

DU (Ph.D) 2019

Q1. The solution to the differential equation,

$$(1+x^2)\frac{df}{dx} + xf(x) = 0$$

is given by, A being an arbitrary constant,

- (a) $\ln(A(x^2+1))$ (b) $\ln(A(x^2+1)^{-1/2})$
 (c) $\cos(A(x^2+1))$ (d) $A(x^2+1)^{-1/2}$

Q2. If $\vec{\nabla} \times \vec{F}(\vec{r}) \neq 0$ but $\vec{\nabla} \times (g(\vec{r})\vec{F}(\vec{r})) = 0$, then

- (a) $\vec{F}(\vec{r}) \cdot (\vec{\nabla} \times \vec{F}(\vec{r})) = 0$ (b) $\vec{\nabla} \times (\vec{\nabla} \times \vec{F}(\vec{r})) = 0$
 (c) $\vec{\nabla} \cdot \vec{F}(\vec{r}) = 0$ (d) $\vec{\nabla} g(\vec{r}) \cdot (\vec{\nabla} \times \vec{F}(\vec{r})) = 0$

Q3. $\frac{d(\delta(y))}{dy}$ can be expressed as

- (a) $\int_{-\infty}^{\infty} \frac{e^{ixy}}{x} dx$ (b) $\frac{1}{2\pi} \int_{-\infty}^{\infty} xe^{ixy} dx$
 (c) $\frac{1}{\pi} \int_{-\infty}^{\infty} xe^{ixy} dx$ (d) $\frac{1}{2\pi} \int_{-\infty}^{\infty} x^2 e^{ixy} dx$

Q4. A system, in three dimensions, is described by the Lagrangian

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \dot{x} \sin(t) - \frac{k}{x^2 + y^2} + x \cos(t)$$

where k is a constant. Of energy (E), linear momentum (\vec{p}) and angular momentum (\vec{J}), which are conserved?

- (a) E, \vec{p}, \vec{J} (b) p_z alone
 (c) \vec{p} alone (d) E, p_z and J_z alone

Q5. The Hamiltonian for a particle in one dimension is given by

$$H(x, p) = \frac{p^2}{2m} + \lambda px + \frac{\lambda}{2} x^2$$

where m, λ are constants. The corresponding Lagrangian is

(a) $L = \frac{m}{2} \dot{x}^2 - \frac{\lambda}{2} x^2$

(b) $L = \frac{m}{2} (\dot{x} - \lambda x)^2 - \frac{\lambda}{2} x^2$

(c) $L = \frac{m}{2} \dot{x}^2 - \lambda m x \dot{x} - \frac{\lambda}{2} x^2$

(d) $L = \frac{m}{2} (\dot{x} - \lambda x)^2 - \lambda m x \dot{x} - \frac{\lambda}{2} x^2$

Q6. A star is pulsating isotropically. Its gravitational force on any body, at distances much larger than its own mean radius, is given by

$$\vec{F}(\vec{r}) = \left(\frac{-k}{r^3} + \frac{a}{r^4} \right) \vec{r}$$

where k and a are positive constants. Which of the following is true about the motion of the body?

(a) Any bounded motion is described by a processing ellipse

(b) Any bounded motion is described by a pulsating ellipse

(c) Any bounded motion is still in an elliptical path, but the parameters of the ellipse are shifted from those in the Newtonian case

(d) No bounded motion exists at all

Q7. A point mass m is attached, through a massless incompressible rod of length l , to a fixed point. The point is allowed to have any motion consistent with the above. If θ be the instantaneous angle of the rod with the vertical, which of the following is necessarily true? (Here $k \geq 0$ is an arbitrary constant)

(a) $\ddot{\theta} + \frac{g}{l} \sin \theta = 0$

(b) $\ddot{\theta} + \frac{g}{l} \theta = 0$

(c) $\ddot{\theta} + \frac{k \sin \theta}{\cos^3 \theta} + \frac{g}{l} \sin \theta = 0$

