DU PhD PAPER 2020

Q1. A small mass m with a charge q is attached to spring of spring constant k and allowed to oscillate at amplitude A. Assuming that the amplitude of the oscillations and the speed of the mass is small, the time averaged power radiated by the system in Gaussian units is

(a) $\frac{q^2k^2A^2}{3c^2m^2}$ (b) $\frac{q^2k^2A^2}{3c^4m^2}$

(c) $\frac{2q^2k^2A^2}{3c^3m^2}$ (d) None of these

Ans. (a)

Q2. A sphere of radius a made of a material of dielectric constant $\in_r = \frac{\in}{\in_0}$ has a uniform charge density potential (ρ) at the center of the sphere is

(a) $V(0) = \frac{\rho a^2}{6\in_0\in_r}(2\in_r + 1)$ (b) $V(0) = 0$

(c) $V(0) = \frac{\rho a^2}{4\pi\in_0}(2\in_r + 1)$ (d) $V(0) = \frac{4\pi a^2\rho}{3\in_0\in_r}$

Ans. (a)

Q3. In the planetary model of the hydrogen atom, the time taken for the electron of charge e and mass m in the first Bohr orbit $\left(a_0 = \frac{\hbar^2}{me^2}\right)$ to spiral into the nucleus is given by

(a) $\frac{m^2c^3a_0^3}{4e^4}$ (b) $\frac{m^2c^3a_0^3}{2e^4}$

(c) $\frac{m^2c^3a_0^3}{2\hbar e^4}$ (d) None of these

Ans. (a)

Q4. A particle of mass m and charge q is accelerated from rest in a uniform electric field $\vec{E} = E\hat{x}$ for a time t. Assuming relativistic motion, the speed of the particle at time t is given by

(a) $\dfrac{qEct}{\sqrt{(qEt)^2 + (mc)^2}}$ (b) $\left(\dfrac{qE}{m}\right)t$

(c) $\dfrac{qEct}{2\sqrt{(qEt)^2 + (mc)^2}}$ (d) $\dfrac{qEct^2}{\sqrt{(qEt)^2 + (mc)^2}}$

Ans. (a)

Q5. A circular air filled parallel plate capacitor of radius R and separation d has an electric field $E(t)$ which varies as $\dfrac{\partial E}{\partial t}$ ignoring edge effects, the magnitude of the magnetic field is given by

(a) $B = \dfrac{R}{2c}\dfrac{\partial E}{\partial t}$ (b) $B = \dfrac{R^2}{2cd}\dfrac{\partial E}{\partial t}$

(c) $B = \dfrac{d^2}{Rc}\dfrac{\partial E}{\partial t}$ (d) $B = \dfrac{R^2}{2d}\dfrac{\partial E}{\partial t}$

Ans. (a)

Q6. The first-order correction to the ground state energy of an isotropic 3-dimensional harmonic oscillator with the perturbation $V = \lambda xyz^2$ is

(a) 0 (b) $\lambda^2\left(\dfrac{\hbar}{2m\omega}\right)$

(c) ∞ (d) $\left(\dfrac{\hbar}{2m\omega}\right)^2\lambda^2$

Ans. (a)

Q7. Consider a particle of mass m constrained in the segment $-a \le x \le a$ and subject to the repulsive potential $V(x) = \lambda\delta(x), \lambda > 0$. Consider $V(x)$ as a perturbation and calculate the 1st order correction $\Delta E_0^{(1)}$ and $\Delta E_1^{(1)}$ to the energies of the ground and first excited states

(a) $\Delta E_0^{(1)} = \frac{\lambda}{a}$ and $\Delta E_1^{(1)} = 0$ (b) $\Delta E_0^{(1)} = 0$ and $\Delta E_1^{(1)} = \frac{\hbar^2\pi^2}{8ma^2}$

(c) $\Delta E_0^{(1)} = \frac{\lambda}{a}$ and $\Delta E_1^{(1)} = \frac{\lambda}{a}$ (d) $\Delta E_0^{(1)} = \frac{\hbar^2\pi^2}{8ma^2}$ and $\Delta E_1^{(1)} = \frac{\lambda}{a}$

Ans. (a)

Q8. If the scattering amplitude $f(\theta) = 4\sin(\theta) + i5\cos\theta$, the total cross-section σ_T is

