GATE - 2014

Q.1-Q.25 carry one mark each.

Q1 The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is

(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

Q2. Which one of the following quantities is invariant under Lorentz transformation?

(a) charge density (b) Charge (c) Current (d)Electric field

Q3. The number of normal Zeeman splitting components of ${}^1P \to D$ transition is

(a) 3 (b)4 (c)8 (d) 9

Q4. If the half -life of an elementary particle moving with speed 0.9c in the laboratory frame is $5\times10^{-8}\,s$, then the proper half-life is _____ $\times10^{-8}\,s.\left(c = 3\times10^8\,m/s\right)$

Q5. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is

(a) 0^o (b) 45^o (c) 90^o (d) 120^o

Q6. Two masses m and $3m$ are attached to the two ends of a massless spring with force constant K. If $m = 100g$ and $K = 0.3N/m$, then the natural angular frequency of oscillation is _____ Hz

Q7. The electric filed of a uniform plane wave propagating in a dielectric non-conducting medium is given by $\vec{E} = x10\cos\left(6\pi\times10^7 t - 0.4\pi z\right)V/m$. The phase velocity of the wave is _____ $10^8\,m/s$

Q8. The matrix

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$ is

(a) orthogonal (b) symmetric (c) anti-symmetric (d) Unitary

Q9. The recoil momentum of an atom is p_A when it emits an infrared photon of wavelength $1500\,nm$, and it is p_B when it emits a photon of visible wavelength $500nm$. The ratio $\frac{p_A}{p_B}$ is

(a) 1:1 (b) $1:\sqrt{3}$ (c) 1 : 3 (d) 3 : 2

Q10. For a gas under isothermal condition its pressure p varies with volume V as $P \propto V^{-5/3}$. The bulk modules B is proportional to

(a) $V^{-1/2}$ (b) $V^{-2/3}$ (c) $V^{-3/5}$ (d) $V^{-5/3}$

Q11. Which one of the following high energy processes is allowed by conservation laws?

(a) $p + \bar{p} \to A^o + A^o$ (b) $\pi^- + p \to \pi^o + n$

(c) $n \to p + e^- + V_e$ (d) $\mu^+ \to e^+ + \gamma$

Q12. The length element ds of an arc is given by, $(ds)^2 = 2(ds^1)^2 + (dx^2)^2 + \sqrt{3}dx^1dx^2$. The metric tensor g_{ij} is

(a) $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & \frac{\sqrt{3}}{2} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

Q13. The ground state and first excited state wave function of a one dimensional infinite potential well are ψ_1 and ψ_2 respectively. When two spin-up electrons are placed in this potential which one of the following with x_1 and x_2 denoting the position of the two electrons correctly represents the space part of the ground state wave function of the system?

(a) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1)-\psi_1(x_2)\psi_2(x_2)]$ (b) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2)+\psi_1(x_2)\psi_2(x_1)]$

(c) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1)+\psi_1(x_2)\psi_2(x_2)]$ (d $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2)-\psi_1(x_2)\psi_2(x_1)]$)

Q14. If the vector potential $\vec{A}=\alpha x\hat{x}+2y\hat{y}-3z\hat{z}$, satisfies the Coulomb gauge, the value of the constant α is _____

Q15. At a given temperature T, the average energy per particle of a non-interacting gas of two-dimensional classical harmonic oscillators is _____k_BT

(k_B is the Boltzmann constant)

Q16. which one of the following is a fermion?

(a) α particle (b) ${}_4Be^2$ nucleus

(c) Hydrogen atom (d)deuteron

Q17. which one of the following three-quark states (qqq) denoted by X. CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript.

