TATA INSTITUTE OF FUNDAMENTAL RESEARCH

GS 2019 ADMISSION TEST

Instructions for candidates for Integrated Ph.D. Programme in Physics

PLEASE READ THESE INSTRUCTIONS CAREFULLY BEFORE YOU ATTEMPT THE QUESTIONS

- This test consists of TWO sections.
  - **SECTION A comprises 25 questions, numbered Q1 - Q25**
    These are questions on basic topics.
  - **SECTION B comprises 15 questions, numbered Q1 - Q15**
    These may require somewhat more thought/knowledge.
- ALL questions are Multiple-Choice Type. In each case, ONLY ONE option is correct. Answer them by clicking the radio button next to the relevant option.
- If your calculated answer does not match any of the given options exactly, you may mark the closest one if it is reasonably close.
- The **grading scheme** will be as follows:
  Section A : **+3** marks if correct; **–1** mark if incorrect; **0** marks if not attempted
  Section B : **+5** marks if correct; **0** marks if incorrect or not attempted,
  i.e. NO negative marks.

SECTION A

(For both Int. Ph.D. and Ph.D. candidates)

This section consists of 25 questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +3 marks, an incorrect answer will get –1 mark.

Q1. Consider the surface defined by $ax^2+by^2+cz+d=0$, where a,b,c and d are constants. If $\hat{n}_1$ and $\hat{n}_2$ are unit normal vectors to the surface at the points $(x,y,z)=(1,1,0)$ and $(0,0,1)$ respectively and $\hat{m}$ is a unit vector normal to both $\hat{n}_1$ and $\hat{n}_2$, then $\hat{m}=$

(a) $\dfrac{-ai+bj}{\sqrt{a^2+b^2}}$ (b) $\dfrac{b\hat{i}-a\hat{j}}{\sqrt{a^2+b^2}}$ (c) $\dfrac{2a\hat{i}+2b\hat{j}-c\hat{k}}{\sqrt{4a^2+4b^2+c^2}}$ (d) $\dfrac{a\hat{i}+b\hat{j}-c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

Q2. The eigenvalues of a 3×3 matrix M are

$$\lambda_1=2 \quad \lambda_2=-1 \quad \lambda_3=1$$

and the eigenvectors are

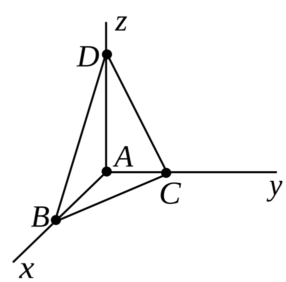
$$e_1=\begin{pmatrix}1\\1\\1\end{pmatrix} \quad e_2=\begin{pmatrix}1\\1\\-2\end{pmatrix} \quad e_3=\begin{pmatrix}1\\-1\\0\end{pmatrix}$$

The matrix M is

(a) $\begin{pmatrix}1&0&1\\0&1&1\\1&1&0\end{pmatrix}$ (b) $\begin{pmatrix}0&1&1\\1&0&0\\1&0&2\end{pmatrix}$ (c) $\begin{pmatrix}1&0&0\\1&0&-1\\0&-1&1\end{pmatrix}$ (d) $\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}$

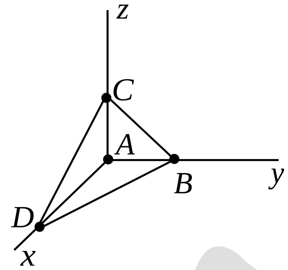
Q3. Which of the following operations will transform a tetrahedron $ABCD$ with vertices as listed below

	x	y	z
A	0	0	0
B	1	0	0
C	0	1	0
D	0	0	2



Into a tetrahedron $ABCD$ with vertices as listed below

	x	y	z
A	0	0	0
B	0	1	0
C	0	0	1
D	2	0	0



Up to suitable translation?

(a) A rotation about x axis by $\frac{\pi}{2}$ then a rotation about z axis by $\frac{\pi}{2}$

(b) A reflection in the xy plane, then a rotation about x axis by $\frac{\pi}{2}$

(c) A reflection in the yz plane, then a rotation about xy plane

(d) A rotation about y axis by $\frac{\pi}{2}$, then a reflection in the xz plane

Q4. A British coin has a portrait of Queen Elizabeth II on the 'heads' side and 'ONE POUND' written on the tails side, while an Indian coin has a portrait of Mahatma Gandhi on the heads side and '10 RUPEES' written on the 'tails' side (see below).

These two coins are tossed simultaneously twice in succession.

The result of the first toss was 'heads' for both the coins. The probability that the result of the second toss had a '10 RUPEES' side is

(a) $\frac{1}{2}$ (b) $\frac{4}{7}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

Q5. A set of polynomials of order n are given by the formula

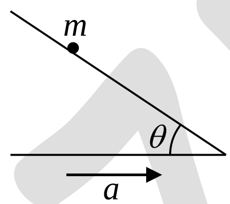
$$p_n(x) = (-1)^n \exp\left(\frac{x^2}{2}\right)\frac{d^n}{dx^n}\exp\left(-\frac{x^2}{2}\right)$$

The polynomial $p_7(x)$ of order $n=7$ is

(a) $x^7 - 21x^5 + 105x^4 + 35x^3 - 105x$
(b) $x^6 - 21x^5 + 105x^4 - 105x^3 + 21x^2 + x$
(c) $x^7 - 21x^5 + 105x^3 - 105x + 21$
(d) $x^7 - 21x^5 + 105x^3 - 105x$

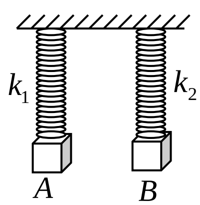
Q6. A particle of mass m is placed on an inclined plane making an adjustable angle θ with the horizontal, as shown in the figure. The coefficient of friction between the particle and the inclined plane is μ.