(a) $\frac{20\pi}{k}$ (b) $\frac{5}{k^2}$ (c) $\frac{4}{k^2}$ (d) 0

Ans. (a)

Q9. Find the first order probability of transition of a harmonic oscillator to go from its ground state $|0>$ to the first excited state $|1>$ for a time–dependent perturbation $H(t) = xe^{\frac{-t}{\tau}}, t \geq 0, \tau > 0$, for $t \to \infty$ late time, and $\tau \to 0$. $P_{0\to 1}^{(1)}$ is therefore equal to

(a) 0 (b) $\frac{1}{2m\hbar\omega^3}$ (c) 1 (d) ∞

Ans. (a)

Q10. The angle between two (hkl) planes corresponding to (100) and (110) is

(a) 45^o (b) 60^o (c) 30^o (d) 15^o

Ans. (a)

Q11. The Madelung constant of a one dimensional crystal consisting of alternate positive and negative ions with interatomic distance R is given by the expression $a = 2\ln 2$. The Madelung constant for a divalent ion can be expressed as:

(a) $\alpha = 8\ln 2$ (b) $\alpha = 4\ln 2$

(c) $\alpha = \ln 2$ (d) 0

Ans. (a)

Q12. The total scattering amplitude of reflection from (h,k,l) plane is given by the expression $F(h,k,l)=\sum_j e^{2\pi i(u_j h+v_j k+w_j l)}$. Where (u_j,v_j,w_j) represent the coordinates of the jth atom. The allowed reflections for (h,k,l) values for a FCC structure are

(a) all odd or all even
(b) all odd
(c) all even
(d) zero

Ans. (a)

Q13. A one dimensional lattice chain consists of periodic arrangement of atoms with lattice spacing 'a'. Each atom is represented by the potential $V(x)=aV_0\delta(x)$, the energy gaps between the bands in the nearly free electron approximation is

(a) $2V_0$
(b) V_0
(c) $\frac{V_0}{2}$
(d) $\sqrt{V_0}$

Ans. (a)

Q14. If an AC current of frequency $1GHz$ is observed through a Josephson junction, then the applied dc voltage is, (Given $h=6.625\times10^{-34}$)

(a) $2.07\,\mu V$
(b) $3.8\,\mu V$
(c) $1\mu V$
(d) $5.48\,\mu V$

Ans. (a)

Q15. Suppose that Newton's theory of gravitation is modified for short range. In this modified theory the potential energy between two masses m_1 and m_2 are given by,

$$V(r)=-\frac{Gm_1m_2}{r}\left(1-ae^{-r/\lambda}\right)$$

where a is a constant and other symbols have their usual physical significance. For short distances $r<<\lambda$ calculate the force between m_1 and m_2.

(a) $F=-Gm_1m_2(1-a)/r^2$
(b) $F=-Gm_1m_2a/\lambda r$
(c) $F=-Gm_1m_2(1+a)/r^2$
(d) $F=-Gm_1m_2a/r^2$

Ans. (a)

Q16. A statistical system is composed of two ultra-relativistic particles moving in a segment of Length L. The Hamiltonian of the system is given by,

$$H(p_1, p_2) = c(|p_1| + |p_2|)$$

where, p_1 and p_2 are the momenta of the particles and c is the speed of light in vaccum. The volume of phase space enclosed by the surface of constant energy E is given by

(a) $\sum(E,L) = \frac{2E^2L^2}{c^2}$ (b) $\sum(E,L) = \frac{E^2L^2}{c^2}$

(c) $\sum(E,L) = \frac{2EL^2}{c^2}$ (d) $\sum(E,L) = 2E^2L^2$

Ans. (a)

Q17. Consider an ensemble of N distinguishable particles distributed in two energy levels ε and $-\varepsilon$, with number of particles in them N_+ and N_-, respectively in equilibrium. The ensemble is isolated and has fixed energy E at temperature T given by, $E = -N\varepsilon \tanh\left(\frac{\varepsilon}{k_B T}\right)$ is the Boltzmann constant.

