,cm$ on a CRO screen in which the time/division control is set at $1\ ms/div$, then the frequency of the signal is:

(a) $1\,kHz$ (b) $5\,kHz$ (c) $50\,Hz$ (d) $500\,Hz$

Q8. The decimal equivalent of the hexadecimal number $(A3B)_{16}$ is:

(a) $(2361)_{10}$ (b) $(2621)_{10}$ (c) $(2619)_{10}$ (d) $(2879)_{10}$

Q9. Simplification of the following Boolean expression

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$ is given by:

(a) $\bar{A}\bar{C} + AC$ (b) $\bar{A}\bar{C}$ (c) C (d) $\bar{C}$

Q10. An astronaut in a spaceship has a mass $75\,kg$ in the rest frame of the spaceship. The spaceship's length in its rest frame is $1150\ m$ while its length as observed from the earth is $325\ m$. The total energy of the astronaut in the earth's frame will be approximately:

(a) $75\,J$ (b) $750\ J$

(c) $2.4\times10^{19}\ J$ (d) $6.8\times10^{18}\ J$

Q11. A thick steel rod rotates with an initial angular velocity $\vec{\omega}$ about one of its endpoints in a horizontal plane. If at time $t=0$ an ant having mass m starts moving outwards from the point of rotation with constant speed then for the system after time $t>0$:

(a) both the angular velocity and the kinetic energy decreases

(b) the angular velocity decreases but kinetic energy increases

(c) both the angular velocity and the kinetic energy remain same

(d) the angular velocity increases but kinetic energy decreases

Q12. A tunnel is dug from the surface of the earth through the center and opens at the other end. A ball is dropped from one end of the tunnel. The acceleration due to gravity on the earth's surface is g and the radius of the earth is R. Assuming that the earth has a constant density, the time taken by the ball to reach the center of the earth is:

(a) $\pi\sqrt{\frac{R}{g}}$ (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $\frac{\pi}{2}\sqrt{\frac{R}{g}}$ (d) $\frac{\pi}{4}\sqrt{\frac{R}{g}}$

Q13. A block of mass $250\,g$ is given an initial speed of $3\ m/s$ up a plane inclined at an angle of 30^0 with the horizontal. The coefficient of friction is $\sqrt{3}/6$. After $1.5\ s$, how far is the block from its original position?

(a) $0.9\ m$ (b) $2.4\ m$ (c) $3.8\ m$ (d) $6.5\ m$

Q14. The Sun is approximately $25,000$ light years away from the centre of the Milky Way and moves around it, in an approximately circular path, in roughly 170 million years. Let M be the mass of the galactic matter within a radius of $25,000$ light years from the galactic centre. Given that sunlight take approximately 8 min to reach the Earth, what is the ratio of M to the mass of the Sun?

(a) 1.5×10^{11} (b) 5×10^{11} (c) 1.5×10^{12} (d) 5×10^{12}

Q15. Two particles of mass m each are connected by a mass less spring of length l and spring constant k. The system is lying (in the relaxed state) on a frictionless horizontal table, A very short impulse P is applied to the first mass (along the axis of the system) at $t=0$. When would the second mass first come to rest?

(a) $t=\pi\sqrt{\frac{2m}{k}}$ (b) $t=2\pi\sqrt{\frac{2m}{k}}$ (c) $t=\pi\sqrt{\frac{m}{2k}}$ (d) $t=\pi\sqrt{\frac{m}{k}}$

Q16. A narrow light beam of wavelength λ is travelling along the x - axis, when it is faced by a barrier with narrow slits at $y=0,\pm d$ (and all at $x=0$). There is a vertical screen at $x=D>>d$. What is the angle θ that the central bright spot and the next-nearest dark spot subtend at the central slit $\left[(x,y)=(0,0)\right]$?