If the inclined plane is moving horizontally with a uniform acceleration $a < \frac{g}{\mu}$ (see figure), the value of θ for which the particle will remain at rest on the plane is



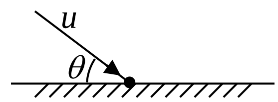
(a) $\theta = \tan^{-1}\left(\frac{\mu g + a}{g + \mu a}\right)$
(b) $\theta = -\cot^{-1}\left(\frac{\mu a + g}{a + \mu g}\right)$
(c) $\theta = \tan^{-1}\left(\frac{\mu g + a}{g - \mu a}\right)$
(d) $\theta = \cot^{-1}\left(\frac{\mu a - g}{a + \mu g}\right)$

Q7. Two bodies A and B of equal mass are suspended from two rigid supports by separate massless springs having spring constants k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal the ratio of the amplitude of oscillations of A to that of B is



(a) $\frac{k_2}{k_1}$
(b) $\frac{k_2}{k_1}$
(c) $\sqrt{\frac{k_1}{k_2}}$
(d) $\sqrt{\frac{k_2}{k_1}}$

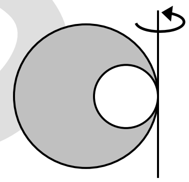
Q8. A particle of mass m is bounced on the ground with a velocity u making an angle of θ with the ground. The coefficient of restitution for collisions between the particle and the ground is ε and frictional effects are negligible both on the ground and in the air. The horizontal distance travelled by the particle from the point of initial impact till it begins to slide along the ground is



(a) $\frac{u^2}{2g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\sin 2\theta$ (b) $\frac{u^2}{g}\left(\frac{1}{1-\varepsilon}\right)\sin\theta$

(c) $\frac{u^2}{g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\tan 2\theta$ (d) $\frac{u^2}{g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\sin 2\theta$

Q9. The three-dimensional object sketch on the right is made by taking a solid sphere of uniform density (shaded) with radius R, and scooping out a spherical cavity (unshaded) as shown, which has diameter R.



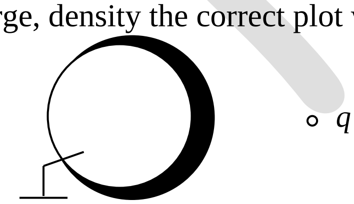
In this object has mass M, its moment of inertia about the tangential axis passing through the point where the spheres tough (as shown in the figure) is

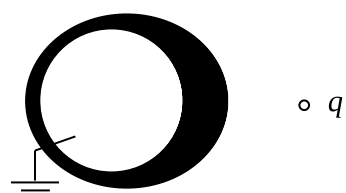
(a) $\frac{3}{16}MR^2$ (b) $\frac{62}{35}MR^2$ (c) $\frac{31}{70}MR^2$ (d) $\frac{31}{20}MR^2$

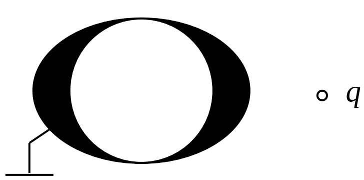
Q10. Consider three straight coplanar, parallel wires of infinite length where the distance between adjacent where is d. Each wire carries a current I in the same direction. The perpendicular distance from the middle wire (on either side) where the magnetic field vanishes is

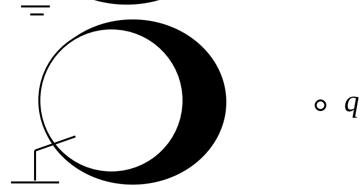
(a) $\frac{d}{\sqrt{3}}$ (b) $\frac{2d}{\sqrt{3}}$ (c) $\frac{d}{3}$ (d) $\frac{2d}{3}$

Q11. A point charge $q<0$ is brought in front of a grounded conducting sphere. If the induced charge density on the sphere is plotted such that the thickness of the black shading is proportional to the charge, density the correct plot will most closely resemble

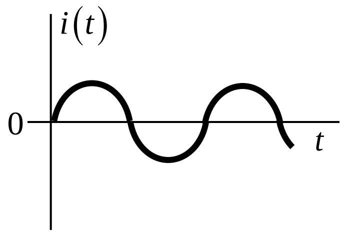
(a)


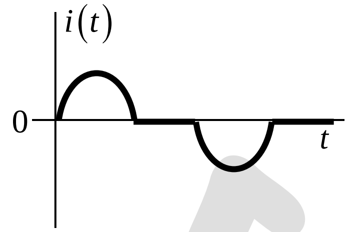
(b)


