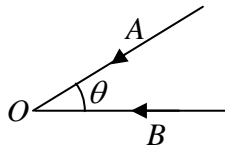


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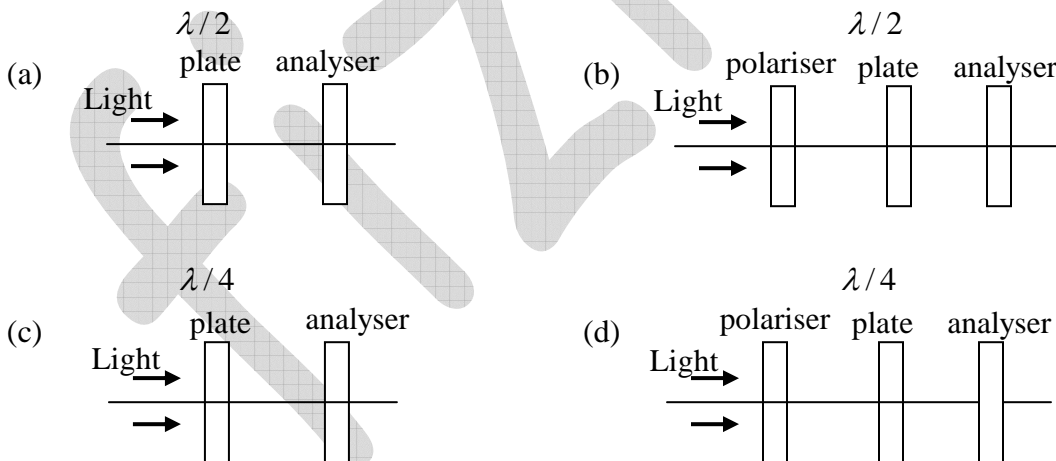
Q1. – Q10. carry one mark each.

- Q1. Let $f(x, y) = x^3 - 2y^3$. The curve along which $\nabla^2 f = 0$ is
- (a) $x = \sqrt{2}y$ (b) $x = 2y$
 (c) $x = \sqrt{6}y$ (d) $x = \frac{-y}{2}$
- Q2. A curve is given by $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$. The unit vector of the tangent to the curve at $t = 1$ is
- (a) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (b) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i} + 2\hat{j} + 2\hat{k}}{3}$ (d) $\frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$
- Q3. There are three planets in circular orbits around a star at distances $a, 4a$ and $9a$, respectively. At time $t = t_0$, the star and the three planets are in a straight line. The period of revolution of the closest planet is T . How long after t_0 will they again be in the same straight line?
- (a) $8T$ (b) $27T$ (c) $216T$ (d) $512T$
- Q4. A current I is flowing through the sides of an equilateral triangle of side a . The magnitude of the magnetic field at the centroid of the triangle is
- (a) $\frac{9\mu_0 I}{2\pi a}$ (b) $\frac{\mu_0 I}{\pi a}$ (c) $\frac{3\mu_0 I}{2\pi a}$ (d) $\frac{3\mu_0 I}{\pi a}$
- Q5. Two vehicles A and B are approaching an observer O at rest with equal speed as shown in the figure. Both vehicles have identical sirens blowing at a frequency f_s . The observer hears these sirens at frequency f_A and f_B , respectively from the two vehicles. Which one of the following is correct?

