HCU Ph.D. 2020

PART - A

Q1. If g is the number of girls in a school foot ball team and $g+30$ is the number of boys, the percentage of girls in the team is

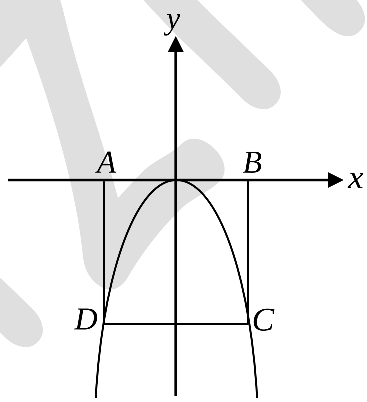
(a) $\frac{g}{(g+30)}\%$ (b) $\frac{g}{200(g+30)}\%$

(c) $\frac{100g}{2g+30}\%$ (d) $\frac{100g}{g+30}\%$

Q2. Consider functions $f(x)=(x-1)^3$ and $g(x)=(x+1)^3$. For how many values of 'x' do the given functions $f(x)$ and $g(x)$ have the same value?

(a) 0 (b) 1 (c) 2 (d) 3

Q3. In the figure shown, the square $ABCD$ intersects the graph $y=h(x)$ at points C and D. Points A and B lie on the x- axis. If the area of $ABCD$ is 36 units, and $h(x)=kx^2$, the value of k is



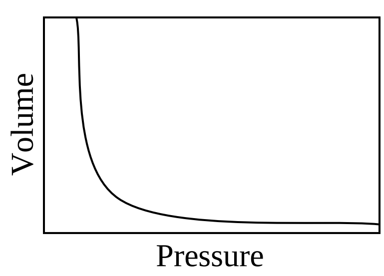
(a) $\frac{1}{3}$

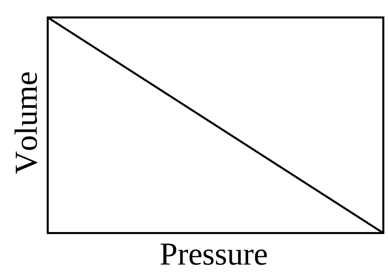
(b) $-\frac{1}{6}$

(c) $-\frac{1}{3}$

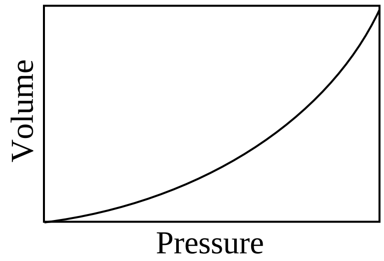
(d) $-\frac{2}{3}$

Q4. A sample of ideal gas is subjected to compression at constant temperature. Which of the following graphs would accurately depict the outcome?

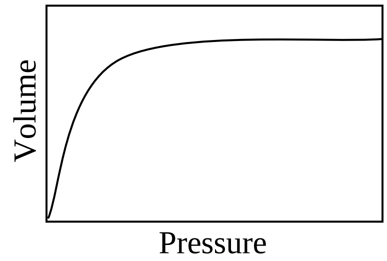
(a)


(b)


(c)



(d)



Q5. A set of 37 numbers were given as data and 3 of them were wrongly entered as $48, 52$ and 40, resulting in a mean 45. If the numbers are corrected to 53, 75 and 49, then the correct mean is

(a) 46 (b) 47 (c) 48 (d) 49

Q6. Ancient Egyptians are experts in building pyramids of different sizes. If they plan to build a pyramid with a square base of $50\,m$ long edges and the four sides as equilateral triangles, the height of the pyramid is

(a) $50\sqrt{3}$ (b) $50\sqrt{2}$ (c) $25\sqrt{3}$ (d) $25\sqrt{2}$

Q7. A survey on TV watching habit of the people was carried out. Among 1000 persons sampled in the survey, 25% were adults and the rest were children. 300 persons enjoyed watching TV. 30% of those who have not watched TV were adults. The number of children who watched TV is

(a) 260 (b) 240 (c) 230 (d) 250

Q8. A box contains 5 blue, 4 red, 3 black pens. If three pens are drawn at random and are not replaced, the probability of drawing two blue pens first and then one black pen is

(a) $\frac{1}{11}$ (b) $\frac{1}{22}$ (c) $\frac{5}{88}$ (d) $\frac{5}{144}$

Q9. A bag contains balls of four different colours. All but 12 balls are yellow, all but 11 balls are red, all but 10 balls are blue and all but 9 balls are green. The number of green balls in the bag are

(a) 3 (b) 4 (c) 5 (d) 6

Q10. Three runners A, B and C participate in a race. Runner "A" is twice as likely to win as "B" and the runner "B" is twice as likely to win as "C". The probabilities of runners A, B and C winning the race are respectively,