(a) X^{++} (b) X^{+} (c) X^{-} (d) X^{--}

Q18. The Hamilton's canonical equation of motion in terms of Poisson Brackets are

(a) $\dot{q}=\{q,H\};\dot{p}=\{p,H\}$ (b) $\dot{q}=\{H,q\};\dot{p}=\{H,p\}$

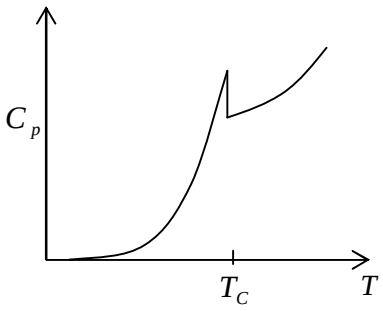
(c) $\dot{q}=\{H,p\};\dot{p}=\{H,p\}$ (d) $\dot{q}=\{p,H\};\dot{p}=\{q,H\}$

Q19. The Miller indices of a plane passing through the three points having coordinates (0, 0, 1) $(1,0,0)\left(\frac{1}{2},\frac{1}{2},\frac{1}{4}\right)$ are

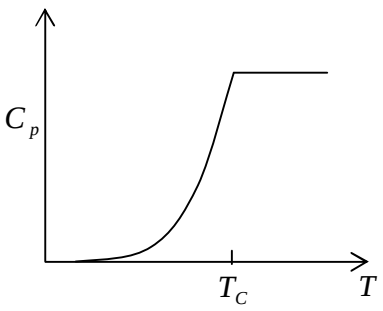
(a) (212) (b)(111) (c) (121) (d) (211)

Q20. The plot of specifies heat versus temperature across the superconducting transition temperature (T_c) is most appropriately represented by

(a)



(b)



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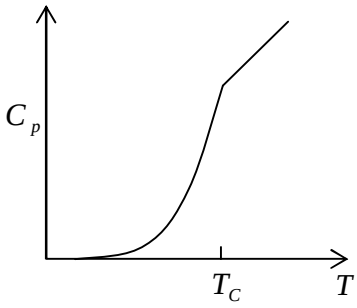

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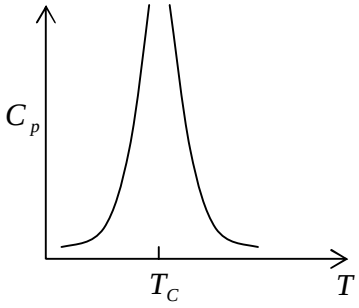
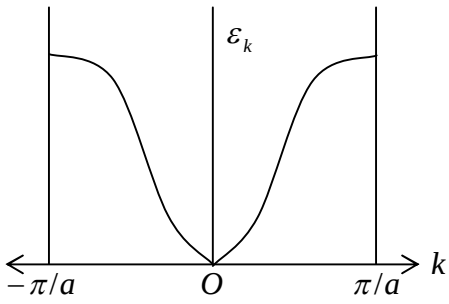


Q21. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}.\vec{S}$ does not commute with

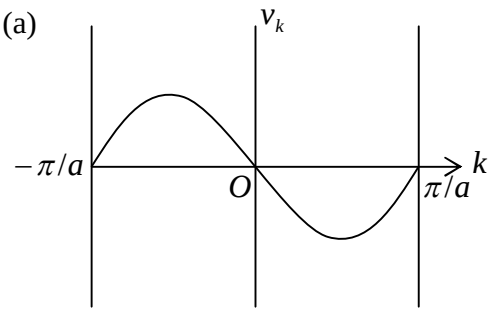
(a) S_z (b) L^2 (c) S^2 (d) $\left(\vec{L}+\vec{S}\right)^2$

Q22. The energy ε_k for band electrons as a function of the wave vector k in the first Brillouin zone $\left(-\frac{\pi}{a} \le k \le \frac{\pi}{a}\right)$ of a one dimensional monatomic lattice is shown as (a is lattice constant)

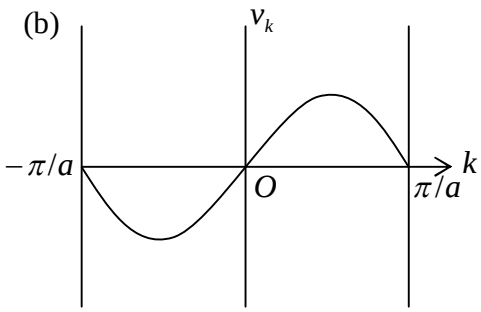


The variation of the group velocity v_k is most appropriately represented by

(a)



(b)