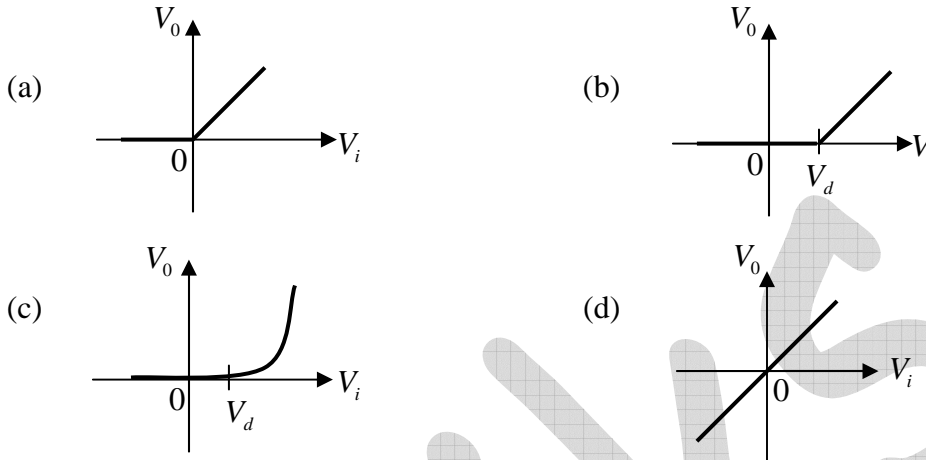
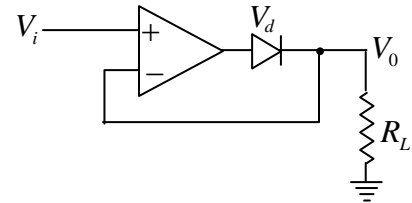


- (a) $f_A = f_B < f_s$ (b) $f_A = f_B > f_s$
 (c) $f_A > f_B > f_s$ (d) $f_A < f_B < f_s$

- Q6. Three infinite plane sheets carrying uniform charge densities $-\sigma, 2\sigma, 3\sigma$ are parallel to the $x-z$ plane at $y = a, 3a, 4a$, respectively. The electric field at the point $(0, 2a, 0)$ is
- (a) $\frac{4\sigma}{\epsilon_0} \hat{j}$ (b) $-\frac{3\sigma}{\epsilon_0} \hat{j}$ (c) $-\frac{2\sigma}{\epsilon_0} \hat{j}$ (d) $\frac{\sigma}{\epsilon_0} \hat{j}$
- Q7. Two boxes A and B contain an equal number of molecules of the same gas. If the volumes are V_A and V_B and λ_A and λ_B denote respective mean free paths, then
- (a) $\lambda_A = \lambda_B$ (b) $\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$ (c) $\frac{\lambda_A}{V_A^{1/2}} = \frac{\lambda_B}{V_B^{1/2}}$ (d) $\lambda_A V_A = \lambda_B V_B$
- Q8. Let T_g and T_e be the kinetic energies of the electron in the ground and the third excited states of a hydrogen atom, respectively. According to the Bohr model, the ratio $\frac{T_g}{T_e}$ is
- (a) 3 (b) 4 (c) 9 (d) 16
- Q9. Which of the following arrangements of optical components can be used to distinguish between an unpolarised light and a circularly polarised light?

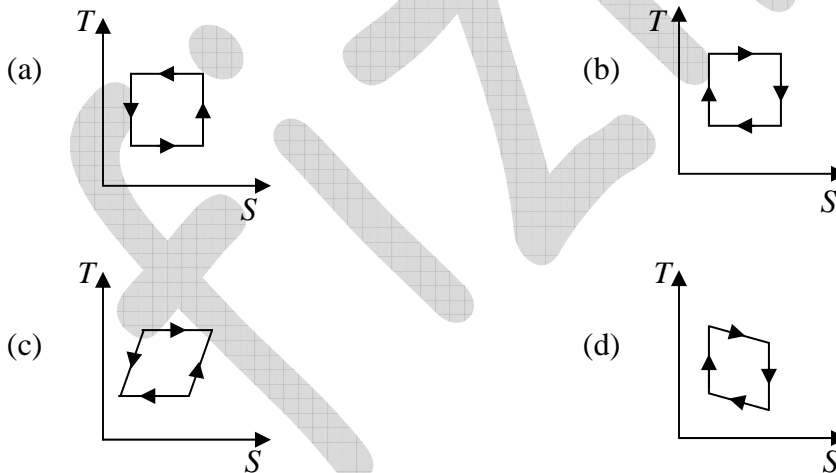


Q10. Which one of the following graphs shows the correct variation of V_0 with V_i ? Here, V_d is the voltage drop across the diode and the OP-Amp is assumed to be ideal.