(d) $\ddot{\theta} - \frac{k \cos \theta}{\sin^3 \theta} + \frac{g}{l} \sin \theta = 0$

- Q11. A theory has equally spaced nondegenerate energy levels starting from $E_{\min} = E_0$ all the way upto ∞ . A system of many such particles is at equilibrium at a temperature. If the average energy squared of the particles is given by

$$\langle E^2 \rangle = 5E_0^2, \text{ what is } T ?$$

- (a) $T = \frac{E_0}{k_B}$ (b) $T = \frac{2E_0}{k_B}$ (c) $T = \frac{\sqrt{5}E_0}{k_B}$ (d) $T = \frac{\sqrt{5}E_0}{2k_B}$

- Q12. A system is composed of two particles moving in a one-dimensional segment of length L . The Hamiltonian is given by $H = c(|p_1| + |p_2|)$, where c is a constant. The volume of the phase space enclosed by a surface of given energy E is

- (a) $\frac{E^2 c^2}{(2L)^2}$ (b) $\frac{E^2 L^2}{c^2}$ (c) $\frac{4E^2 L^2}{c^2}$ (d) $\frac{2E^2 L^2}{c^2}$

- Q13. A crystal contains a paramagnetic impurity whose energy levels, in the presence of a magnetic field, are $\pm E$. The impurity's contribution to the specific heat is

- (a) $\frac{E^2}{k_B T^2}$ (b) $\frac{E^2}{k_B T^2} \sec h^2 \left(\frac{E}{k_B T} \right)$
 (c) $\frac{E}{T} \tanh h^2 \left(\frac{E}{k_B T} \right)$ (d) $\frac{E^2}{k_B T^2} \exp \left(\frac{-E}{k_B T} \right)$

- Q14. Consider an ensemble of N distinguishable particles allowed only two states with energies $\pm \epsilon$, where $\frac{\epsilon}{K} = k_B \ln 8$. If at equilibrium, the higher state has one-third the number of particles, the temperature is

- (a) $2K$ (b) $4K$ (c) $6K$ (d) $8K$

- Q15. The interatomic potential between H - atoms has a range of approximately 4 \AA . If a gas of H -atoms is in thermal equilibrium, what is the temperature T below which atom - atom scattering is overwhelmingly dominated by the s - wave amplitude?

- (a) $1^\circ K$ (b) $10^\circ K$ (c) $20^\circ K$ (d) $50^\circ K$

Q16. Consider an atom in a flame emitting in optical wavelengths. What would the typical Doppler broadening of a line be?

- (a) 10^{12} Hz (b) 10^3 Hz (c) 10^6 Hz (d) 10^9 Hz

Q17. In a Hydrogen atom (Bohr radius a), consider the proton to be a uniformly charged thin spherical shell of radius $R \ll a$. What is the lowest order change in the binding energy (as compared to the case of a point-particle proton)?

- (a) $\frac{2e^2 R}{a^2}$ (b) $\frac{e^2}{2R}$ (c) $\frac{2e^2 R^2}{3a^3}$ (d) $\frac{3e^2 R^2}{4a^4}$

Q18. A quantum-mechanical particle of mass m and charge q is subjected to a potential of the form $(\vec{r}) = \frac{m\omega^2 r^2}{2}$, where ω is a constant. An electric field $\vec{E} = E_0 \hat{x}$ is now switched on (E_0 being a constant). What is the consequent change, upto second order in E_0 , in the energy of the second excited state?