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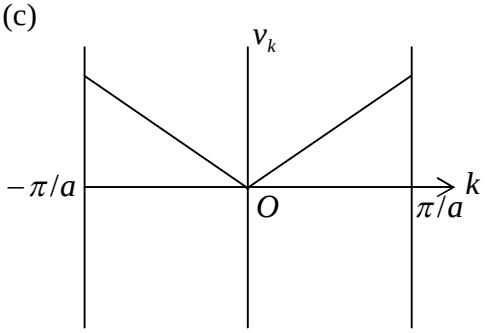

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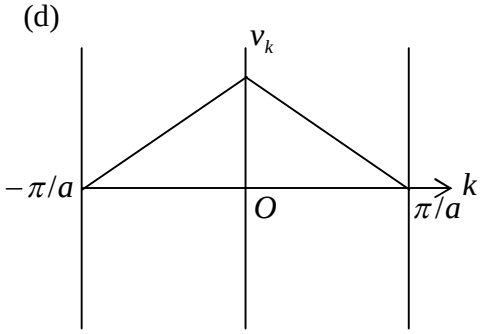
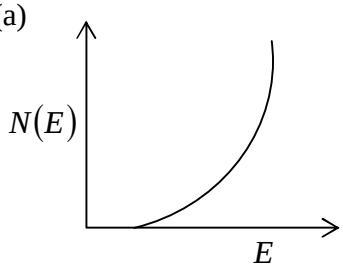
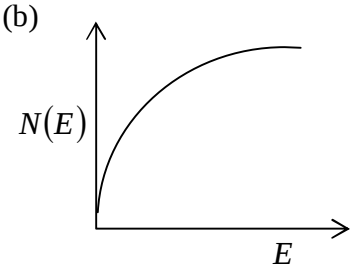


Q23. for a free electron gas in two dimensions the variations of the density of states. $N(E)$as a function of energy E, is best represented by

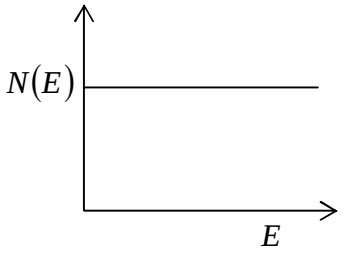
(a)



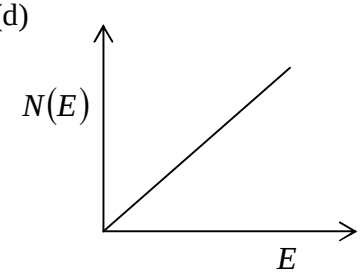
(b)



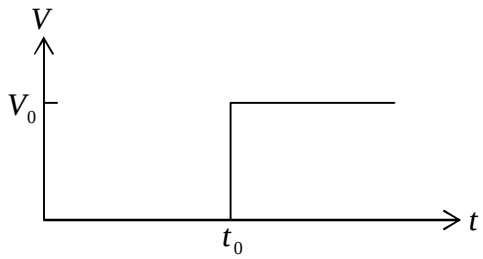
(c)



(d)

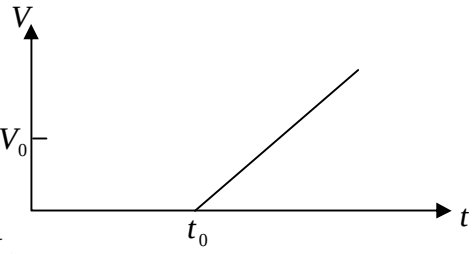


Q24. The input given to an ideal OP-AMP integrator circuit is

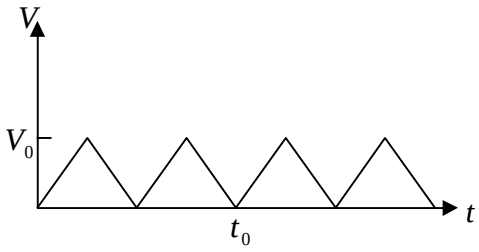


The correct output of the integrator circuit is

(a)



(b)



