D U PhD 2017

Q1. A harmonic oscillator is in the energy eigenstate $|n\rangle$. A time independent perturbation $\lambda\left(a^{\dagger}a\right)^2$ is applied to it, where λ a constant of is suitable dimension and $a, a^{\dagger}$ are the lowering and raising operators. Then the first order energy shift is given by:

(a) λn (b) $2\lambda n$ (c) λn^2 (d) $-\lambda^2 n$

Q2. To a particle with charge q and mass m in a 1-D harmonic oscillator potential of angular frequency ω, a weak electric field is applied, which changes its potential energy by $-qEx$. The lowest order non-zero correction to ground state energy is:

(a) $\frac{qE^2}{2m\omega^2}$ (b) $\frac{q^2E^2}{m\omega^2}$ (c) $\frac{q^2E^2}{2m\omega^2}$ (d) $-\frac{q^2E^2}{2m\omega^2}$

Q3. The Hamiltonian in the matrix form is:

$$H = V_0\begin{pmatrix} 1-\in & 0 & 0 \\ 0 & 1 & \in \\ 0 & \in & 2 \end{pmatrix}$$

Using degenerate perturbation theory one obtains the first order correction to the two initially degenerate eigenvalues to be

(a) $(-V_0 \in, 0)$ (b) $(V_0 \in, 0)$ (c) $(V_0 \in, V_0 \in)$ (d) $(0,0)$

Q4. Given the trial wave function $\psi_{trial} = A\exp\left(-\alpha r^2\right)$ and the ground state energy E_0 of the hydrogen atom. The expected value $\langle E\rangle = \frac{\langle\psi|H|\psi\rangle_{trial}}{\langle\psi|\psi\rangle_{trial}}$ is given by:

(a) $\langle E\rangle = E_0$ (b) $\langle E\rangle < E_0$ (c) $\langle E\rangle > E_0$ (d) $\langle E\rangle = -E_0$

Q5. The Hamiltonian assumed by Dirac is

$$H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2$$

Where c is the speed of light $\vec{p}$ is the momentum and m is the rest mass. If $\vec{\alpha}$, β, I are constant matrices independent of space, time and momentum, then the relations satisfied by $\vec{\alpha}$, β, I are:

(a) $\alpha_i\beta + \beta\alpha_i = 0$ and $\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}I$

(b) $\alpha_i\beta - \beta\alpha_i = 0$ and $\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}I$

(c) $\alpha_i\beta + \beta\alpha_i = 0$ and $\alpha_i\alpha_j - \alpha_j\alpha_i = 2\delta_{ij}I$

(d) $\alpha_i\beta - \beta\alpha_i = 0$ and $\alpha_i\alpha_j - \alpha_j\alpha_i = 2\delta_{ij}I$

Q6. An electron of charge e and mass m moves under the influence of an electromagnetic field given by $\vec{A} = \lambda x\hat{y}$, where λ is a constant. If n is a positive integer, the energy eigenvalues of the electron are:

(a) $\frac{P_z^2}{2m} + \left(n+\frac{1}{2}\right)\hbar\frac{\lambda|e|}{m}$

(b) $\frac{P_x^2}{2m} + \left(n+\frac{1}{2}\right)\hbar\frac{\lambda|e|}{m}$

(c) $\frac{P_y^2}{2m} + \left(n+\frac{1}{2}\right)\hbar\frac{\lambda|e|}{m}$

(d) $\frac{P_z^2}{2m} + \frac{P_y^2}{2m}\left(n+\frac{1}{2}\right)\hbar\frac{\lambda|e|}{m}$

Q7. The commutator of two Hermitian operators A and B is given by $[A,B] = \lambda$, where λ is a constant. Then $e^{A}Be^{-A}$ is:

(a) $e^{B+\lambda}$ (b) $e^{B}+\lambda$ (c) $B+\lambda$ (d) $A+\lambda$

Q8. The Hamiltonian for a quantum system is given by $H = A^2 + B^2$ where A and B are Hermitian operators. If $[A,B] = -i\alpha I$, where $\alpha > 0$ and I the identity operator. Then which of the following statements is true?