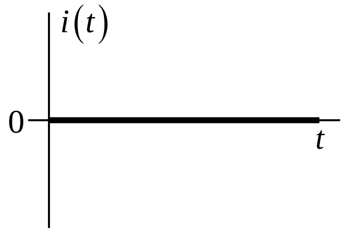
(c)


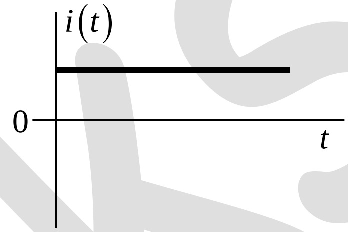
(d)


Q12. A circular coil of conducting wire of radius a and n turns, is placed in a uniform magnetic field $\vec{B}$ along the axis of the coil and is then made to undergo simple harmonic oscillations along the direction of the axis. The current through the coil will be best described by

(a) 

(b) 

(c) 

(d) 

Q13. A plane electromagnetic wave travelling through vacuum has electric field $\vec{E}$ and magnetic field $\vec{B}$ defined as

$$\vec{E} = \left(\hat{i} + \hat{j}\right) E_0 \exp i\left(\omega t - \vec{k} \cdot \vec{x}\right) \qquad \vec{B} = \left(\hat{i} - \hat{j} - \hat{k}\right) B_0 \exp i\left(\omega t - \vec{k} \cdot \vec{x}\right)$$

where E_0 and B_0 are real constants. The time-averaged pointing vector will be given by

(a) $\vec{S} = -\frac{2}{\sqrt{\in_0 \mu_0}} E_0 B_0 \left(\hat{i} - \hat{j} + 2\hat{k}\right)$

(b) $\vec{S} = -\frac{1}{2}\sqrt{\frac{3\in_0}{\mu_0}} E_0^2 \left(\hat{i} - \hat{j} + 2\hat{k}\right)$

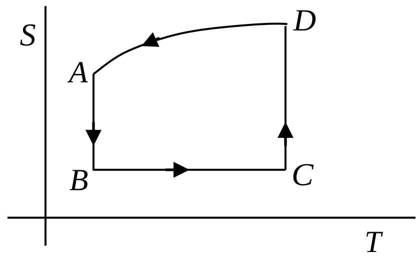
(c) $\vec{S} = \sqrt{\frac{\in_0}{6\mu_0}} E_0^2 \left(-\hat{i} + \hat{j} - 2\hat{k}\right)$

(d) $\vec{S} = \frac{1}{2}\sqrt{\frac{\in_0}{\mu_0}} B_0^2 \left(\hat{i} - \hat{j} + 2\hat{k}\right)$

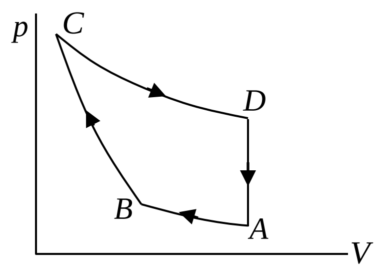
Q14. A thermally insulated coffee mug contains $500\,g$ of warm coffee at $80^0 C$. Assuming that the heat capacity of this liquid is $1 cal\, g^{-1}\, {}^0C^{-1}$ and the latent heat of fusion for ice is $80 cal\, g^{-1}$, the amount of ice that must be dropped into the cup to convert it into cold coffee at $5^0 C$ is about

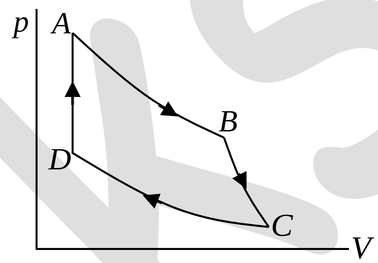
(a) $421 g$ (b) $441 g$ (c) $469 g$ (d) $471 g$

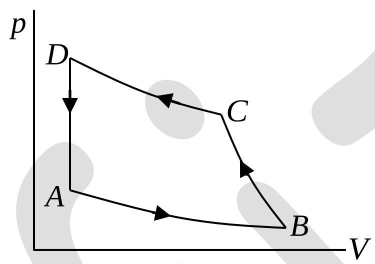
Q15. An ideal gas engine is run according to the cycle shown in the $s-T$ diagram below, where the process from D to A is known to be isochoric (i.e. maintaining $V=$constant).

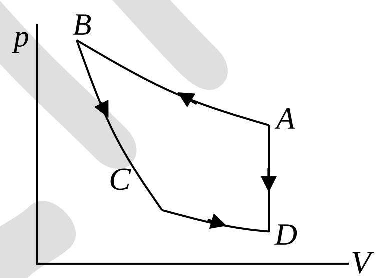


the corresponding cycle in the $p-V$ diagram will most closely resemble

(a)


(b)


(c)


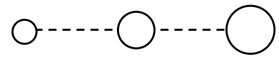
(d)


Q16. Consider a thermal ensemble at temperature T, which is composed of identical quantum harmonic oscillator of frequency ω_0 with non-overlapping wave function. The probability that there will be an even number of energy quanta in the system is

(a) $\frac{1}{\exp\left(-\hbar\omega_0/k_BT\right)+1}$ (b) $\frac{1}{\exp\left(\hbar\omega_0/k_BT\right)+1}$

(c) $\frac{1}{\exp\left(-\hbar\omega_0/k_BT\right)-1}$ (d) $\tan h\left(\hbar\omega_0/2k_BT\right)$

Q17. Consider the following linear model of a molecule of hydrogen cyanide (HCN) depicted below.