If $\varepsilon = k_B \ln 2$, find out the temperature at which $\frac{N_+}{N_-} = \frac{1}{2}$. [Given, $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$]

(a) $+2k$ (b) $-2k$ (c) $+1k$ (d) $-4k$

Ans. (a)

Q18. Primary advantage of a crystal oscillator is that

(a) it can oscillate at any frequency

(b) it gives a high output voltage

(c) its frequency of oscillation remains almost constant

(d) it gives a constant a $d.c$ output voltage

Ans. (c)

Q19. In the spectrum of a frequency-modulated wave

(a) the carrier frequency disappears when the modulation index is large

(b) the amplitude of any sideband depends on the modulation index

(c) the total number of sidebands depends on the modulation index

(d)the carrier frequency cannot disappear

Ans. (b)

Q20. The largest value of output voltage from an 8-bit digital-to-analog converter that produces $1.0V$ for a digital input of 00110010 is

(a) $5.1V$ (b) $10.2V$ (c) $20.4V$ (d) $2.5V$

Ans. (a)

Q21. Which of the following statement is NOT correct for a Depletion type n-channel MOSFET

(a) Channel width can be increased

(b) Channel width can be decreased

(c) can work with both positive and negative gate bias

(d) Initially the channel between drain and source is completely blocked by a p-region

Ans. (d)

Q22. If an inverter is placed between the inputs of an $S-R$ Flip-Flop, the resulting Flip-Flop is a

(a) D Flip-Flop (b) $J-K$ Flip Flop

(c) Master Slave Flip Flop (d) Romains a $S-R$ Flip Flop

Ans. (a)

Q23. The Carnot engines X and Y are operating in series. The first engine X receives heat at $1200K$ and rejects to a reservoir at temperature T. The second engine Y receives the heat rejected by X, and thereafter re-ejects to heat reservoir at $300K$. Calculate the temperature (in kelvin) for the situation, when the work output of the two engines is equal

(a) $750K$ (b) $600K$ (c) $0K$ (d) $450K$

Ans. (a)

Q24. The quantum mechanical energy states of an atom are described by the energy states such as 0 and $\in$ at the thermal equilibrium temperature T. Now the system has partition function Q such that its total internal energy will be:

(a) $U = \dfrac{\in}{e^{\frac{\in}{kT}} + 1}$ (b) $U = \dfrac{2\in}{e^{\frac{2\in}{kT}} + 1}$

(c) $U = \dfrac{kT}{e^{\frac{\in}{kT}} + 1}$ (d) $U = \dfrac{kT}{e^{\frac{2\in}{kT}} + 1}$

Ans. (a)

Q25. $1kg$ of water at $273K$ is brought in contact with a heat reservoir at $373K$. Now after the transfer of heat to the heat reservoir, there is a change of entropy in the system when the after reaches $373K$. What is the change in entropy.

[Given specific heat $s = 10^3\ cal/kg - K$]

(a) $2.303\log_{10}\left(\dfrac{373}{273}\right) cal/K$ (b) $10^3 \times 2.303\log_{10}\left(\dfrac{373}{273}\right) cal/K$

(c) $10^3 \times \log_{10}\left(\dfrac{373}{273}\right) cal/K$ (d) None of these

Ans. (b)

Q26. Roughing vacuum range is

(a) $10^{-7} - 10^{-5}\ mbar$ (b) $10^{-11} - 10^{-9}\ mbar$

(c) $10^{-3} - 10^{-1}\ mbar$ (d) $10^{3} - 10^{1}\ mbar$

Ans. (c)

Q27. Pirani gauge works in pressure range of

(a) $10^{5} - 10^{1}$ Torr (b) $10^{-4} - 10^{-1}$ Torr

(c) $10^{-8} - 10^{-4}$ Torr (d) $10^{-12} - 10^{-3}$ Torr

Ans. (b)

Q28. 3 Isospin (I) of elementary particle Ω - is

(a) $\dfrac{1}{2}$ (b) $\dfrac{3}{2}$ (c) 1 (d) 0

Ans. (d)

Q29. Which one of the following particle has a strangeness quantum number 1?

(a) π^+ (b) $\wedge^0$ (c) K^+ (d) Ω^-

Ans. (c)

Q30. Hypercharge (Y) of elementary particle K^+ is

(a) 0 (b) +1 (c) −1 (d) −2

Ans. (b)

Q31. Quark structure of elementary particle $\sum^+$ is

(a) uus (b) uds (c) sds (d) sus

Ans. (a)

Q32. Total number of down quarks in ${}_3^7Li$ are

(a) 9 (b) 10 (c) 11 (d) 12

Ans. (c)

Q33. If the probability that a problem will be solved by three students is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$, then what is the probability that the problem will be solved?