(a) The energy eigenvalues of the system are positive but the spacing between them is non-uniform.

(b) All the energy eigenstates of the system are degenerate with degeneracy 2

(c) The energy eigenvalues of the system are strictly negative.

(d) There exists some λ for which $A|\psi_0\rangle = \lambda B|\psi_0\rangle$, where $|\psi_0\rangle$ is the ground state of the system.

Q9. In a Hydrogen atom (Bohr radius a), consider the proton to be a uniformly charged thin spherical shell of radius $R << a$. What is the lowest order change in the binding energy (as compared to the case of a point-particle proton)?

(a) $\frac{2e^2R}{4\pi\epsilon_0 a^2}$ (b) $\frac{2e^2R^2}{12\pi\epsilon_0 a^3}$ (c) $\frac{3e^2R^4}{16\pi\epsilon_0 a^4}$ (d) $\frac{e^2}{8\pi\epsilon_0 R}$

Q10. The atomic density (number of atoms per unit area) in the $[111]$ plane of copper (FCC) with lattice parameter of $3.6\overset{0}{A}$ is:

(a) 0.568×10^{19} (b) 1.780×10^{19} (c) 2.200×10^{19} (d) 5.278×10^{19}

Q11. The energy gap of germanium is $0.75\,eV$. The fraction of electrons excited into its conduction band at room temperature ($300\,K$) is :

(a) 1.1×10^{-7} (b) 4.9×10^{-7} (c) 6.1×10^{-7} (d) 9.1×10^{-7}

Q12. The fraction of volume occupied by the atoms (hard spheres) in a body-cantered cubic unit cell of lattice parameter a is:

(a) $\frac{\sqrt{2}\pi}{4}$ (b) $\frac{\sqrt{2}\pi}{8}$ (c) $\frac{\sqrt{3}\pi}{4}$ (d) $\frac{\sqrt{3}\pi}{8}$

Q13. The ratio of electron densities in the conduction bands of Germanium (Ge) $(E_g = 0.7\,eV)$ and silicon (Si) $(E_g = 1.14\,eV)$ at 127^0C is:

(a) 1587 (b) 794 (c) 588 (d) 294

Q14. The Lagrangian of a system is, $L = q\dot{q}^2 - \dot{q}\sin(q)$. The corresponding Hamiltonian then, given by:

(a) $(p+\sin q)^2$ (b) $\frac{1}{q}(p+\sin q)^2$

(c) $\frac{1}{4q}(p+\sin q)^2$ (d) $\frac{1}{2q}p^2$

Q15. The Lagrangian of a system is given to be,

$$L = \frac{1}{2}\dot{q}^2 - \dot{q}\exp(-q)$$

The corresponding Hamilton-Jacobi equation is:

(a) $\frac{\partial S}{\partial t} + \left(\frac{\partial S}{\partial q} - \exp(-q)^2\right) = 0$

(b) $\frac{\partial S}{\partial t} + \frac{1}{2}\left(\frac{\partial S}{\partial q} + \exp(-q)^2\right) = 0$

(c) $\frac{\partial S}{\partial t} + \frac{1}{2}\left(\frac{\partial S}{\partial q} + 2\exp(-q)^2\right) = 0$

(d) $\frac{\partial S}{\partial t} + \frac{1}{2}\left(\frac{\partial S}{\partial q} + \exp(-q)\right) = 0$

Q16. Given the position vector $\vec{r} = x_1\hat{x} + x_2\hat{y} + x_3\hat{z}$ and the canonical momentum vector $\vec{p} = p_1\hat{x} + p_2\hat{y} + p_3\hat{z}$, along with two constant vectors $\vec{u}$ and $\vec{w}$, the Poisson bracket $[\vec{r}.\vec{u}.\vec{p}.\vec{w}]$ is equal to:

(a) $\frac{1}{2}(\vec{u}.\vec{w})(\vec{r}.\vec{p})$

(b) $(\vec{u}.\vec{w})(\vec{r}.\vec{p})$

(c) $(\vec{u}.\vec{p})(\vec{r}.\vec{w})$

(d) $\vec{u}.\vec{w}$

Q17. A particle of mass m moves on a plane under the influence of a potential given by $V = \frac{(a + b\dot{r}^2)}{r}$, where $\dot{r} = \frac{dr}{dt}$ and a, b are constants, which of the following statements is true?

(a) Angular momentum is conserved but energy is not conserved.

(b) Linear momentum along the radial direction is conserved.

(c) Angular momentum is not conserved.

(d) Both energy and angular momentum are conserved.

Q18. A particle in three dimensions moves under the influence of a potential $V(r) = -\frac{k}{r^n}$, where k a positive constant is and n a non-zero integer. The particle can have stable circular orbits:

(a) For $n < 2$

(b) Only for $n = 2$

(c) For $n > 3$

(d) Only for $n = 3$

Q19. A solid sphere of radius R is rotating with a constant angular velocity ω. It is then lowered onto a horizontal plane with zero horizontal velocity such that the axis of rotation is parallel to the plane. The coefficient of friction between the sphere and the plane is μ. The time after which the sphere starts rolling without slipping is:

(a) $\frac{1}{3}\frac{R\omega}{\mu g}$ (b) $\frac{1}{6}\frac{R\omega}{\mu g}$ (c) $\frac{2}{7}\frac{R\omega}{\mu g}$ (d) $\frac{2}{5}\frac{R\omega}{\mu g}$

Q20. A particle of mass m and charge q having initial velocity v (at infinite distance) collides head on with an identical particle initially at rest. The distance of closet approach between the two particles, classically, will be:

(a) $\frac{4}{mv^2}\left(\frac{q^2}{4\pi\in_0}\right)$ (b) $\frac{3}{mv^2}\left(\frac{q^2}{4\pi\in_0}\right)$

(c) $\frac{2}{mv^2}\left(\frac{q^2}{4\pi\in_0}\right)$ (d) $\frac{1}{mv^2}\left(\frac{q^2}{4\pi\in_0}\right)$

Q21. A particle of mass m is bounded by a linear potential $V = kr$. If the orbit of the particle is a circle of radius r about the origin, the energy of the particle is:

(a) kr (b) $2kr$ (c) $\frac{2kr}{3}$ (d) $\frac{3kr}{2}$

Q22. In CE configuration of a bi-junction transistor, the voltage drop across a $5k\Omega$ resistor connected in the collector circuit is $5V$. If $\beta = 50$ (β is the small signal current gain), the value of the base current is:

(a) $0.02\ A$ (b) $0.02\ mA$ (c) $0.02\ \mu A$ (d) $0.02\ nA$

Q23. When a reverse gate voltage of $20V$ is applied to a field effect transistor (FET), the gate current is $1.6\times10^{-3}\mu A$. The resistance between the gate and the source is:

(a) 1250Ω (b) $12,500\ \Omega$ (c) $12,500\ k\Omega$ (d) $12,500\ M\Omega$

Q24. The signals applied to the inverting and non-inverting terminals of a differential amplifier are respectively $-0.04\ mV$ and $-0.42\ mV$. The differential gain A_d and the common mode rejection ratio (CMRR) are 10^5 and $80\,dB$, respectively. The total output voltage is:

(a) $-2.004\,mV$ (b) $+2.004\,mV$ (c) $-2.004\ V$ (d) $+2.004\ V$

Q25. The high input impedance of a field effect transistor (FET) is due to:

(a) its very low gate current.

(b) the pinch-off voltage.

(c) the source and drain being far apart.

(d) the geometry of FET.