(a) $\frac{1}{7}, \frac{2}{7}, \frac{4}{7}$ (b) $\frac{2}{7}, \frac{1}{7}, \frac{4}{7}$ (c) $\frac{4}{7}, \frac{1}{7}, \frac{2}{7}$ (d) $\frac{4}{7}, \frac{2}{7}, \frac{1}{7}$

Q11. There are 35 students in a hostel. Due to the admission of 7 new students the expenses of the mess increases by Rs. 42 per day while the average expenditure per student decreases by Rs. 1. The original expenditure of the mess, per day, is

(a) Rs. 320 (b) Rs. 420 (c) Rs. 450 (d) Rs. 350

Q12. In a certain class, the male students outnumber female students by a ratio $2{:}1$. If 4 males and 6 females get additionally admitted, the new ratio of males to females is 1. Then the total strength of the class now will be

(a) 8 (b) 12 (c) 14 (d) 16

Q13. The missing (*) number in the sequence $2, 8, 18, *, 50$ is

(a) 28 (b) 30 (c) 32 (d) 36

Q14. An island has two kinds of inhabitants; "T" s, who always tell the truth and their opposites, "L"s, who always lie. If one randomly encounters two people A and B in the island and then if A says "B is a T" and B says "The two of us are opposite types?"

(a) A is "T" and B is "L" (b) A is "L" and B is "T"

(c) Both A and B are "T"s (d) Both A and B are "L"s

Q15. If CORONA is DOSOOB, then VIRUS is

(a) VJSVT (b) VJSUT (c) WISUT (d) WITUT

Q16. The last digit of 3^{2019} is

(a) 1 (b) 3 (c) 7 (d) 9

Q17. The coefficient of a^2b^5d in the expansion of $(a+b-c-d)^8$ is

(a) $+168$ (b) -168 (c) $+336$ (d) -336

Q18. If 1, ω and ω^2 are the cube roots of unity, then $(1+\omega^2)^4$ is

(a) 1 (b) ω (c) ω^2 (d) 4

Q19. Given $S = 1^2 - 2^2 + 3^2 - 4^2 + ... + 13^2$, the value of S is

(a) -9 (b) 91 (c) 90 (d) 119

Q20. In an election between two candidates, one candidate got 55% of the total valid votes with 20% of the total votes are declared invalid. If the total number of votes polled was 7500, the number of valid votes that the other candidate got is

(a) 2700 (b) 2900 (c) 3000 (d) 3100

PART - B

Q21. The residue of $f(z) = z(z-1)^{-1}(z+1)^{-2}$ at $z = -1$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) 1

Q22. The eigen values of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

(a) $(1,4,9)$ (b) $(0,7,7)$ (c) $(0,1,13)$ (d) $(0,0,14)$

Q23. The function ϕ is a scalar satisfying Laplace equation $\nabla^2\phi = 0$, in a region $\mathbb{R}^3$. Then the function $\nabla\phi$ is

(a) only solenoidal

(b) only irrotational

(c) both solenoidal and irrotational

(d) neither solenoidal nor irrotational

Q24. The number of cyclic coordinates for the Lagrangian of a free particle in 2 dimensions for the Cartesian-coordinates system and plane-polar coordinate system respectively, are

(a) 1,1 (b) 1,2 (c) 2,1 (d) 2,2

Q25. Given the Hamiltonian $H = \frac{p^2}{2} - \frac{a}{q^2}$, where a is any arbitrary constant. The numerical coefficient α such that $S = \frac{pq}{2} - \alpha Ht$, is a constant of motion, is

(a) $+1$ (b) -1 (c) $+\frac{1}{2}$ (d) $-\frac{1}{2}$

Q26. The quantity that commutes with zp_z is

(a) L_x (b) L_y (c) L_z (d) Both $L_z, \vec{L}^2$

Q27. The state of a hydrogen atom is given by the wavefunction $\psi = \frac{1}{6}\left(4\psi_{100} + 3\psi_{211} - \psi_{210} + \sqrt{10}\psi_{21-1}\right)$. (In the notation ψ_{nlm}, nlm represent the principal, orbital angular momentum, and azimuthal quantum numbers respectively). If $E_0 = -13.6\,eV$ is the ground state energy of the hydrogen atom, the expectation value of $\frac{H}{E_0}$, is

(a) $\frac{7}{18}$ (b) $\frac{7}{3}$ (c) $\frac{7}{12}$ (d) $\frac{5}{6}$

Q28. Consider the wave function

$$\psi_a(x) = \exp\left(\frac{-ia\hat{p}}{\hbar}\right)\psi_0(x)$$

where ψ_0 is the ground state function of linear harmonic oscillator. If the expectation value of $\hat{x}, \hat{x}^2$ is calculated for $\psi_a(x)$, which among the following depends on the parameter a?