(a) $\theta=\sin^{-1}\frac{3\lambda}{d}$ (b) $\theta=\sin^{-1}\frac{2\lambda}{d}$ (c) $\theta=\sin^{-1}\frac{\lambda}{2d}$ (d) $\theta=\sin^{-1}\frac{\lambda}{d}$

Q17. Two tuning forks with natural frequencies v_1 and v_2, respectively, are struck at the same time with equal force. The intensity, of the resulting sound, beats with a frequency of $1\ Hz$ while the frequency of the sound is $512\ Hz$. Hence, v_1 and v_2, respectively, are:

(a) $512.0\ Hz$ and $511.5\ Hz$ (b) $512.0\ Hz$ and $511.0\ Hz$

(c) $512.5\ Hz$ and $511.0\ Hz$ (d) $512.5\ Hz$ and $511.5\ Hz$

Q18. During normal to superconducting transition:

(a) Entropy increases and thermal conductivity decreases

(b) Entropy and thermal conductivity decreases

(c) Entropy and thermal conductivity increases

(d) Entropy decreases and thermal conductivity increases

Q19. A silicon wafer of thickness $200\ \mu m$ is doped with 10^{23} phosphorus atoms per m^3. If the sample is kept in a field of 0.2 Weber/m^3 and a current of $1\ mA$ is passed through the sample, the Hall voltage produced is:

(a) $-6.25\ \mu V$ (b) $6.25\ \mu V$ (c) $-62.5\ \mu V$ (d) $62.5\ \mu V$

Q20. In the Kronig-Penney model, the discontinuities in the E vs k curve occur on either side of the centre for a k - value equal to (where $n = 1, 2, 3, ...$)

(a) $\frac{n\pi}{a}$ (b) $\frac{n\pi}{2a}$ (c) $\left(\frac{n\pi}{2a}\right)^{1/2}$ (d) $\frac{2n\pi}{a}$

Q21. The fraction of volume occupied by the atoms (hard spheres) in a simple cubic unit cell of lattice parameter a is:

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

Q22. The energy gap of Indium (In) which starts superconducting at $3.4\ K$ is;

(a) $2.07\ \mu eV$ (b) $2.07\ meV$ (c) $1.035\ \mu eV$ (d) $1.035\ meV$

Q23. The energies of the $4s$ and $4p$ states of potassium are $-4.339\,eV$ and $-2.73\,eV$ respectively. The ratio of Z_{eff} of $4s$ and $4p$ is:

(a) 0.89 (b) 1.26 (c) 1.78 (d) 2.18

Q24. In helium atom the total wave function of the ortho state is a:

(a) singlet with spin anti-symmetric wave functions

(b) singlet with spin symmetric wave functions

(c) triplet with spin anti-symmetric wave function

(d) triplet with spin symmetric wave function

Q25. The ^7Be nucleus undergoes decay to ^7Li. Which Beta decay process is possible in this decay? (Given $M(^7Be) = 7.016929\ amu$ and $M(^7Li) = 7.016004\ amu$)

(a) Beta minus decay only

(b) Beta plus decay only

(c) Both Beta plus and Electron capture

(d) Electron capture only

Q26. An unstable nucleus $^{44}_{20}Ca$ undergoes decay by positron emission. The daughter product of the decay is:

(a) $^{43}_{20}Ca$ (b) $^{43}_{19}K$ (c) $^{44}_{19}K$ (d) $^{45}_{20}Ca$

Q27. The metric tensor g_{ik}, is defined by $ds^2 = g_{ij}dx^i dx^j$. If

$$ds^2 = dr^2 + r^2 (d\theta)^2 + r^2 \sin^2\theta (d\phi)^2$$

and $\Gamma_{ij,k} = \frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^k}\right]$

then, $\Gamma_{22,1}$ is:

(a) $-r$ (b) r (c) r^2 (d) $2r$

Q28. The solution of $\frac{\partial u}{\partial t} = 4\frac{\partial u}{\partial x}$ is:

(a) Ae^{kx+t} (b) $Ae^{k\left(\frac{x}{2}+t\right)}$ (c) $Ae^{(x+t)}$ (d) $Ae^{k\left(\frac{x}{4}+t\right)}$

Q29. If C is the unit circle traversed, then the integral

$$\int_C \frac{dz}{z^2-3z}$$

is:

(a) 0 (b) $\frac{\pi}{3}i$ (c) $\frac{2\pi}{3}i$ (d) $-\frac{2\pi}{3}i$

Q30. The Fourier series of a periodic function (with period 2π) is given by:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos nx+b_n\sin nx\right)$$