Q11. – Q30. carry two marks each.

Q11. Which one of the figures correctly represents the $T - S$ diagram of a Carnot engine?

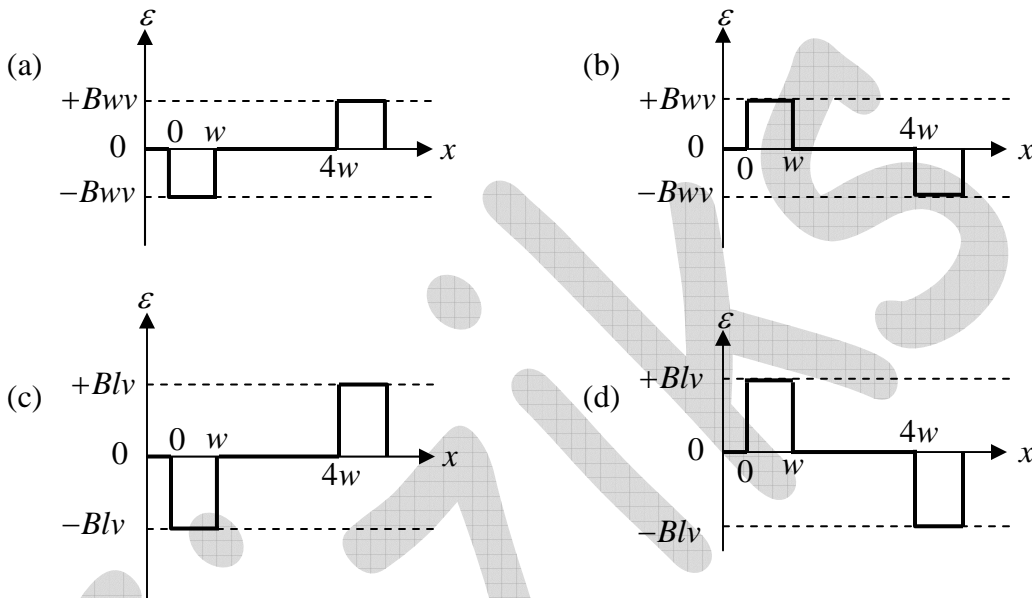
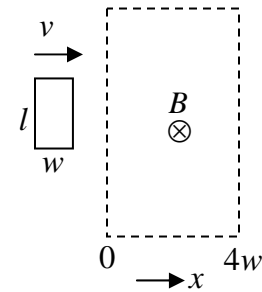


Q12. The plane of polarisation of a plane polarized light rotates by 60° after passing through a wave plate. The pass-axis of the wave plate is at an angle α with respect to the plane of polarization of the incident light. The wave plate and α are

- (a) $\frac{\lambda}{4}, 60^\circ$ (b) $\frac{\lambda}{2}, 30^\circ$ (c) $\frac{\lambda}{2}, 120^\circ$ (d) $\frac{\lambda}{4}, 30^\circ$

Given, $2\alpha = 60^\circ \Rightarrow \alpha = 30^\circ$.

Q13. A rectangular loop of dimensions l and w moves with a constant speed of v through a region containing a uniform magnetic field B directed into the paper and extending a distance of $4w$. Which of the following figures correctly represents the variation of emf (ε) with the position (x) of the front end of the loop?