- (a) $\frac{q^2 E_0^2}{m\hbar\omega}$ (b) $\frac{q^2 E_0^2}{2m\omega^2}$ (c) $\frac{2q^2 E_0^2}{m\omega^2}$ (d) $\frac{q^2 E_0^2 (x^2)}{(m\hbar\omega)^2}$

Q19. In perturbed harmonic oscillators, where the potential energy is:

$$V(x) = \frac{1}{2} kx^2 + ax^3 + bx^4$$

The first order change in the energy level is:

(where α and n have their usual meaning)

- (a) $a \frac{3}{4\alpha^2} (2n^2 + 2n + 1)$ (b) $(a + b) \frac{3}{4\alpha^2} (2n^2 + 2n + 1)$
 (c) 0 (d) $b \frac{3}{4\alpha^2} (2n^2 + 2n + 1)$

Q20. A particle of mass m moves in a one dimensional potential given by $V(x) = \frac{1}{2} m\omega^2 x^2$.

It is given that the particle is in a state such that $\langle \psi | \hat{\Pi} | \psi \rangle = 0$, where $\hat{\Pi}$ is a parity operator. The lowest possible expectation value for such a state is

- (a) $2\hbar\omega$ (b) $\frac{1}{2} \hbar\omega$ (c) $\frac{3}{2} \hbar\omega$ (d) $\hbar\omega$

Q21. Consider a three state system characterized by the Hamiltonian

$$H_0 = -(|1\rangle\langle 1| + |1\rangle\langle 2| + |1\rangle\langle 3| + |2\rangle\langle 1| + |2\rangle\langle 2| + |2\rangle\langle 3| + |3\rangle\langle 1| + |3\rangle\langle 2| + |3\rangle\langle 3|)$$

If a small perturbation of the form $\Delta H = \lambda|1\rangle\langle 1|$, where λ is a constant such that $\lambda \ll 1$ is applied to the system. The first order correction to the ground state energy is

- (a) $\frac{\lambda}{2}$ (b) $\frac{-2\lambda}{3}$ (c) $\frac{-\lambda}{2}$ (d) $\frac{\lambda}{3}$

Q22. The Hamiltonian for a spinless particle having orbital angular momentum $l = 2$ is given by

$$\bar{H} = \frac{3\epsilon}{2\hbar} \bar{L}_z - \frac{\epsilon}{\hbar^2} (\bar{L}_x^2 + \bar{L}_y^2); (\epsilon > 0)$$

The energy of the first excited state of the particle is

- (a) -3.5ϵ (b) -5ϵ (c) -6.5ϵ (d) -6ϵ

Q23. Suppose that a machine prepares the states $|\psi_p\rangle = \frac{1}{\sqrt{2}}(e^{i\pi p}|1\rangle + e^{-i\pi p}|2\rangle)$ with a probability $2p$, ($0 \leq p \leq 1$), where the states $\{|1\rangle, |2\rangle\}$ are orthogonal and are elements of a two dimensional Hilbert space. Now consider an operator $A = \mu(|1\rangle\langle 2| + |2\rangle\langle 1|)$. If A is measured for every state the machine prepares, what is the fraction of times the outcome is the maximum value?

- (a) $\frac{1}{2}$ (b) $\frac{1}{2p}$ (c) $\frac{1}{4}$ (d) $\frac{1}{4p}$

Q24. A particle having mass m is initially in the ground state of the potential $U(x)$. At time $t=0$ the barrier in the potential $U(x)$ is suddenly removed so that potential for $t>0$ is given by $V(x)$

$$U(x) = \begin{cases} \frac{m\omega^2}{2}x^2 & \text{for } 0 < x < \infty \\ = 0 & \text{for } x \leq 0 \end{cases} \quad V(x) = \begin{cases} \frac{m\omega^2}{2}x^2 & \text{for } -\infty < x < \infty \end{cases}$$

Suppose that the particle is initially in the ground state $\phi_0(x)$ for $t < 0$, the probability

that the particle is in the new ground state $\psi_0(x) = \frac{\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}}{4} e^{-\frac{m\omega x^2}{2\hbar}}$ for $t > 0$ is

- (a) $\sin(\omega t)$ (b) 0 (c) $\frac{1}{\pi} \sin(\omega t)$ (d) $\frac{1}{\pi}$