It follows that the molar specific heat of hydrogen cyanide gas at constant pressure must be

(a) $6R$ (b) $4.5R$ (c) $5R$ (d) $5.5R$

Q18. A beam of high energy neutrons is scattered from a metal lattice, where the spacing between nuclei is around $0.4\,nm$. In order to see quantum diffraction effects, the energy of the neutrons must be around

(a) $7.85\ MeV$ (b) $5.11\ meV$ (c) $511\ keV$ (d) $78.5\ meV$

Q19. A particle of mass m, moving in one dimension, satisfies the modified Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+i\hbar u\frac{d\psi}{dx}=i\hbar\frac{d\psi}{dt}$$

Where u is the velocity of the substrate? If, now, this particle is treated as a Gaussian wave packet peaked at wave number k, its group velocity will be $v_g =$

(a) $\frac{hk}{2m}-u$ (b) $\frac{hk}{m}+u$ (c) $\frac{hk}{m}-u$ (d) $-\frac{hk}{m}+u$

Q20. In a one-dimensional system, the boundary condition that the derivative of the wavefunction $\psi'(x)$ should be continuous at every point is applicable whenever

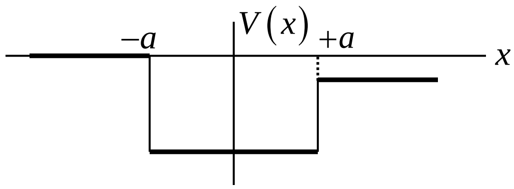
(a) The wavefunction $\psi(x)$ is itself continuous everywhere.

(b) There is a bound state and the potential is piecewise continuous.

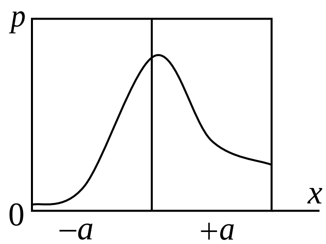
(c) There is a bounded state and the potential has no singularity anywhere.

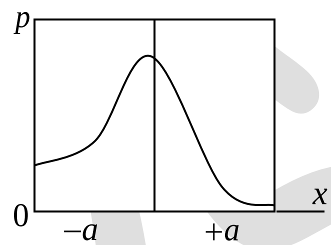
(d) There are bound or scattering states with definite momentum.

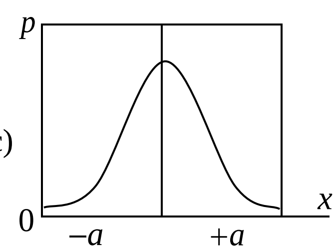
Q21. A particle moving in one dimension, is placed in an asymmetric square well potential $V(x)$ as sketched below.



The probability density $p(x)$ in the ground state will most closely resemble

(a)


(b)


(c)


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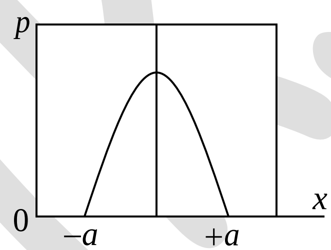
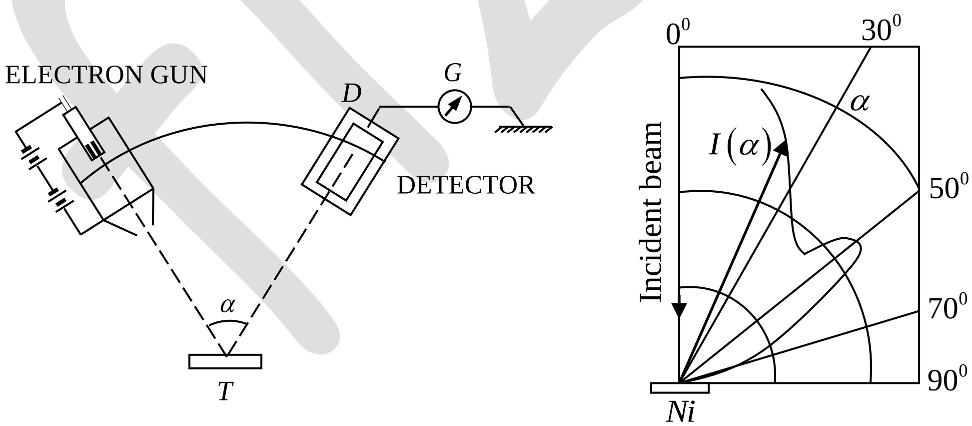


Q22. The sketch shown below illustrates the apparatus and results for a famous experiment. The graph on the right is a polar plot of the number of electrons received in the detector.



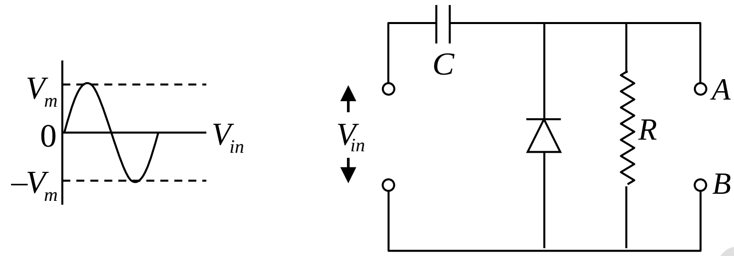
(a) The energy levels of atoms in a metal are quantized.