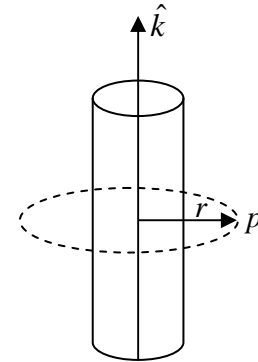
Q14. The equation of state for one mole of a non-ideal gas is given by $PV = A\left(1 + \frac{B}{V}\right)$, where the coefficient A and B are temperature dependent. If the volume changes from V_1 to V_2 in an isothermal process, the work done by the gas is

- (a) $AB\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$ (b) $AB\ln\left(\frac{V_2}{V_1}\right)$
 (c) $A\ln\left(\frac{V_2}{V_1}\right) + AB\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$ (d) $A\ln\left(\frac{V_2 - V_1}{V_1}\right) + B$

Q15. An ideal gas consists of three dimensional polyatomic molecules. The temperature is such that only one vibrational mode is excited. If R denotes the gas constant, then the specific heat at constant volume of one mole of the gas at this temperature is

- (a) $3R$ (b) $\frac{7}{2}R$ (c) $4R$ (d) $\frac{9}{2}R$

Q16. A long solenoid is carrying a time dependent current such that the magnetic field inside has the form $\vec{B}(t) = B_0 t^2 \hat{k}$, where \hat{k} is along the axis of the solenoid. The displacement current at the point P on a circle of radius r in a plane perpendicular to the axis



- (a) is inversely proportional to r and radially outward
- (b) is inversely proportional to r and tangential
- (c) increases linearly with time and is tangential.
- (d) is inversely proportional to r^2 and tangential

Q17. Consider an ensemble of thermodynamic systems each of which is characterized by the same number of particles, pressure and temperature. The thermodynamic function describing the ensemble is

- (a) Enthalpy
- (b) Helmholtz free energy
- (c) Gibbs free energy
- (d) Entropy

Q18. Given a spherically symmetric charge density $\rho(r) = \begin{cases} kr^2, & r < R \\ 0, & r > R \end{cases}$ (k being a constant),

the electric field for $r < R$ is (take the total charge as Q)

- (a) $\frac{Qr^3}{4\pi\epsilon_0 R^5} \hat{r}$
- (b) $\frac{3Qr^2}{4\pi\epsilon_0 R^4} \hat{r}$
- (c) $\frac{5Qr^3}{8\pi\epsilon_0 R^5} \hat{r}$
- (d) $\frac{Q}{4\pi\epsilon_0 R^5} \hat{r}$

Q19. An infinitely long solenoid, with its axis along \hat{k} , carries a current I . In addition there is a uniform line charge density λ along the axis. If \vec{S} is the energy flux, in cylindrical coordinates $(\hat{\rho}, \hat{\phi}, \hat{k})$, then

- (a) \vec{S} is along $\hat{\rho}$
- (b) \vec{S} is along \hat{k}
- (c) \vec{S} has non zero components along $\hat{\rho}$ and \hat{k}
- (d) \vec{S} is along $\hat{\rho} \times \hat{k}$

Q20. Consider two waves $y_1 = a \cos(\omega t - kz)$ and $y_2 = a \cos[(\omega + \Delta\omega)t - (k + \Delta k)z]$. The group velocity of the superposed wave will be ($\Delta\omega \ll \omega$ and $\Delta k \ll k$)

- (a) $\frac{(\omega - \Delta\omega)}{(k - \Delta k)}$ (b) $\frac{(2\omega - \Delta\omega)}{(2k + \Delta k)}$ (c) $\frac{\Delta\omega}{\Delta k}$ (d) $\frac{(\omega + \Delta\omega)}{(k + \Delta k)}$

Q21. Consider a convex lens of focal length f . A point object moves towards the lens along its axis between $2f$ and f . If the speed of the object is V_o , then its image would move with speed V_I . Which of the following is correct?

- (a) $V_I = V_o$; the image moves away from the lens.
 (b) $V_I = -V_o$; the image moves away from the lens.
 (c) $V_I > V_o$; the image moves away from the lens.
 (d) $V_I < V_o$; the image moves away from the lens.