Q26. When heat Q flows into a monoatomic gas, the volume increases keeping the pressure constant. The fraction of the heat energy used for the gas expansion is:

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{2}{3}$ (d) $\frac{2}{5}$

Q27. The amount of heat that must be added to a system at 27^0C for the number of accessible states to increase by a factor of 10^8 is:

(a) $7.63\times10^{-18}\ J$ (b) $7.63\times10^{-20}\ J$

(c) $7.63\times10^{-26}\ J$ (d) $7.63\times10^{-31}\ J$

Q28. The number of ways of removing n atoms from N sites of a perfect crystal having N lattice sites and M interstitial sites is:

(a) $\frac{n!(N-n)!}{N!}$ (b) $\frac{N!}{n!(M-n)!}$

(c) $\frac{M!}{n!(N-n)!}$ (d) $\frac{N!}{n!(N-n)!}$

Q29. Consider a gas of identical but distinguishable quantum particles of mass m each and subjected to the three-dimensional potential

$$V(\vec{r})=\frac{1}{2}m\omega^2\vec{r}^2$$

Where ω is a constant. The gas is in thermal equilibrium. What must the temperature be so that the number of particles in the first excited state is equal to that in the second excited state?

(a) $T=0$ (b) $T=\infty$ (c) $T=\frac{\hbar\omega}{k_B\ln 2}$ (d) $T=\frac{2\hbar\omega}{k_B\ln 2}$

Q30. A system consists of two non-interacting particles and energy levels $E_n = n\in, n = 0,1,2,..$ The $n-$th energy level has a degeneracy of $g(n) = 2n+1$. If the particles are spin zero bosons, then the total number of microstates having total energy $N\in$ ($N\equiv$ odd integer, also given that $\sum_0^N n^2 = \frac{N(N+1)(2N+1)}{6}$) is:

(a) $\frac{N^3}{3}+2N^2+\frac{5}{3}N+1$

(b) $\frac{N^3}{3}+N^2+\frac{7}{6}N+\frac{1}{2}$

(c) $\frac{N^3}{3}+N^2+\frac{5}{3}N$

(d) $\frac{N^3}{3}+2N^2+\frac{7}{3}N$

Q31. The Hamiltonian for a system is given by $H = \varepsilon_0\begin{pmatrix} 0 & 1+i \\ 1-i & 0 \end{pmatrix}$. The single particle partition function is given by:

(a) $2\cosh\left(\beta\varepsilon_0\sqrt{2}\right)$

(b) $2\sinh\left(\beta\varepsilon_0\sqrt{2}\right)$

(c) $2\cosh\left(\beta\varepsilon_0\sqrt{2}\right)+\sinh\left(\beta\varepsilon_0\sqrt{2}\right)$

(d) $\cosh\left(\beta\varepsilon_0\sqrt{2}\right)-\sinh\left(\beta\varepsilon_0\sqrt{2}\right)$

Q32. The number of photons per second emitted by a $7.5\ mW$ CO_2 laser having a wavelength of $10.6\ \mu m$ is approximately:

(a) 8×10^{17} (b) 4×10^{17} (c) 2×10^{17} (d) 1×10^{15}

Q33. Which of the following states is NOT possible for a two electron configuration $3d^1 4d^1$:

(a) ${}^1S_0$ (b) ${}^2D_2$ (c) ${}^3D_2$ (d) ${}^1F_3$

Q34. The interatomic potential between H-atomshas a range of approximately $4\overset{0}{A}$. If a gas of H-atoms is in thermal equilibrium, what is the temperature T (in Kelvin) below which atom-atom scattering is overwhelmingly dominated by the s-wave amplitude?