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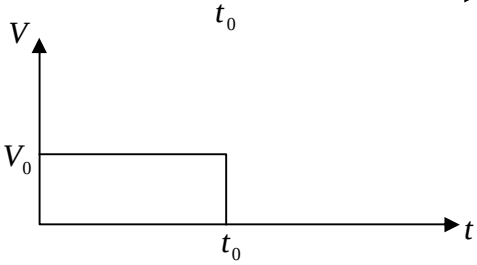
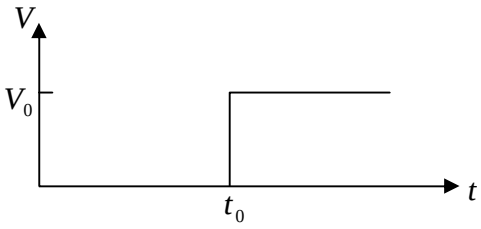
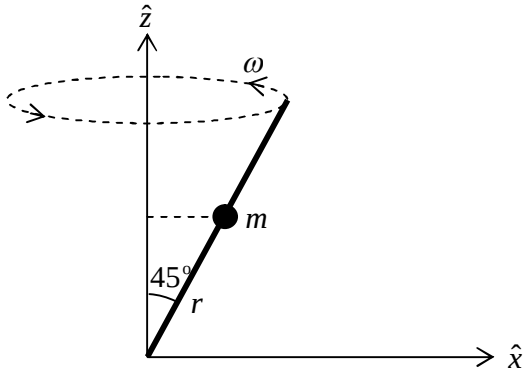


(d)



Q25. The minimum number of flip-flops required to construct a mod-75 counter is _____

Q26. A bead of mass m can slide without friction along a massless rod kept at 45^{o} with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant r is the distance of the bead from the origin. The momentum conjugate to r is



(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$ (c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

Q27. An electron in the ground state of the hydrogen atom has the wave function

$$\psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\left(\frac{r}{a_0}\right)}$$

where a_0 is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is

(Hint: $\int_0^\infty e^{-ar} r^n dr = \frac{r(n)}{a^{n+1}} = \frac{(n-1)!}{a^{n+1}}$)

(a) $\frac{-a_0^2}{2}$ (b) $-a_0^2$ (c) $\frac{-3a_0^2}{2}$ (d) $-2a_0^2$

Q28. For Nickel the number density is $8\times10^{23}\ atoms/cm^3$ and electronic configuration is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic state is _____$\times10^9\ A/m$.

(Given the value of Bohr magneton $\mu_B = 9.21\times10^{-21}\ Am^2$)

Q29. A particle of mass m is in a potential given by

$$V(r)=\frac{a}{r}+\frac{ar_n^2}{3r^2}$$

when a and r are positive constants. When disturbed slightly from its stable equilibrium position it undergoes a simple harmonic oscillation. The time period of oscillation is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{m\,r_0^{\,3}}{a}}$ (c) $2\pi\sqrt{\frac{2m\,r_0^{\,3}}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

Q30. The donar concentration in a sample of $n-$type silicon is increased by a factor of 100. the shift in the position of the Fermi level at 300K. assuming the sample to non degenerate is _____ meV

$\left(k_BT=25meV\,at\,300K\right)$

Q31. A particle of mass m is subjected to a potential

$$V(x,y)=\frac{1}{2}m\omega^2\left(x^2+y^2\right),-\infty\le x\le\infty,-\infty\le y\le\infty$$

The state with energy $4\hbar\omega$ is $g-$fold degenerate. The value of g is_____

Q32. A hydrogen atom is in the state

$$\psi=\sqrt{\frac{8}{21}}\psi_{200}-\sqrt{\frac{3}{7}}\psi_{310}+\sqrt{\frac{4}{21}}\psi_{321}'$$

where n,l,m in ψ_{nlm} denote the principal. Orbit and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of L^2 is _____ $\hbar^2$

Q33. A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r)=-\frac{k}{r}$, where k is a positive constant. The orbit angular momentum of the planet is