Q22. A disc of radius R_1 having uniform surface density has a concentric hole of radius $R_2 < R_1$. If its mass is M , the principal moments of inertia are

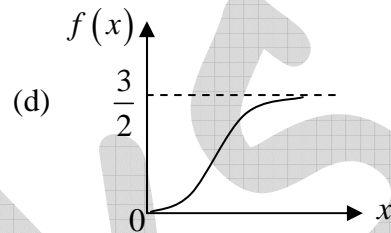
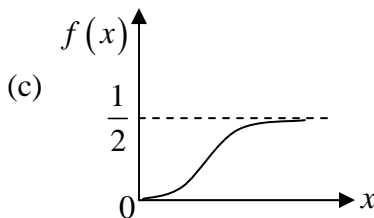
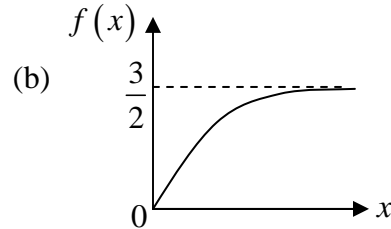
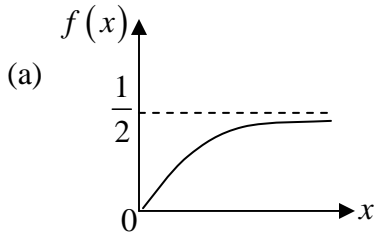
- (a) $\frac{M(R_1^2 - R_2^2)}{2}, \frac{M(R_1^2 - R_2^2)}{4}, \frac{M(R_1^2 - R_2^2)}{4}$
 (b) $\frac{M(R_1^2 + R_2^2)}{2}, \frac{M(R_1^2 + R_2^2)}{4}, \frac{M(R_1^2 + R_2^2)}{4}$
 (c) $\frac{M(R_1^2 + R_2^2)}{2}, \frac{M(R_1^2 + R_2^2)}{4}, \frac{M(R_1^2 + R_2^2)}{8}$
 (d) $\frac{M(R_1^2 - R_2^2)}{2}, \frac{M(R_1^2 - R_2^2)}{4}, \frac{M(R_1^2 - R_2^2)}{8}$

Q23. The function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ is expanded as a Fourier series of the form

$a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$. Which of the following is true?

- (a) $a_0 \neq 0, b_n = 0$ (b) $a_0 \neq 0, b_n \neq 0$
 (c) $a_0 = 0, b_n = 0$ (d) $a_0 = 0, b_n \neq 0$

Q24. Which one of the following curves correctly represents (schematically) the solution for the equation $\frac{df}{dx} + 2f = 3$: $f(0) = 0$?



Q25. The mean momentum \bar{p} of a nucleon in a nucleus of mass number A and atomic number Z depends on A, Z as

- (a) $\bar{p} \propto A^{\frac{1}{3}}$ (b) $\bar{p} \propto Z^{\frac{1}{3}}$ (c) $\bar{p} \propto A^{-\frac{1}{3}}$ (d) $\bar{p} \propto (AZ)^{\frac{2}{3}}$

Q26. The Boolean expression $(\overline{AB})(\overline{A+B})(A+B)$ can be simplified to

- (a) $A+B$ (b) \overline{AB} (c) $\overline{A+B}$ (d) AB

Q27. Consider the transformation to a new set of coordinates (ξ, η) from rectangular Cartesian coordinates (x, y) , where $\xi = 2x + 3y$ and $\eta = 3x - 2y$. In the (ξ, η) coordinate system, the area element $dxdy$ is

- (a) $\frac{1}{13} d\xi d\eta$ (b) $\frac{2}{13} d\xi d\eta$ (c) $5d\xi d\eta$ (d) $\frac{3}{5} d\xi d\eta$

Q28. A particle of mass m is in a one dimensional potential $V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$. At some

instant its wave function is given by $\psi(x) = \frac{1}{\sqrt{3}}\psi_1(x) + i\sqrt{\frac{2}{3}}\psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ are the ground and the first excited states, respectively. Identify the correct statement.