(b) Electrons in a beam can behave as waves.

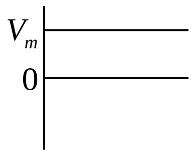
(c) Electrons have spin half.

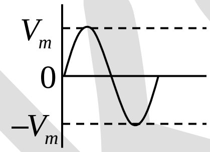
(d) There are magnetic domains inside a nickel sample.

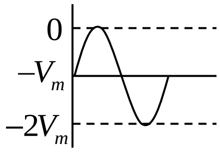
Q23. The signal shown on the left side of the figure below is fed into the circuit shown on the right side.

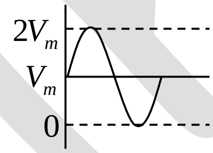


If the signal has time period τ_s and the circuit has a natural frequency τ_{RC}, then in the case when $\tau_s << \tau_{RC}$, the steady-state output will resemble

(a)


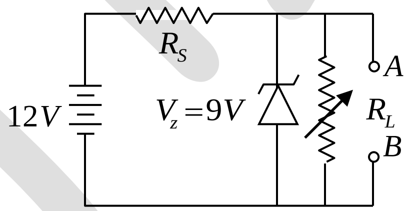
(b)


(c)


(d)


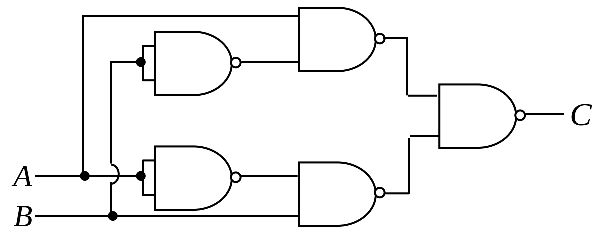
Q24. Drawing power from a $12V$ car battery a $9V$ stabilized DC voltage is required to power a car stereo system, attached to the terminals A and B, as shown in the figure.

If a Zener diode with ratings $V_2 = 9V$ and $P_{\max} = 0.27W$, is connected as shown in the figure, for the above purpose, the minimum series resistance R_S must be



(a) 111Ω (b) 103Ω (c) 100Ω (d) 97Ω

Q25. The circuit shown below uses only NAND gates



The final output at C is

(a) A AND B (b) A OR B (c) A XOR B (d) A NOR B

SECTION B

(only for Int.-Ph.D. candidates)

This Section consists of 15 Questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

Q1. An array T has elements T_{ijkl} where $i, j, k, l = 1, 2, 3, 4$. It is given that

$$T_{ijkl} = T_{jikl} = T_{ijlk} = -T_{klij}$$

for all values of i, j, k, l. The number of independent components in this array is

(a) 256 (b) 55 (c) 1 (d) 45

Q2. The differential equation $x\frac{dy}{dx} - xy = \exp(x)$, where $y = e^2$ at $x = 1$, has the solution $y =$

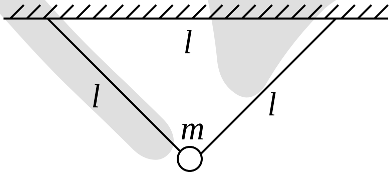
(a) $\exp(x^2 + x)$

(b) $(1-x)\exp(x) + \exp(1+x)$

(c) $\exp(1+x)(1+\ln x)$

(d) $\exp(x)\ln x + \exp(1+x)$

Q3. A pendulum is created by hanging a heavy bob of mass m from a rigid support (taken as zero level of potential) symmetrically using two massless inextensible strings each of length l, making an equilateral triangles as shown in the figure below.



A correct Lagrangian for the angular oscillations of the bob in the plane perpendicular to the paper is

(a) $L = \frac{3}{8}ml^2\dot{\theta}^2 - \sqrt{3}4mgl\sin^2\frac{\theta}{2}$

(b) $L = \sqrt{3}ml^2\dot{\theta}^2 + 4mgl\sin^2\frac{\theta}{2}$

(c) $L = \frac{3}{4}m\dot{\theta}^2 - \sqrt{3}\frac{mg}{l}\sin^2\frac{\theta}{2}$

(d) $L = \frac{3}{8}ml^2\dot{\theta}^2 + mgl\dot{\theta}\sec^2\theta + \sqrt{3}mgl\sin^2\frac{\theta}{2}$

Q4. A perfectly straight tunnel is dug between any two points on the surface of the Earth, which can be treated as static sphere of uniform density ρ. The tunnel does not necessarily pass through the centre of the Earth. If a particle is allowed to slide without friction in this tunnel under the action of the Earth's gravity. It will execute simple harmonic motion with time period

(a) $\sqrt{\frac{3}{2\rho G}}$ (b) $\sqrt{\frac{3}{2\pi\rho G}}$ (c) $\sqrt{\frac{6\pi}{\rho G}}$ (d) $\sqrt{\frac{3\pi}{\rho G}}$

Q5. In a futuristic experiment, two rocket ships, each containing one astronaut Rakesh and Sunita respectively, blasted off from the Earth's surface simultaneously, and travelled into space in straight lines in opposite directions at uniform speeds of $0.3c$ and $0.5c$ respectively. They both travelled in straight lines for some time, then reversed direction smoothly and returned along the same paths with the same speeds. It was found that they returned simultaneously after exactly 9.0 years. Which of the following a statement is correct?