Q25. Consider a quantum system consisting of a harmonic oscillator that is coupled to a spin- $\frac{1}{2}$ particle. The Hamiltonian is given by $H = H_1 + H_2$, where

$$H_1 = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega(|+\rangle\langle+| - |-\rangle\langle-|) \quad \text{and} \quad H_2 = g(a|+\rangle\langle-| + a^\dagger|-\rangle\langle+|)$$

The eigenstates of the noninteracting system ($g = 0$) are labeled by $|n, \pm\rangle$, where n is an integer and $|0, \pm\rangle = |\pm\rangle$ in H_1 . Which of the following statements is true?

- (a) All the eigenstates of H_1 are non-degenerate
 (b) For ($g \neq 0$) the states that mix due to the presence of interaction are $|n-1, +\rangle$ and $|n+1, -\rangle$
 (c) All the eigenstates of H are degenerate
 (d) The commutator of H_1 and H_2 vanishes

Q26. Consider a two-level system with basis states $|A\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|B\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and a time evolution operator

$$U(t) = \begin{pmatrix} \cos \frac{\omega t}{2} & -\sin \frac{\omega t}{2} \\ \sin \frac{\omega t}{2} & \cos \frac{\omega t}{2} \end{pmatrix}$$

At $t = 0$, the system is in the state $|A\rangle$ and can be found in the state $|B\rangle$ for the first time at $t = \tau$. If N successive measurements are made at times $t_1 = \frac{\tau}{N}, t_2 = \frac{2\tau}{N}, \dots, t_N = \tau$, the probability that each measurement gives the state $|B\rangle$ is

- (a) $\cos^2 \frac{\pi}{N} \cos^2 \frac{2\pi}{N} \dots \cos^2 \pi$ (b) $\sin^2 \frac{\pi}{2N} \sin^2 \frac{2\pi}{2N} \dots \sin^2 \frac{\pi}{2}$
 (c) $\cos^{2N} \frac{\pi}{2N}$ (d) $\cos^N \frac{\pi}{2N}$

Q27. Two identical non-relativistic spinless bosons of mass m are confined to one dimension. Each particle moves under the influence of the potential $V(x) = \frac{1}{2}kx^2, (k > 0)$. In addition there is an attractive potential between the particles given by $W(x_1, x_2) = -\lambda kx_1x_2$. The exact energy eigenvalues for the two particle system are

$$E(r, s) = \hbar\omega \left[\sqrt{1-\lambda} \left(r + \frac{1}{2} \right) + \sqrt{1+\lambda} \left(s + \frac{1}{2} \right) \right] \text{ with } \omega = \sqrt{\frac{k}{m}}. \text{ Then}$$

- (a) r can take any integer value but s must be an even number
 (b) s can take any integer value but r must be an even number
 (c) Both r and s can take all integer values
 (d) Both r and s must be odd integer

Q28. For Boolean variables, simplifying

$$\overline{a+d} \cdot \overline{b+c} \cdot \overline{c+d}$$

yields

- (a) $a \cdot \bar{b} + c \cdot \bar{d}$ (b) $a \cdot b \cdot c \cdot \bar{d}$ (c) $a \cdot \bar{c} \cdot b \cdot \bar{d}$ (d) $a \cdot \bar{b} \cdot c \cdot \bar{d}$

Q29. Consider a line of $2n$ ions with alternate charges $\pm q$. Nearest neighbours feel a repulsive potential $\frac{b}{r^n}$ apart from the Coulomb attraction. If R be the equilibrium separation, the equilibrium energy is (here, a is the Maelung constant)

- (a) $na \left(\frac{b}{R} - \frac{q^2}{R} \right)$ (b) $\frac{(1-n)aq^2}{R}$
 (c) $2na \left(\frac{b}{R} - \frac{q^2}{R} \right)$ (d) $\frac{naq^2}{R}$