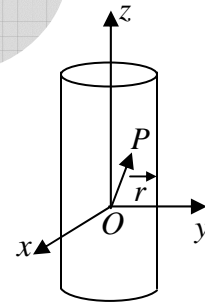
- (a) $\langle x \rangle = \frac{L}{2}; \langle E \rangle = \frac{\hbar^2}{2m} \frac{3\pi^2}{L^2}$ (b) $\langle x \rangle = \frac{2L}{3}; \langle E \rangle = \frac{\hbar^2}{2m} \frac{\pi^2}{L^2}$
 (c) $\langle x \rangle = \frac{L}{2}; \langle E \rangle = \frac{\hbar^2}{2m} \frac{8\pi^2}{L^2}$ (d) $\langle x \rangle = \frac{2L}{3}; \langle E \rangle = \frac{\hbar^2}{2m} \frac{4\pi^2}{3L^2}$

Q29. A raindrop falls under gravity and captures water molecules from atmosphere. Its mass changes at the rate $\lambda m(t)$, where λ is a positive constant and $m(t)$ is the instantaneous mass. Assume that acceleration due to gravity is constant and water molecules are at rest with respect to earth before capture. Which of the following statements is correct?

- (a) The speed of the raindrop increases linearly with time
- (b) The speed of the raindrop increases exponentially with time
- (c) The speed of the raindrop approaches a constant value when $\lambda t \gg 1$
- (d) The speed of the raindrop approaches a constant value when $\lambda t \ll 1$

Q30. A particle P of mass m is constrained to move on the surface of cylinder under a force $-k\vec{r}$ as shown in figure (k is the positive constant). Which of the following statements is correct? (Neglect friction.)

- (a) Total energy of the particle is not conserved.
- (b) The motion along z direction is simple harmonic.
- (c) Angular momentum of the particle about O increases with time.
- (d) Linear momentum of the particle is conserved.



SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q31. – Q40. carry two marks each.

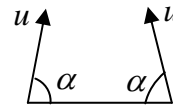
Q31. Let matrix $M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$. If $\det(M) = 0$, then

- (a) M is symmetric (b) M is invertible
(c) one eigenvalue is 13 (d) Its eigenvectors are orthogonal

Q32. Let $f(x) = 3x^6 - 2x^2 - 8$. Which of the following statements is (are) true?

- (a) The sum of all its roots is zero
(b) The product of its roots is $-\frac{8}{3}$
(c) The sum of all its roots is $\frac{2}{3}$
(d) Complex roots are conjugates of each other.

Q33. Two projectiles of identical mass are projected from the ground with same initial angle (α) with respect to earth surface and same initial velocity (u) in the same plane. They collide at the highest point of their trajectories and stick to each other. Which of the following statements is (are) correct?



- (a) The momentum of the combined object immediately after collision is zero.
(b) Kinetic energy is conserved in the collision
(c) The combined object moves vertically downward.
(d) The combined object moves in a parabolic path.

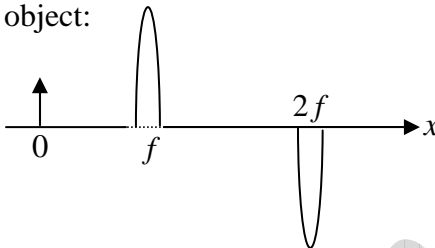
Q34. Two beams of light in the visible range ($400 \text{ nm} - 700 \text{ nm}$) interfere with each other at a point. The optical path difference between them is 5000 nm . Which of the following wavelengths will interfere constructively at the given point?

- (a) 416.67 nm (b) 555.55 nm (c) 625 nm (d) 666.66 nm

Q35. Which of the following relations is (are) true for thermodynamic variables?