(a) $2r_0km$ (b) $\sqrt{2r_0km}$ (c) r_0km (d) $\sqrt{r_0km}$

Q34. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$

(c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

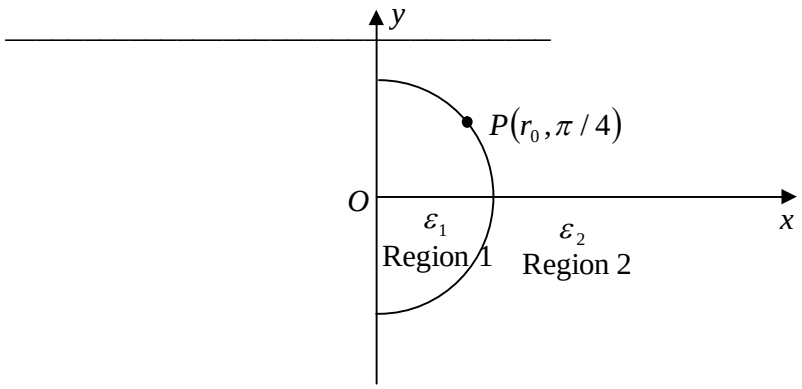
Q35. The value of the integral

$$\oint_C \frac{z^2}{e^z + 1} dz$$

where C is the circle $|z| = 4$, is

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

Q36. A ray of light inside Region 1 in the $xy-$plane is incident at the semicircular boundary that carries no free charges. The electric field at the point $P\left(r_0 \frac{\pi}{4}\right)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$, where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in Region 2 has the electric field $\vec{E}_2$ parallel to $x-$axis. If ε_1 and ε_2 are the dielectric constants of Region 1and Region 2 respectively then $\frac{\varepsilon_2}{\varepsilon_1}$ is



Q37. The solution of the differential equation

$$\frac{d^2y}{dt^2} - y = 0$$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$ is

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$

(c) $\cos t - \sin t$ (d) $\cosh t - \sin t$

Q38. Given that the linear transformation of a generalized coordinate q and the corresponding momentum p,

$$Q = q + 4ap$$

$$P = q + 2p$$

is canonical, the value of the constant a is _____

Q39. The value of the magnetic field required to maintain non-relativistic protons of energy $1MeV$ in a circular orbit of radius 100mm is_____Tesla

(Given: $m_p = 1.67 \times 10^{-27}\, kg.e = 1.6 \times 10^{-19} C$)

Q40. For a system of two bosons each of which can occupy any of the two energy levels 0and ε the mean energy of the system at temperature T with $\beta = \frac{1}{k_\beta T}$ is given by

(a) $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (b) $\frac{1 + \varepsilon e^{-\beta\varepsilon}}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(c) $\frac{2\varepsilon e^{-\beta\varepsilon} + \varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

Q41. In an interference pattern formed by two coherent sources, the maximum and minimum of the intensities are $9I_0$ and I_0, respectively. The intensities of the individual wave are

(a) $3I_0$ and I_0 (b) $4I_0$ and I_0

(c) $5I_0$ and $4I_0$ (d) $9I_0$ and I_0

Q42. ψ_1 and ψ_2 are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that

$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix},$$

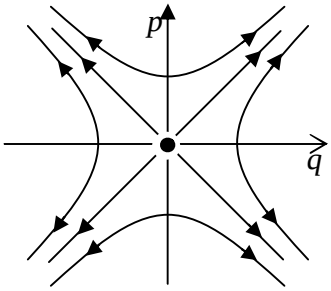
where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represent the spin-up and spin-down states, respectively. When the system is in the state ψ_2 its probability to be in the spin-up state is _____

Q43. Neutrons moving with speed $10^3\ m/s$ are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is 30^o the interplannar spacing of the crystal is _____ $\overset{0}{A}$

(Given: $m_n = 1.675\times10^{27}\ kg\ h = 6.626\times10^{-34}\ J.s$)

Q44. The Hamiltonian of particle of mass m is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$.which one of the following figure describes the motion of the particle in phase space?

(a)



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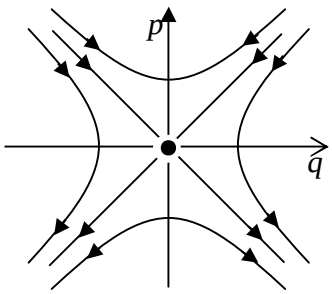

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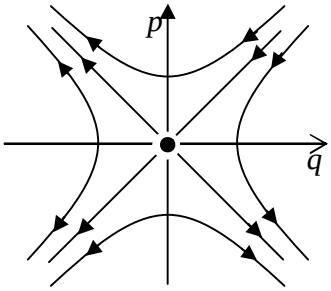