(a) $\frac{13}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{18}$ (d) None of these

Ans. (a)

Q34. Find the eigenvalues of $4A^{-1}+3A+2I$, where I is the identity matrix and $A=\begin{pmatrix}1 & 0\\ 2 & 4\end{pmatrix}$

(a) 9,15 (b) 9,36 (c) 7,28 (d) None of these

Ans. (a)

Q35. If $u=x^2+y^2+z^2$ and $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ then $div(u\vec{r})$ is

(a) u (b) $2u$ (c) $4u$ (d) $5u$

Ans. (d)

Q36. The value of complex integral $\oint \frac{z}{z^2+9}dz$ with the closed contour $|z-2i|=4$ is

(a) πi (b) $2\pi i$ (c) $3\pi i$ (d) $4\pi i$

Ans. (a)

Q37. The Fourier transform of $f(x)=\begin{cases} 0, & x\le 0 \\ e^{-ax}, & x>0 \end{cases}$

(a) $\frac{1}{2\pi is+a}$ (b) $\frac{1}{2\pi is+2a}$

(c) $\frac{1}{2\pi is-a}$ (d) None of these

Ans. (a)

Q38. Given the operator $\vec{J}=\hat{J}_x\hat{i}+\hat{J}_y\hat{j}+\hat{J}_z\hat{k}$, where the commutator $\left[\hat{J}_j,\hat{J}_k\right]=i\sum_{l=1}^{3}\in_{jkl}\hat{J}_l$, as well as two constant vector $\vec{u}$ and $\vec{v}$ then the commutator $\left[\vec{u}.\hat{\vec{J}},\vec{v}.\hat{\vec{J}}\right]$ is equal to,

(a) $i(\vec{u}\times\vec{v}).\hat{\vec{J}}$ (b) $i\left(\sum_{k=1}^{3}u_k v_k \hat{J}_k\right)$

(c) $i\sum_{k=1}^{3}u_k v_k \hat{J}_k$ (d) $\sum_{k=1}^{3}u_k v_k \hat{\vec{J}}.\hat{\vec{J}}$

Ans. (a)

Q39. For $-1\le x\le +1$, the series $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$ is equal to:

(a) $\tan^{-1}x$ (b) $\frac{x\exp x}{\pi}$

(c) $\sin^2 x$ (d) $\cos^2 x$

Ans. (a)

Q40. The integral, $\int_{-\infty}^{+\infty}\frac{d(\delta(y))}{dy}\sin y\,dy$ is equal to

(a) -1 (b) $\cos y$ (c) $+1$ (d) π

Ans. (a)

Q41. The solution of the differential equation, $\left(1+x^2\right)\frac{df}{dx}+xf\left(x\right)=0$ is given by, a being an arbitrary constant,

(a) $A\left(x^2+1\right)^{-1/2}$ (b) $\ln\left(A\left(x^2+1\right)\right)$

(c) $\ln\left(A\left(x^2+1\right)^{-1/2}\right)$ (d) $\cos\left(A\left(x^2+1\right)\right)$

Ans. (a)

Q42. If $\vec{\nabla}\times\vec{F}\left(\vec{r}\right)\neq 0$ but $\vec{\nabla}\times\left(g\left(\vec{r}\right)\vec{F}\left(\vec{r}\right)\right)=0$ then,

(a) $\vec{F}\left(\vec{r}\right).\left(\vec{\nabla}\times\vec{F}\left(\vec{r}\right)\right)=0$ (b) $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\left(\vec{r}\right)\right)=0$

(c) $\vec{\nabla}.\vec{F}\left(\vec{r}\right)=0$ (d) $\vec{\nabla}g\left(\vec{r}\right).\left(\vec{\nabla}\times\vec{F}\left(\vec{r}\right)\right)=0$

Ans. (a)

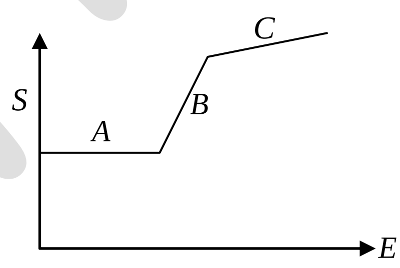
Q43. $\frac{d\left(\delta\left(y\right)\right)}{dy}$ equals to:

(a) $\frac{i}{2\pi}\int_{-\infty}^{+\infty}xe^{ixy}dx$ (b) $\int_{-\infty}^{+\infty}\frac{e^{ixy}}{x}dx$

(c) $\frac{1}{\pi}\int_{-\infty}^{+\infty}xe^{ixy}dx$ (d) $\frac{i}{2\pi}\int_{-\infty}^{+\infty}x^2e^{ixy}dx$

Ans. (d)

Q44. The entropy S of a thermodynamic system as a function of energy E is given by the following graph



If T_A, T_B and T_C are the temperatures for the phases A, B and C respectively, then

(a) $T_B > T_C > T_A$ (b) $T_A > T_B > T_C$

(c) $T_C > T_A > T_B$ (d) $T_C > T_B > T_A$

Ans. (a)

Q45. If the half-life of radium is about 1600 years, then for a given ball of pure radium weighing $4\,gm$ now, what would be the amount of time required to have only $0.125\,gm$ of radium to be left,

(a) 9600 years (b) 4800 years (c) 7200 years (d) 8000 years

Ans. (d)

Q46. The Hamiltonian for a 1-dimensional system is given to be $H(x,p)=\alpha p+\beta x$, where α and β are positive real numbers, respectively. The phase space trajectory in the position-momentum $(x-p)$ plane is given by,

(a) An ellipse

(b) A straight line with a positive slope

(c) A parabola

(d) A straight line with a negative slope

Ans. (d)

Q47. The Lagrangian for a system is given by $L=\alpha e^{-bt}\dot{x}^2-e^{-bt}\beta x$, where α and β are positive real numbers. The constant b is also a positive real number. The equation of motion that follows from this Lagrangian is

(a) $2\alpha\ddot{x}-b\dot{x}\beta e^{-bt}=0$ (b) $e^{-bt}(\alpha\ddot{x}-2b\dot{x})+\beta=0$

(c) $\alpha(\ddot{x}+b\dot{x})+\beta=0$ (d) $2\alpha(\ddot{x}-b\dot{x})+\beta=0$

Ans. (d)

Q48. The Hamiltonian of a system is given by,

$$H=ap^3+bp+x^2$$

where a and b are positive constants. The corresponding Lagrangian is

(a) $\pm\frac{2}{\sqrt{3a}}(\dot{x}-b)^2-x^2$ (b) $\frac{2}{3\sqrt{3a}}(\dot{x}-bx)^{3/2}-x^2$

(c) $\pm(\dot{x}-b)^{3/2}+x^2$ (d) $\pm\frac{2}{3\sqrt{3a}}(\dot{x}-b)^{3/2}-x^2$

Ans. (d)

Q49. Consider the transformation,

$$q \to Q = a_1 q + \beta_1 p$$

$$p \to P = \alpha_2 q + \beta_2 p$$

where, $\alpha_1, \alpha_2, \beta_1$ and β_2 are real constants. This transformation is:

(a) Always canonical as it is a linear transformation

(b) Never a canonical transformation since it is linear

(c) A canonical transformation if $\beta_1 = 1$ and $\alpha_2 = 1$ while $\alpha_1 = 0$ and $\beta_2 = 0$

(d) A canonical transformation if $\alpha_1\beta_2 - \beta_1\alpha_2 = 1$

Ans. (d)

Q50. A free-particle moving in 1-dimension is described by the wavefunction

$$\psi(x,t)\left[Ae^{\frac{ipx}{\hbar}} + Be^{\frac{-ipx}{\hbar}}\right]e^{\frac{-ip^2t}{2m\hbar}},$$

Which of the following options is correct?

(a) $\psi(x,t)$ is an eigenstate of the momentum operator

(b) $\psi(x,t)$ is not a solution of the Schrodinger equation, but is an eigenstate of the Hamiltonian

(c) $\psi(x,t)$ is an eigenstate of the momentum operator as well as an eigenstate of the Hamiltonian

(d) $\psi(x,t)$ is a solution of the Schrodinger equation and is an eigenstate of the Hamiltonian

Ans. (d)