(a) 30 (b) 20 (c) 10 (d) 1

Q35. The gap between the ${}^1p_{3/2}$ and ${}^1d_{5/2}$ neutron shells for nuclei with mass $A\simeq16$ is (Given $B.E.\left({}^{15}O\right)=111.9556\ MeV, B.E.\left({}^{16}0\right)=127.6193\ MeV, B.E.\left({}^{17}O\right)=131.76237\ MeV$)

(a) 19.8 MeV (b) 15.7 MeV (c) 11.5 MeV (d) 4.1MeV

Q36. If the mean range of 10 MeV protons in lead (Pb) is 0.316 mm, the mean range of 20 MeV deuterons and 40 MeV alpha particles are:

(a) 0.316 mm and 2.538 mm (b) 0.447 mm and 0.316 mm

(c) 0.632 mm and 0.316 mm (d) 0.632 mm and 1.264 mm

Q37. In the Geiger Muller (GM) region, when the applied voltage is increased, which of the following happens:

(a) The pulse amplitude increases but the counting rate remains nearly constant.

(b) The pulse amplitude remains nearly constant .and the counting rate increases.

(c) Both the pulse amplitude and counting rate increases.

(d) Both the pulse amplitude and counting rate remain nearly constant

Q38. A radioactive source undergoes positron decay and these positrons then get annihilated. The resulting photons undergo Compton scattering. The maximum kinetic energy of the Compton scattered electrons, approximately, is:

(a) 171 keV (b) 256 keV (c) 341 keV (d) 511 keV

Q39. Which of the following statements is correct concerning the group of continuous rotations R in 3-dimensions?

(a) R is a non-Abelian group and the subgroup consisting of all rotations about y-axis is Abelian.

(b) R is a non-Abelian group with a non-Abelian subgroup of R consisting of all rotations about the z-axis.

(c) R is a non-Abelian group and it does not any Abelian subgroup

(d) R is a non-Abelian group with a non-Abelian subgroup of R consisting of all rotations about the unit vector $\frac{1}{3}(\hat{x}+\hat{y}+\hat{z})$

Q40. The value of the integral $\int_{-\infty}^{\infty} \frac{x\sin(kx)}{x^2+1} dx, k \equiv \text{costant}$ is:

(a) πe^{-k} (b) πe^{k} (c) $2\pi e^{-k}$ (d) $\pi e^{k/2}$

Q41. If $\left[x\right]$ denotes the largest integer less than or equal to x, and $\{x\}=x-[x], \int_1^n [x]\{x\}\,dx$ equal.

(a) n^2 (b) $\frac{n(n-1)}{4}$ (c) $\frac{(n+1)}{2}$ (d) $\frac{(n+1)(n+2)}{2}$

Q42. If the Fourier transform of e^{-ax^2} is $\frac{e^{-k^2/4a}}{\sqrt{a}}$, then the Fourier transform of $f(x)=x^2e^{-ax^2}$ is:

(a) $F(k)=\frac{-k^2+2a}{4a^{5/2}}e^{-k^2/4a}$ (b) $F(k)=\frac{k^2+2a}{4a^{3/2}}e^{-k^2/4a}$

(c) $F(k)=\frac{-k^2+2a}{4a^{3/2}}e^{-k^2/4a}$ (d) $F(k)=\frac{k^2-2a}{4a^{5/2}}e^{-k^2/4a}$

Q43. The solution to the differential equation $\frac{dy}{dx}+2xy=x$ given that for $y(x=0)=0$ is:

(a) $y=\frac{1}{2}\left(1-e^{-x}\right)$ (b) $y=\frac{1}{2}\left(1-e^{-x^2}\right)$

(c) $y=\frac{1}{4}\left(1-e^{-x^2}\right)$ (d) $y=\left(1-e^{-x^3}\right)$

Q44. A mobile phone has dimensions $\sim 10\ cm$. The frequency of the GSM signal used by the phone is $2\,GHz$. Two people A and B are at a distance of $15\ cm$ and $3\,m$ respectively from the phone. Which of the following statements is true?

(a) A is not in the radiation zone limit while B is in the radiation zone limit

(b) A is in the radiation zone limit while B is not in the radiation zone limit

(c) Neither A nor B are in the radiation zone limit

(d) Both A and B are in the radiation zone limit.