(a) Only $< x >$

(b) Only $< x^2 >$

(c) Both $< x >, < x^2 >$

(d) Neither $< x >, < x^2 >$

Q29. A spherical conductor having uniform surface density bas electrostatic field energy $0.6\,keV$. If the total amount of the surface charge is $1.6\times10^{-14}\,C$, then the radius of the sphere is [assume $\frac{1}{(4\pi\in_0)} \approx 9\times10^9$ SI units]

(a) $12.0\,mm$ (b) $14.0\,mm$ (c) $16.0\,mm$ (d) $20.0\,mm$

Q30. Two vector potential are given $A_I = (-yB_0, 0, 0), A_{II} = \left(-\frac{y}{2}B_0, \frac{x}{2}B, 0\right)$ where B_0 is a constant. Which of the following statement is correct?

(a) Only A_I represent magnetic field $\vec{B} = B_0\hat{z}$

(b) Only A_{II} represent magnetic field $\vec{B} = B_0\hat{z}$

(c) Both A_I and A_{II} represent magnetic field $\vec{B} = B_0\hat{z}$

(d) Neither A_I nor A_{II} represent magnetic field $\vec{B} = B_0\hat{z}$

Q31. A random walker takes a step of unit length in the positive direction with probability $\frac{2}{3}$ and a step of unit length in the negative direction with probability $\frac{1}{3}$. The mean displacement of the walker after n steps is

(a) $\frac{n}{3}$ (b) $\frac{n}{8}$ (c) $\frac{2n}{3}$ (d) 0

Q32. Consider a rigid lattice of N number of spin $\frac{1}{2}$ atoms in an external constant magnetic field $\vec{B} = B\hat{k}$. Each atom has two energy eigenstates of energies $-\mu_B B$ and $\mu_B B$ for spins up and down, respectively, relative to $\vec{B}$. The system is at a temperature T. As $T \to \infty$, the specific heat per particle (c_v) of such a system at constant volume tends to

(a) 0 (b) $\frac{k_B}{2}$ (c) k_B (d) $k_B \sec h^2\left(\frac{\mu_B B}{k_B T}\right)$

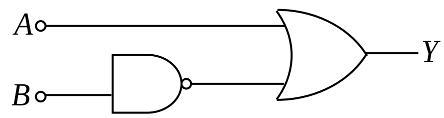
Q33. Given speed of sound in a crystalline solid is $c/1000$ (where c is speed of light) and number density of the constituent atoms $10^{27}/m^3$, then Debye temperature is of the order of

(a) $9\,K$ (b) $90\,K$ (c) $900\,K$ (d) $9000\,K$

Q34. X-ray of wavelength $\lambda = a$ is reflected from the (111) plane of a simple cubic lattice. If the lattice constant is a, the corresponding Bragg angle (in radians) is

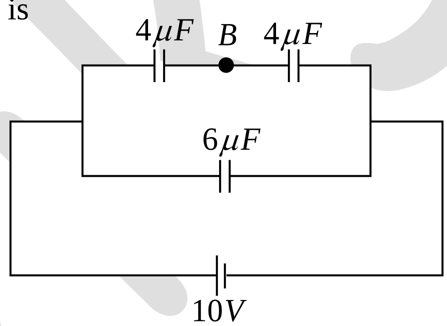
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{8}$

Q35. The output 'Y' for the given logic circuit is



(a) A (b) B (c) 0 (d) 1

Q36. Three capacitors are connected to a $10V$ power supply as shown in the figure. The charge on the $4\mu F$ capacitor on top left is



(a) $10\mu C$

(b) $20\mu C$

(c) $40\mu C$

(d) $60\mu C$

Q37. Among the following (1) $e^- + p \rightarrow \nu_e + \pi^0$ and (2) $\gamma + p \rightarrow \pi^+ + n$, the allowed processes are

(a) only (1) (b) only (2)

(c) both (1) and (2) (d) neither (1) nor (2)

Q38. A relativistic particle of rest mass m_0 is moving in a free-space with a speed v. The value of v at which the relativistic kinetic energy is double to its rest mass energy is (c is speed of light in vacuum)

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{2}{3}c$ (c) $\frac{2\sqrt{2}}{3}c$ (d) $\frac{2\sqrt{\sqrt{2}}}{3}c$

Q39. The electric field of an EM wave propagating in vacuum is given by $\vec{E} = \hat{y}E_0 \exp\left[i\left(hz - \omega t\right) - kx\right]$. The relationship between the real parameters h, k and ω is

(a) $h^2 + k^2 = \frac{\omega^2}{c^2}$ (b) $h^2 - k^2 = \frac{\omega^2}{c^2}$

(c) $h^2 - k^2 = -\frac{\omega^2}{c^2}$ (d) $h^2 + k^2 = -\frac{\omega^2}{c^2}$

Q40. The electric field of an EM wave is given by $\vec{E} = E_0 \cos\left(kz - \omega t\right)\hat{x} + E_0 \sin\left(kz + \omega t\right)\hat{y}$.

The polarization of this field at $z = \frac{\lambda}{4}$ is

(a) linear (b) circular (c) elliptical (d) unpolarized