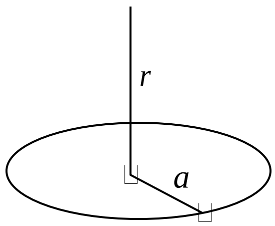
(a) The age of Rakesh increased by about 8.6 years and that of Sunita by about 7.8 years

(b) The ages of both Rakesh and Sunita increased by 3.06 years

(c)The age of Rakesh increased by about 9.4 years and that of Sunita by about 10.4 years.

(d) The ages of both Rakesh and Sunita increased by 9.0 years

Q6. Consider a hydrogenic atom in its ground state ass conceived in Bohr's theory, where an electron of charge $-e$ is rotating about a central nucleus of charge $+Ze$ in a circular orbit of radius $a = 4\pi \in_0 \hbar^2 / Ze^2$. In this model, the magnetic field at a distance r from the nucleus, perpendicular to the orbit will be



(a) $\frac{Ze^2\mu_0}{16\pi^2\hbar \in_0}\frac{1}{a}\left(1+\frac{r^2}{a^2}\right)^{-1/2}$

(b) $\frac{Ze^2\mu_0}{4\pi^2\hbar \in_0}\frac{1}{a}\left(1+\frac{r^2}{a^2}\right)^{-1/2}$

(c) $\frac{Ze\mu_0}{\hbar \in_0}\left(1+\frac{r^2}{a^2}\right)^{-3/2}$

(d) $\frac{Ze^2\mu_0}{8\pi^2\hbar \in_0}\frac{1}{a}\left(1+\frac{r^2}{a^2}\right)^{-3/2}$

Q7. A dielectric interface is formed by two homogeneous and isotropic dielectrics 1 and 2 with dielectric constants $\frac{4}{3}$ and 1 respectively and it carries no residual free charge. A linearly polarized electromagnetic wave is incident on the interface from dielectric1 at a point where the unit normal to the surface is

$$\hat{n} = \frac{1}{2}\left(\sqrt{3}i + \hat{k}\right)$$

Pointing into the dielectric1. The incident wave, which is incident from 1 into 2, just before it reaches the interface, has electric vector

$$\vec{E}_1 = iE_0 \exp i\omega\left(t + \frac{\sqrt{\in_1}}{c}z\right)$$

where E_0 is a real constant. The electric vector just after it crosses the interface is rotated from $\vec{E}_1$ by an angle

(a) $\frac{\pi}{6}$ (b) $\tan^{-1}\frac{2}{5\sqrt{3}}$ (c) $\sin^{-1}\frac{1}{2\sqrt{19}}$ (d) $\csc^{-1}3\sqrt{\frac{4}{19}}$

Q8. In an experiment to measure the Earth's mean albedo (i.e. fraction of solar energy reflected back into space), the solar constant (i.e., flux of solar energy incident upon the Earth), was measured as $1.37\,kWm^{-2}$. Assuming the Earth to behave as a perfect blackbody at a uniform surface temperature of -18^0C the albedo is about

(a) 0.30 (b) 0.18 (c) 0.46 (d) 0.06

Q9. The molar equation of state of a gas at temperature T, pressure P and volume V is given by

$$p = \frac{RT}{V-b} - \frac{a}{TV^2}$$

where a and b are two constants and R is the gas constant. The critical temperature and pressure for the gas will be

(a) $T_c = \sqrt{\frac{a}{27Rb}}, P_c = \frac{RT_c}{b}$ (b) $T_c = \sqrt{\frac{8a}{27Rb}}, P_c = \frac{RT_c}{8b}$

(c) $T_c = \sqrt{\frac{4a}{27Rb}}, P_c = \frac{RT_c}{4b}$ (d) $T_c = \sqrt{\frac{8a}{3Rb}}, P_c = \frac{RT_c}{8b}$

Q10. An excited gas, consisting of spinless charged particles, is confined in an infinite square well potential of width a, is found to radiate a spectrum whose α line (largest wavelength) has wavelength 816 nm. If the width a of the well is halved to $\frac{a}{2}$, the wavelength of the δ line (fourth-largest wavelength) will be

(a) 26.112 μm (b) 1.224 μm (c) 1.088 $g\,\mu m$ (d) 0.306 μm

Q11. The wave function of a non-relativistic particle of mass m in a one-dimensional potential $V(x)$ has the form

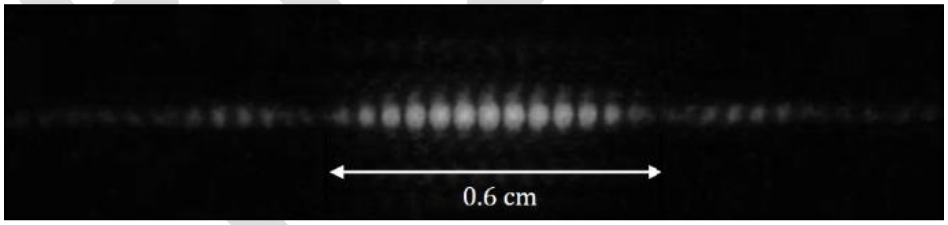
$$\psi(x) = \sqrt{a}e^{-a|x|}$$

Where $|x|$ denotes the absolute value of the coordinate x. The potential is $V(x) =$