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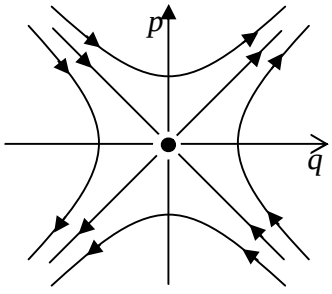


Q45. The intensity of a laser in free space is $150 mW/m^2$. The corresponding amplitude of the electric field of the laser is _____ $\frac{V}{m}$ $\left(\varepsilon_0 = 8.854\times10^{-12} C^2/N.m^2\right)$

Q46. The emission wavelength for the transition $D_2 \to F_3$ is 3122 ´. The ratio of population of the final to the initial states at a temperature $5000K$ is $\left(h = 6.626\times10^{-34} J.s, c = 3\times10^8 m/s\, k_B = 1.380\times10^{-23} J/K\right)$

(a) 2.03×10^{-5} (b) 4.02×10^{-5} (c) 7.02×10^{-5} (d) 9.83×10^{-5}

Q47. Consider a system of 3 fermions which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(a) $k_B \ln 2$ (b) $2k_B \ln 2$ (c) $2k_B \ln 4$ (d) $3k_B \ln 4$

Q48. A particle is confined to a one dimensional potential box, with the potential

$$V(x) = 0, \quad 0 < x < a$$
$$= \infty, \quad \text{otherwise}$$

If particle is subjected to a perturbation within the box. $W = \beta x$. Where β is small constant, the first order correction to the ground state energy is

(a) 0 (b) $a\beta/4$ (c) $a\beta/2$ (d) $a\beta$

Q49. consider the process $\mu^+ + \mu \to \pi^+ + \pi^-$. The minimum kinetic energy of the muons (μ) in the centre of mass frame required to produce the pion (π) pairs at rest is _____ Mev.

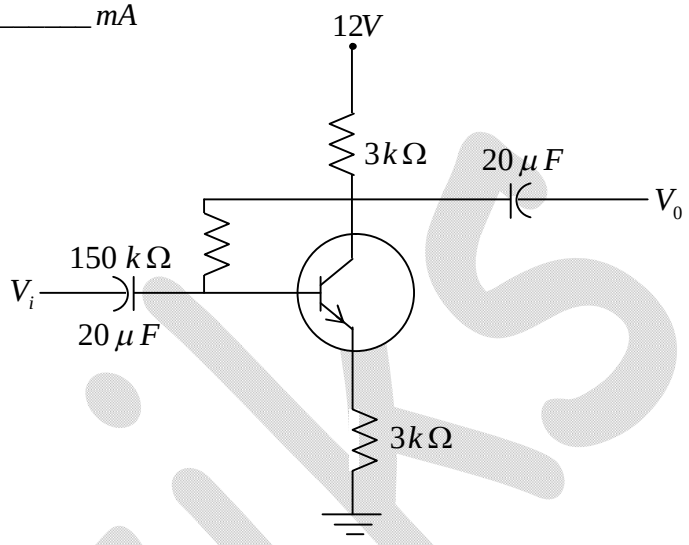
Q50. A one dimensional harmonic oscillator is in the superposition of number state $|n\rangle$ given by $|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$

The average energy of the oscillator in the given state is_____ $\hbar\omega$.

Q51. A nucleus X undergoes a first forbidden $\beta-$decay to nucleus Y. If the angular momentum (I) and parity (P), denoted by I^P as $\frac{7^-}{2}$ for X which of the following is a possible I^P value for Y?

(a) $\frac{1^+}{2}$ (b) $\frac{1^-}{2}$ (c) $\frac{3^+}{2}$ (d) $\frac{3^-}{2}$

Q52. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current I_c is_____ mA



Q53. In order to measure a maximum of $1V$ with a resolution of $1mV$ using a $n-$bit $\frac{A}{D}$ converter working under the principle of ladder network the minimum value of n is _____

Q54. If L_+ and L_- are the angular momentum ladder operators then the expectation value of $(L_+L_- + L_-L_+)$ in the state $|l=1, m=1\rangle$ of an atom is _____ $\hbar$

Q55. A low pass liter is formed by a resistance R and a capacitance C. At the cut-off angular frequency $\omega_c = \frac{1}{RC}$, the voltage gain and the phase of the output voltage relative to the input voltage respectively are

(a) 0.71 and 45^o (b) 0.71 and -45^o

(c) 0.5 and -90^o (d) 0.5 and 90^o

Q.1-Q.5 carry one mark each.