Q30. A binary alloy consists of N_A atoms of type A and N_B atoms of type B where

$N_{A,B} \ll 1$. If $\frac{N_A}{N_B} \approx 1+x$ where $x \ll 1$, the entropy of mixing is

- (a) $Nk_B [\ln 2 - x^2]$ (b) $Nk_B [\ln 2 - 2x]$
 (c) $Nk_B [\ln 2 + 2x^2]$ (d) $Nk_B [\ln 2 + x]$

Q31. In case of the wave function $\psi(r) = \frac{e^{ikr}}{r}$ the probability current density is

- (a) $\frac{\hbar k}{mr}$ (b) $\frac{\hbar k}{mr^2}$ (c) $\frac{\hbar k}{mr^3}$ (d) $\frac{\hbar k}{m}$

Q32. Consider the statement: "Given a group, if $g \in G$, then there exists an integer n such that g^n is the identity element." For this to hold, which of the following must be true?

- (a) G has an even number of elements (b) G is any group and n is positive
 (c) G is finite (d) G is infinite

- Q33. The Lyman- α spectral line for Hydrogen has a wavelength of 122 nm . When the light coming from opposite ends of the Sun's equator is examined, the spectral lines show a difference of $2 \times 10^{12}\text{ m}$ in their wavelengths. What is the approximate rotational speed of a Hydrogen atom at the equator?
- (a) 1200 m/s (b) 2500 m/s (c) 4900 m/s (d) 6500 m/s
- Q34. A cylindrical conductor, of radius $r = \sqrt{x^2 + y^2}$, has magnetic vector potential $\vec{A} = r^2 \hat{z}$, due to a constant current density \vec{J} whose magnitude is numerically
- (a) $|\vec{J}| = 2$ (b) $|\vec{J}| = 4$ (c) $|\vec{J}| = 3$ (d) $|\vec{J}| = 6$
- Q35. In a Lorentz frame, the electric and magnetic field vectors are given by $\vec{E} = \hat{x} + \hat{z}$ and $\vec{B} = 2\hat{x} + \hat{y}$ respectively. If in another Lorentz frame S' , the transformed electric and magnetic field vectors \vec{E}' and \vec{B}' are parallel to each other, then their magnitudes would be (in Gaussian units)
- (a) $|\vec{E}'| = 2$ and $|\vec{B}'| = 1$ (b) $|\vec{E}'| = \sqrt{2}$ and $|\vec{B}'| = \sqrt{5}$
(c) $|\vec{E}'| = 1$ and $|\vec{B}'| = 2$ (d) $|\vec{E}'| = \sqrt{5}$ and $|\vec{B}'| = \sqrt{2}$
- Q36. Consider two positive charges q and $2q$ at positions $r=0$ and $r=R$ respectively, in free space. Another charge Q is placed at $r=r_0$, where $0 < r_0 < R$. The system of the three charges
- (a) will be in equilibrium when $Q = (2\sqrt{3} - 4)q$
(b) will be in equilibrium when $Q = (3\sqrt{3} - 6)q$
(c) will be in equilibrium when $Q = (4\sqrt{2} - 6)q$
(d) will never be in equilibrium in 3 dimensions
- Q37. A pion of energy E decays into two photons. The opening angle θ between the photons in the limits of (i) $E \approx m_n$ and (ii) $E \gg m_n$ are given by
- (a) $0, n$ (b) $n, 0$ (c) $n, \frac{n}{2}$ (d) $\frac{n}{2}, 0$