(a) $-\frac{\hbar^2}{m}\delta\left(\frac{x}{a}\right)$ (b) $-\frac{\hbar^2 a}{m}\left\{a+\frac{1}{2}\delta(x)\right\}$

(c) $\frac{\hbar^2 a}{m}\delta(x)$ (d) 0

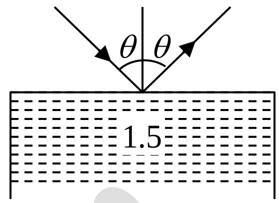
Q12. Te diffraction pattern due to a double slit experiment with two identical slits is recorded in infra-red laser radiation of wavelength 1.2 μm on a specially prepared photographic plate at a distance of 1.8 m from the centre of the slits. A photograph of the observed diffraction pattern is given below.



The distance between the slits can be calculated as

(a) 4.32 mm (b) 5.04 mm (c) 5.76 mm (d) 9.36 mm

Q13. In an experiment performed to determine the width of a thin transparent film of refractive index 2.6 deposited uniformly on the upper surface of a thick transparent glass block of refractive index 1.5, a monochromatic laser beam of wavelength $550\ nm$ was shone on the film at a variable angle θ with the normal to the surface. If the intensity of the reflected beam was maximum when $\theta = 5^0 24'$, the thickness of the film may be inferred to be



(a) $1.12\, g\,\mu m$ (b) $13.2\,\mu m$ (c) $0.48\,\mu m$ (d) $2.92\,\mu m$

Q14. In a sample of germanium, at a temperature $77\ K$, optical excitation create an average density of 10^{12} conduction electrons per cm^3. At this temperature, the electron and hole nobilities are equal and given by

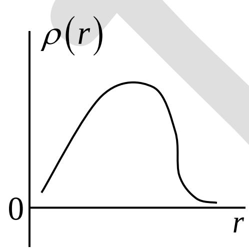
$$\mu = 0.5\times10^4\ cm^2 s^{-1} V^{-1}$$

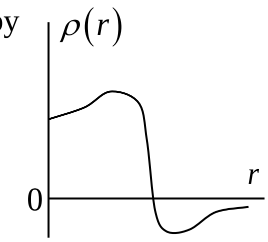
The value of the Einstein diffusion coefficient for the electrons and holes is

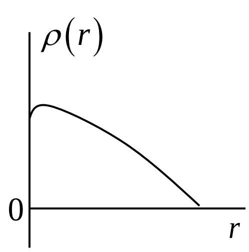
(a) $3.3\times10^{-2}\ m^2 s^{-1}$ (b) $1.65\times10^{-3}\ m^2 s^{-1}$

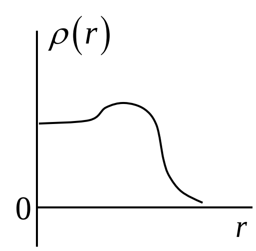
(c) $3.3\times10^{-3}\ m^2 s^{-1}$ (d) $6.6\times10^{-3}\ m^2 s^{-1}$

Q15. The electric charge density $\rho(r)$ inside a heavy spherical nucleus as a function of distance r from the centre may be approximated most closely by

(a)


(b)


(c)


(d)


SECTION B

(Only for Ph.D. candidates)

This Section consists of 15 Questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

Q1. The integral

$$I = \int_0^\infty dx\, e^{-x} \delta(\sin x)$$

where $\delta(x)$ denotes the Dirac delta function, is

(a) 1 (b) $\frac{\exp\pi}{\exp\pi+1}$ (c) $\frac{\exp\pi}{\exp\pi-1}$ (d) $\frac{1}{\exp\pi-1}$

Q2. Consider the complex function

$$f(x,y) = u(x,y) + iv(x,y)$$

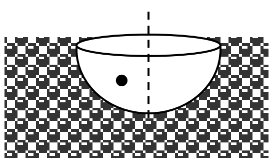
where
$$u(x,y) = x^2(2+x) - y^2(2+3x)$$
$$v(x,y) = y(\lambda x + 3x^2 - y^2)$$

and λ is real. If it is known that $f(x,y)$ is analytic in the complex plane of $z = iy$, then it can be written

(a) $f = z^2(2+z)$ (b) $f = \bar{z}(2+\bar{z}^2)$

(c) $f = 2z\bar{z} + z^2 - \bar{z}^2$ (d) $f = z^2 + z^3$

Q3. A bead of mass of mass m slides under the influence of gravity along the frictionless interior of a hemispherical cup of radius a sunk vertically into the ground with its open side level with the ground (see sketch on the right). In terms of spherical polar coordinates (θ, φ) set up with the centre of the upper circle as origin, the Hamiltonian H for this system will be



(a) $H = \frac{m}{2}\left(a^2\dot{\theta}^2 + a^2\dot{\varphi}^2 \csc^2\theta\right) + 2mga\sin^2\frac{\theta}{2}$