- (a) $TdS = C_v dT + T \left(\frac{\partial P}{\partial T} \right)_v dV$ (b) $TdS = C_p dT - T \left(\frac{\partial V}{\partial T} \right)_p dP$
(c) $dF = -SdT + PdV$ (d) $dG = -SdT + VdP$

- Q36. Consider a convex lens of focal length f . The lens is cut along a diameter in two parts. The two lens parts and an object are kept as shown in the figure. The images are formed at following distances from the object:



- (a) $2f$ (b) $3f$ (c) $4f$ (d) ∞
- Q37. Let the electric field in some region R be given by $E = e^{-y^2} \hat{i} + e^{-x^2} \hat{j}$. From this we may conclude that
- (a) R has a non-uniform charge distribution
 (b) R has no charge distribution
 (c) R has a time dependent magnetic field.
 (d) The energy flux in R is zero everywhere.
- Q38. In presence of a magnetic field $B\hat{j}$ and an electric field $(-E)\hat{k}$, a particle moves undeflected. Which of the following statements is (are) correct?
- (a) The particle has positive charge, velocity $= -\frac{E}{B} \hat{i}$
 (b) The particle has positive charge, velocity $= \frac{E}{B} \hat{i}$
 (c) The particle has negative charge, velocity $= -\frac{E}{B} \hat{i}$
 (d) The particle has negative charge, velocity $= \frac{E}{B} \hat{i}$
- Q39. In a pn junction, dopant concentration on the p -side is higher than that on the n -side. Which of the following statements is (are) correct, when the junction is unbiased?
- (a) The width of the depletion layer is larger on the n -side.
 (b) At thermal equilibrium the Fermi energy is higher on the p -side.
 (c) In the depletion region, number of negative charges per unit area on the p -side is equal to number of positive charges per unit area on the n -side
 (d) The value of the built-in potential barrier depends on the dopant concentration.

Q46. Consider a monoatomic ideal gas operating in a closed cycle as shown in the $P-V$ diagram given below. The ratio $\frac{P_1}{P_2}$ is _____.

(Specific your answer upto two digits after the decimal point)

Q47. Consider the first order phase transition of the sublimation of zinc. Assume the vapour to be an ideal gas and the molar volume of solid to be negligible. Experimentally, it is found that $\log_{10}(P) = -\frac{c_1}{T} + c_2$ where P is the vapour pressure in Pascal, T is in K , $c_1 = 6790 K$ and $c_2 = 9$. The latent heat of sublimation of zinc from the Clausius-Clapeyron equation is _____ $kJ/mole$. ($R = 8.314 J/mole.K$)

(Specific your answer in $kJ/mole$ upto one digit after the decimal point.)

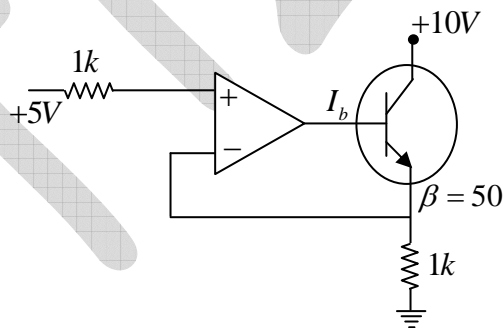
Q48. A system of 8 non-interacting electrons is confined by a three dimensional potential $V(r) = \frac{1}{2} m \omega^2 r^2$. The ground state energy of the system in units of $\hbar \omega$ is _____

(Specify your answer as an integer.)

Q49. For the given circuit, value of the base current (I_b) of the nnp transistor will be _____ mA.

(β is the current gain and assume Op-Amp as ideal.)

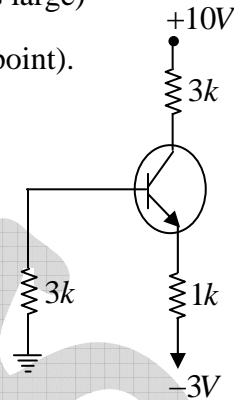
(Specific your answer in mA upto two digits after the decimal point.)



Q50. The lattice constant of unit cell of $NaCl$ crystal is $0.563 nm$. X-rays of wavelength $0.141 nm$ are diffracted by this crystal. The angle at which the first order maximum occurs is _____ degrees.

Q51. – Q60. carry two marks each.

- Q51. For the following circuit, the collector voltage with respect to ground will be _____
 V . (Emitter diode voltage is $0.7V$ and β_{DC} of the transistor is large)
 (Specify your answer in volts upto one digit after the decimal point).