Q45. Consider a parallel plate capacitor with square plates of dimensions $L \times L$ each. The plates have a charge Q and are separated by a distance Δx. The plate with the positive charge has a small hole in the middle through which an electron of mass m and charge $-e$ is shot through. The minimum speed that the electron must have to reach the negative plate is:

(a) $v = \sqrt{\dfrac{eQ\Delta x}{m \in_0 L^2}}$ (b) $v = \sqrt{\dfrac{e\,Q\,\Delta x}{m \in_0 L^2}}$ (c) $v = \left(\dfrac{-Q\,e\,\Delta x}{m \in_0 L^2}\right)^2$ (d) $v = \dfrac{e\,Q^2\Delta x}{m \in_0 L^2}$

Q46. Two particles of mass m_1 and m_2 move along thee x axis with relativistic speeds u_1 and u_2, respectively $(u_1 > u_2)$. They collide and stick together. The speed of the resulting particle is $\left(\gamma = \sqrt{\dfrac{1}{1-\beta^2}}, \beta = \dfrac{u}{c}\right)$:

(a) $U = \dfrac{m_1\gamma_1 u_1 - m_2\gamma_2 u_2}{m_1\gamma_1 - m_2\gamma_2}$

(b) $U = \dfrac{m_1\gamma_1 u_1 + m_2\gamma_2 u_2}{m_1\gamma_1 - m_2\gamma_2}$

(c) $U = \dfrac{m_1\gamma_1 u_1 + m_2\gamma_2 u_2}{m_1\gamma_1 + m_2\gamma_2}$

(d) Cannot be determined from the information supplied

Q47. In an inertial frame S, the ratio of the magnitude of the electric field $\vec{E}$ and magnetic field $\vec{B}$ is 2:1 and the angle between them is 45 degrees. We make a Lorentz transformation to another frame S'. Which of the following statements can be true for the electric field $\vec{E}$, and magnetic field $\vec{B}'$ (with magnetic E' and B', respectively) in S'?

(a) The ratio $\dfrac{E'}{B'}$ is greater than 1 and the angle between them is greater than 90 degrees.

(b) The ratio $\dfrac{E'}{B'}$ is less than 1 and the angle between them is greater than 90 degrees

(c) The ratio $\dfrac{E'}{B'}$ is greater than 1 and the angle between them is less than 90 degrees

(d) The ratio $\dfrac{E'}{B'}$ is less than 1 and the angle between them is less than 90 degrees

Q48. What is the direction of the Poynting vector around a discharging cylindrical parallel-plate capacitor with circular plates?

(a) Directed towards the axis of the capacitor

(b) Directed away from the axis of the capacitor

(c) From the positive plate to the negative plate.

(d) From the negative plate to the positive plate.

Q49. The time averaged potential of a neutral hydrogen atom is given by

$$\phi(r) = \frac{q}{4\pi \epsilon_0} \frac{e^{-\alpha r}}{r}\left(1 + \frac{\alpha r}{2}\right)$$

the charge distribution $\rho(r)$ which gives this potential is

(a) $\rho(r) = -\frac{q\alpha^2 e^{-\alpha r}}{8\pi}$

(b) $\rho(r) = -\frac{q\alpha^2 e^{-\alpha r}}{8\pi r}$

(c) $\rho(r) = -\frac{q^2\alpha^2 e^{-\alpha r}}{8\pi r}$

(d) $\rho(r) = -\frac{q\alpha^2 e^{-\alpha r}}{8\pi} + q\delta^3(r)$

Q50. A body of rest mass m_0 moving at speed v collide and stick to an identical body at rest. The rest mass M of the final clump is $\left(\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$

(a) $M = 2m_0$

(b) $M = 2\gamma m_0$

(c) $M = m_0\sqrt{2(1+\gamma)}$

(d) $M = m_0\sqrt{2(1-\gamma)}$