Q1. A student is required to demonstrate a high level of comprehension of the subject, especially in the social sciences.

The word closest in meaning to comprehension is

(a) understanding (b) meaning

(c) concentration (d) stability

Q2. choose the most appropriate word from the options given below to complete the following sentence.

One of his biggest _____was his ability to forgive

(a) vice (b) virtues (c) choices (d) strength

Q3. Rajan was not happy that Sajan decided to do the project on his own on observing his unhappiness. Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(a) Rajan has decided to work only in a group

(b) Rajan and Sajan were formed into a group against their wishes

(c) Sajan had decided to give in to Rajan's request to work with him

(d) Rajan had believed that Sajan and he would be working together

Q4. If $y = 5x^2 + 3$, then the tangent at $x = 0, x = 3$

(a) passes through $x = 0, y = 0$ (b) has a slope of +1

(c) is parallel to the $x-$axis (d) has a slope of -1

Q5. A foundry has a fixed daily cost of Rs 50,000 whenever it operates and a variable cost of Rs $800Q$, where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes?

Q.6-Q.10 carry two mark each.

Q6. Find the odd one in the following group: $ALRVX, EPVZB, ITZDF, OYEIK$

(a) $ALRVX$ (b) $EPVZB$ (c) $ITZDF$ (d) $OYEIK$

Q7 Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even numbered floor, Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

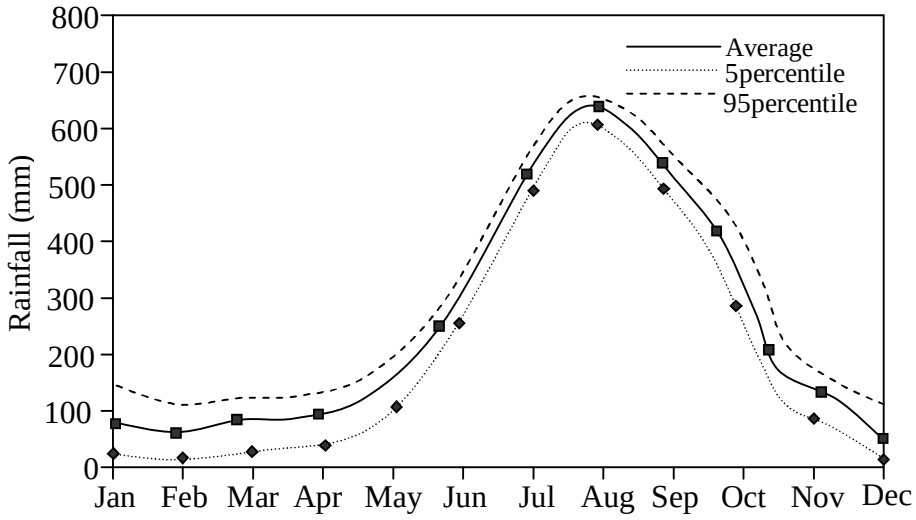
	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(A)	6	2	5	1	3	4
(B)	2	6	5	1	3	4
(C)	4	2	6	3	1	5
(D)	2	4	6	1	3	5

Q8 The smallest angle of a triangle is equal to two third of the smallest angle of quadrilateral. The ratio between the angles of the quadrilateral is 3:4:5:6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

Q9. One percent of the people of country X are taller than 6 ft. two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6ft?

(a) 3.0 (b) 2.5 (c) 1.5 (d) 1.25

Q10. The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? (k percentile is the value such that k percent of the data fall below the value)



(i) On average, it rains more in July than in December

(ii) Every year, the amount of rainfall in August is more than that in January

(iii) July rainfall can be estimated with better confidence than February rainfall

(iv) In August, there is at least 500 mm of rainfall

(a) (i) and (ii) (b) (i) and (iii) (c) (ii) and (iii) (d) (iii) and (iv)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