If $f(x)$ is defined by :

$$f(x)=\begin{cases} x & \text{if} \quad -\pi/2<x<\pi 2 \\ \pi-x & \text{if} \quad \pi/2<x<3\pi/2 \end{cases}$$

then a_0 is equal to:

(a) 0 (b) 2π (c) 1 (d) -2π

Q31. Given the position vector $\vec{r}=x\hat{x}+y\hat{y}+z\hat{z}$ and $r^2=\vec{r}\cdot\vec{r}$, the value of $\vec{\nabla}\cdot\left(r^2\vec{r}\right)$ is given by:

(a) $6r^2$ (b) $5r^2$ (c) $4r^2$ (d) $3r^2$

Q32. The solution to the differential equation $\frac{dy}{dx}+2xy=x$ given that $y(x=0)=0$ is:

(a) $y=\left(1-e^{-x^3}\right)$ (b) $y=\frac{1}{4}\left(1-e^{-x^2}\right)$

(c) $y=\frac{1}{2}\left(1-e^{-x^2}\right)$ (d) $y=\frac{1}{2}\left(1-e^{-x}\right)$

Q33. If $A=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ and I is a 2×2 identity matrix, then e^A is:

(a) $I+\left(\frac{e^2-1}{2}\right)A$ (b) $I+\left(\frac{e-1}{2}\right)A$

(c) $I+\left(\frac{e^2+1}{2}\right)A$ (d) $I+\left(\frac{e^2+1}{2}\right)A$

Q34. If $z = x + iy$, then the real part of $\cos(z)$ is:

(a) $\cos(x)\cosh(y)$ (b) $\cos(x)\cos(y)$

(c) $\sin(x)\sinh(y)$ (d) $-\sin(x)\sinh(y)$

Q35. The heat extracted in each cycle from its heat source of a Carnot engine having an efficiency of 59% and performing $2.5\times10^4\ J$ of work in each cycle is:

(a) $4.24\times10^2\ J$ (b) $2.12\times10^2\ J$

(c) $4.24\times10^4\ J$ (d) $2.12\times10^4\ J$

Q36. The probability that an allowed state is occupied if it lies above the Fermi level by $2k_BT$ is

(a) 24% (b) 12% (c) 2.4% (d) 1.2%

Q37. One mole of a monoatomic perfect gas is initially at a constant temperature of T_0. It expands from a volume V_0 to $2V_0$ under constant pressure. The heat absorbed by the gas is:

(a) RT_0 (b) $2RT_0$ (c) $\frac{3}{2}RT_0$ (d) $\frac{5}{2}RT_0$

Q38. To a pure substance undergoing an infinitesimal reversible process. Which of the following relations is not a Maxwell equation?

(a) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ (b) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial V}{\partial S}\right)_P$

(c) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ (d) $\left(\frac{\partial T}{\partial P}\right)_S = -\left(\frac{\partial V}{\partial S}\right)_P$

Q39. A self-contained machine has, as its input, two equal steady streams of a fluid at temperatures T_1 and $4T_1$, respectively. The only output is a single high speed jet of the same fluid at temperatures T. The heat capacity per unit mass of the fluid is C (and is independent of the temperature). What is the maximum possible speed of the jet?

[Hint: The process is irreversible]

(a) $\sqrt{\frac{CT_1}{4}}$ (b) $\sqrt{4CT_1}$ (c) $\sqrt{2CT_1}$ (d) $\sqrt{CT_1}$

Q40. A freezer in a fridge is maintained at a temperature of $-15^0 C$ where the ambient air temperature is $30^0 C$. If heat leaks into the freezer at the constant rate of $1.75\ kJ/\sec$, then the power needed to pump this heat out continuously is:

(a) $-0.13\ kJ/\sec$ (b) $0.31 kJ/\sec$ (c) $1.49\ kJ/\sec$ (d) $2.06\ kJ/\sec$

Q41. Consider a random walker in one dimension. Assume that at every time step the walker takes steps of unit length to the left or right. The probability of taking a step to the left is $\frac{1}{3}$ and to the right is $\frac{2}{3}$. If the walker starts at $x = 0$, then the probability of its return to its starting point after four steps is given by:

(a) $\frac{2^2}{3^2}$ (b) $\frac{2^3}{3^3}$ (c) $\frac{2}{3^3}$ (d) $\frac{1}{3^3}$

Q42. Consider a system of 3 non-interacting identical bosons. Each particle can be in any one of three states whose energies are $-\varepsilon, 0, +\varepsilon$. Find the entropy of the system if its total energy $E = 0$.