- Q38. A proton of momentum $1.0\text{ GeV}/c$ is passing through a gas. What is the minimum index of refraction of the gas so that the proton may emit Cerenkov radiation?
 (a) 1.21 (b) 1.37 (c) 1.53 (d) 1.80
- Q39. Consider a particle of mass m and charge q moving in the magnetic field of a static hypothetical magnetic monopole. The field due to the magnetic monopole is given by $\vec{B} = g\hat{r}/r^2$ (where g is a constant). If the quantity $\vec{J} = \vec{L} + \vec{f}$ is a constant of motion (\vec{L} being the usual angular momentum of the particle) then \vec{f} equals
 (a) $-qg\vec{r}$ (b) $2qg\vec{r}$ (c) $2qg\vec{r}$ (d) $-qg\vec{r}$
- Q40. At the present time, the temperature of the universe (i.e., the microwave radiation background) is about 3 K . When the temperature was 12 K , typical objects in the universe, such as galaxies, were
 (a) one-half as distant as they are today
 (b) one-quarter as distant as they are today
 (c) separated by about the same distance as they are today
 (d) two times as distant as they are today
- Q41. Which of the following attributes are applicable to the mediators of the four fundamental interactions:
 I. Any integer spin
 II. Nonzero, but integer spin
 III. Zero mass
 IV. Zero charge
 (a) I alone (b) I and III (c) II alone (d) II and IV
- Q42. The thermonuclear fusion

$$p + p \rightarrow d + e^+ + \nu_e$$
 (where d is the deuteron) occurring in the core of the sun is a
 (a) electromagnetic interaction (b) gravitational interaction
 (c) strong interaction (d) weak interaction

- Q43. Which of the following decays is allowed:
- (a) $\mu^+ \rightarrow e^+ + \bar{\nu}_e + \nu_\mu$ (b) $\mu^+ \rightarrow e^+ + \nu_\mu + \nu_e$
(c) $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$ (d) $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \bar{\nu}_e$
- Q44. Consider mirror nuclei ${}^{26}_{12}\text{Mg}$ and ${}^{26}_{14}\text{Si}$, if the first excited 2^+ state in ${}^{26}_{12}\text{Mg}$ is at approximately $E = 1.8\text{MeV}$ with respect to the ground state, what is the energy of the first excited 2^+ state in ${}^{26}_{14}\text{Si}$
- (a) 3.6MeV (b) 1.8MeV
(c) 2.0MeV (d) the information is incomplete
- Q45. Three electrons enter a GM tube. The energies of electrons are (1) 100keV , (2) 200keV and (3) 500keV . All of them are absorbed in the GM tube. What is the correct order of the pulse height of the signals thus created?
- (a) $1 > 2 > 3$ (b) $1 < 2 > 3$ (c) $1 = 2 < 3$ (d) $1 = 2 = 3$
- Q46. In a certain place, it rains on one third of the days. The local newspaper (early city edition) attempts to predict whether or not it will rain during the day. Three quarters of rainy days and three fifths of dry days are correctly predicted. Given that today's paper predicts rain, what is the probability that it will actually do so?
- (a) 0.48 (b) 0.51 (c) 0.61 (d) 0.73
- Q47. There are ten modules in a detector. Of these, eight are working properly and two are defective. Modules which are working properly, give readings each of which, independently, has a probability of 0.05 of being wrong. For the defective modules, this probability is 0.2. A module is chosen at random and ten readings recorded by it are examined. What is the probability that the module chosen is defective given that, of the ten cases examined, two are imperfect and eight are perfect?
- (a) 0.05 (b) 0.2 (c) 0.8 (d) 0.9
- Q48. A JK flip flop has $t_{pd} = 3.5\text{ns}$. The largest modulus of a ripple counter using these flip flops and operating at 30MHz is
- (a) 1024 (b) 128 (c) 512 (d) 64

- Q49. A classical charged particle moves in a circle at constant speed. Which of the following is true?
- (a) It emits both electric dipole and magnetic dipole radiation
 - (b) It emits electric dipole and electric quadrupole radiation
 - (c) It emits electric dipole radiation
 - (d) It emits magnetic dipole radiation
- Q50. In a bi-stable multivibrator, commutating capacitors are used to:
- (a) increase the base storage charge
 - (b) change the frequency of the output
 - (c) increase the speed of response
 - (d) provide a.c. coupling