- Q52. A body of mass 1 kg is moving under a central force in an elliptic orbit with semi major axis 1000 m and semi minor axis 100 m . The orbital angular momentum of the body is $100\text{ kg m}^2\text{ s}^{-1}$. The time period of motion of the body is _____ hours.
 (Specify your answer upto two digits after the decimal point)
- Q53. The moon moves around the earth in a circular orbit with a period of 27 days. The radius of the earth (R) is $6.4 \times 10^6\text{ m}$ and the acceleration due to gravity on the earth surface is 9.8 ms^{-2} . If D is the distance of the moon from the center of the earth, the value of $\frac{D}{R}$ will be _____. (Specify your answer upto two digits after the decimal point)
- Q54. A syringe is used to exert 1.5 atmospheric pressure to release water horizontally. The speed of water immediately after ejection is _____ (take 1 atmospheric pressure = 10^5 Pascal, density of water = 10^3 kg m^{-3}) (Specify your answer in ms^{-1} as an integer)
- Q55. Consider a slit of width $18\text{ }\mu\text{m}$ which is being illuminated simultaneously with light of orange color (wavelength 600 nm) and of blue color (wavelength 450 nm). The diffraction pattern is observed on a screen kept at a distance in front of the slit. The smallest angle at which only the orange color is observed is θ_1 and the smallest angle at which only the blue color is observed is θ_2 . The angular difference $\theta_2 - \theta_1$ (in degrees) is _____. (Specify your answer upto two digits after the decimal point)

Q56. A particle of mass m is moving in a circular orbit given by $x = R \cos \omega t$; $y = R \sin(\omega t)$, as observed in an inertial frame S_1 . Another inertial frame S_2 moves with uniform velocity $\vec{v} = \omega R \hat{i}$ with respect to S_1 . S_1, S_2 are related by Galilean transformation, such that the origins coincide at $t = 0$. The magnitude of the angular momentum of the particle at $t = \frac{2\pi}{\omega}$, as observed in S_2 about its origin is expressed as $(mR^2\omega)x$. Then x is _____.

(Specify your answer upto two digits after the decimal point)

Q57. Rod R_1 has a rest length $1m$ and rod R_2 has a rest length of $2m$. R_1 and R_2 are moving with respect to the laboratory frame with velocities $+v\hat{i}$ and $-v\hat{i}$, respectively. If R_2 has a length of $1m$ in the rest frame of R_1 , $\frac{v}{c}$ is given by _____

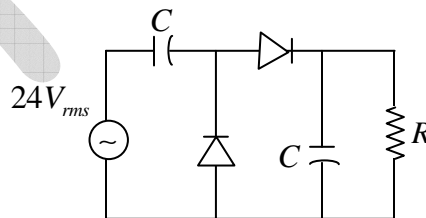
(Specify your answer upto two digits after the decimal point)

Q58. Two events E_1 and E_2 take place in an inertial frame S with respective time space coordinates (in SI units): $E_1(t_1 = 0, \vec{r}_1 = 0)$ and $E_2(t_2 = 0, x_2 = 10^8, y_2 = 0, z_2 = 0)$. Another inertial frame S' is moving with respect to S with a velocity $\vec{v} = 0.8c\hat{i}$. The time difference $(t'_2 - t'_1)$ as observed in S' is _____ s.

(Specify your answer in seconds upto two digits after the decimal point)

Q59. In the following circuit, the time constant RC is much greater than the period of the input signal. Assume diode as ideal and resistance R to be large. The dc output voltage across resistance R will be _____ V.

(Specify your answer in volts upto one digit after the decimal point)



Q60. For a metal, the electron density is $6.4 \times 10^{28} m^{-3}$. The Fermi energy is _____ eV.
($h = 6.626 \times 10^{-34} J s$, $m_e = 9.11 \times 10^{-31} kg$, $1 eV = 1.6 \times 10^{-19} J$)

(Specify your answer in electron volts (eV) upto one digit after the decimal point)