(a) 0 (b) $k \ln 8$ (c) $k \ln 4$ (d) $k \ln 2$

Q43. A rectangular coil of length $0.05\ m$ and breadth $0.08\ m$ and having 420 turns is placed horizontally in a uniform magnetic field $B_1 = 0.14 T$ going into the plane of the coil. The magnetic field is increased over a 0.20 second interval and this induces an emf of $1.8\ V$ in the coil. The final magnetic field B_2 is approximately:

(a) $0.80\ T$

(b) $0.35\ T$

(c) $0.14\ T$

(d) Cannot be determined from the information supplied

Q44. Consider a plane electromagnetic wave travelling in free space. The magnetic field is given by $\vec{B} = B_1(\hat{x} - \hat{z})$ and electric field is given by $\vec{E} = E_1(-\hat{x} + 2\hat{y} - \hat{z})$. Then the direction of the propagation of the wave is:

(a) $-\hat{x} - \hat{y} - \hat{z}$ (b) $\hat{x} + 2\hat{y} - 2\hat{z}$ (c) $2\hat{x} + \hat{y} + 2\hat{z}$ (d) None of these

Q45. A particle of mass m and q is released from rest in a region with $\vec{E} = E_0\hat{x}$ and $\vec{B} = B_0\hat{x}$, where E_0, B_0 are constants. The particle's motion is best described by which of the following?

(a) It moves in a circle with a speed, $v = \frac{qE_0 t}{m}$

(b) It moves along the $\hat{x}$ axis with an acceleration $\frac{qE_0}{m}$

(c) It moves in a helical path in the $x - z$ plane with an increasing radius

(d) It moves along the $\hat{z}$ direction with an acceleration $\frac{qvE_0}{m}$

Q46. A sphere of radius a has a charge density which varies with distance as $\rho = A\sqrt{r}$. The electric field at a distance $r < a$ varies with distance r as:

(a) $E \propto r^{3/2}$ (b) $E \propto r^{1/2}$ (c) $E \propto r^{-1/2}$ (d) $E \propto r^{-2}$

Q47. A long cylindrical conductor kept along the z -axis carries a current density $\vec{J} = j_0 r\hat{z}$, where j_0 is a constant, and r is the radial distance from the axis of the cylinder. The magnetic induction $\vec{B}$ inside the conductor at a distance d from the axis of the cylinder is:

(a) $\mu_0 \hat{J}_0 \hat{\phi}$ (b) $-\frac{1}{2}\mu_0 \hat{J}_0 d\hat{\phi}$ (c) $\frac{1}{3}\mu_0 j_0 d^2\hat{\phi}$ (d) $-\frac{1}{4}\mu_0 j_0 d^3\hat{\phi}$

Q48. Which of the following statements is true for a series LCR circuit with an AC source (where R is a resistance, L is an inductor and C is a capacitor):

(a) The impedance offered by the capacitor decreases as the frequency of the AC voltage increases

(b) The voltage and current are always in phase

(c) The total impedance of the circuit will be less than R

(d) The total impedance of the circuit approaches R as the frequency become very small

Q49. What is the direction of the displacement current inside a discharging cylindrical capacitor having circular positive and negative plates parallel to reach other?

(a) From positive plate to the negative plate

(b) Directed towards the axis of the capacitor

(c) Directed away from the axis of the capacitor

(d) From the negative plate to the positive plate

Q50. Two dice are thrown together. Let S be the sum of the two outcomes. Which of the following statements is false?

(a) The probability that $S=1$ is 0

(b) The probability that $S=6$ is $\frac{1}{6}$

(c) The probability that $S=2$ is $\frac{1}{36}$

(d) The probability that $S=3$ is $\frac{1}{18}$