(b) $H = \frac{m}{2}\left(a^2\dot{\theta}^2 + a^2\dot{\varphi}^2 \sin^2\theta\right) + 2mga\sin^2\frac{\theta}{2}$

(c) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\varphi^2 \sin^2\theta\right) + 2mga\sin^2\frac{\theta}{2}$

(d) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\varphi^2 \csc^2\theta\right) + 2mga\sin^2\frac{\theta}{2}$

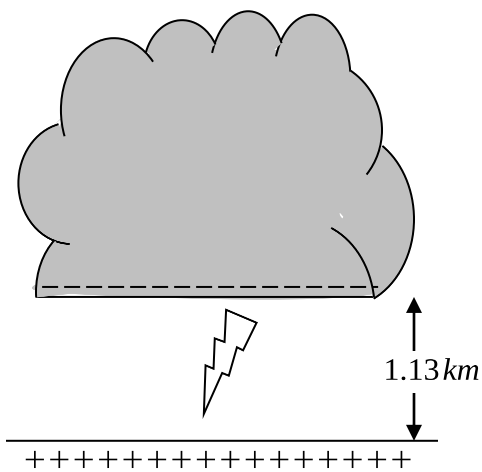
Q4. On a compact stellar object the gravity is so strong that a body failing from rest will soon acquire a velocity comparable with that of light. If the force on this body is $F = mg$ where m is the relativistic mass and g is a constant, the velocity of this failing body will vary with time as

(a) $v = \frac{c}{1-\frac{2c}{gt}}$ (b) $v = c\tanh\frac{gt}{c}$

(c) $v = \frac{2c}{\pi}\tan\frac{gt}{c}$ (d) $v = c\left\{1-\exp\left(-\frac{gt}{c}\right)\right\}$

Q5. A monsoon cloud has a flat bottom of surface area $125\,km^2$. It floats horizontally over the ground at a level such that the base of the cloud is $1.13\,km$ above the ground (see figure). Due to friction with the air below, the base of the cloud acquires a uniform electric charge density. This keeps increasing slowly with time.

When the uniform electric field below the cloud reaches the value $2.4\,MV\,m^{-1}$ a lightning discharge occurs, and the entire charge of the cloud passes to the Earth below-which, in this case, behaves like a grounded conductor.



Neglecting edge effects and inhomogeneities inside the cloud and the air below, the energy released in this lighting discharge can be estimated, in kilowatt-hours (kWh) as about

(a) 10^9 (b) 10^5 (c) 10 (d) 10^{-1}

Q6. The magnetic vector potential corresponding to a uniform magnetic field $\vec{B}$ is often taken as

$$\vec{A} = \frac{1}{2}\vec{B}\times\vec{x}$$

This choice is

(a) valid in the Lorenz gauge

(b) valid in the Coulomb gauge

(c) valid in the Weyl gauge

(d) gauge invariant

Q7. An ideal monatomic gas at chemical potential $\mu = -1eV$ and a temperature given by $k_BT = 0.1eV$ is in equilibrium with an adsorbing metal surface, i.e., there are isolated sites distributed randomly on the metal surface where the gas atom can get bound. Each such binding site can adsorb $0, 1$ or 2 atoms with the released energy being $0, -1eV$ and $-1.9eV$ respectively. The average number of adsorbed molecules at each site would be

(a) $\frac{1+e}{1-e}$

(b) $\frac{1+e}{1+2e}$

(c) $\frac{2+e}{1+2e}$

(d) $\frac{1+2e}{1+e}$

Q8. Consider N non-interacting distinguishable particles in equilibrium at an absolute temperature T. Each particle can only occupy one of two possible states of energy 0 and ε respectively $(\varepsilon > 0)$. The entropy of the system, in terms of $\beta = \frac{\in}{k_BT}$ is

(a) $Nk_B\left\{\ln\left(1+e^{-\beta}\right) - \frac{e^{-\beta}}{1+e^{-\beta}}\right\}$

(b) $Nk_B\left\{\ln\left(1-e^{-\beta}\right) - \frac{\beta e^{-\beta}}{1+e^{-\beta}}\right\}$

(c) $Nk_B\left\{\ln\left(1+e^{-\beta}\right) + \frac{\beta e^{-\beta}}{1+e^{-\beta}}\right\}$

(d) $Nk_B\left\{\ln\left(1+e^{-\beta}\right) - \frac{e^{-\beta}}{1-e^{-\beta}}\right\}$

Q9. An electron in a hydrogen atom is in a state described by the wavefunction:

$$\psi(\vec{r}) = \frac{1}{\sqrt{10}}\psi_{100}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{210}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{211}(\vec{x}) - \frac{1}{\sqrt{10}}\psi_{21,-1}(\vec{x})$$

where $\psi_{nlm}(\vec{x})$ denotes a normalized wavefunction of the hydrogen atom with the principal quantum number n, angular quantum number ℓ and magnetic quantum number m.

Neglecting the spin-orbit intersection, the expectation values of $\hat{L}_z$ and $\hat{L}^2$ for this state are

(a) $\frac{3\hbar}{10}, \frac{9\hbar^2}{5}$

(b) $\frac{3\hbar}{4}, \frac{9\hbar^2}{25}$

(c) $\frac{3\hbar}{5}, \frac{9\hbar^2}{10}$

(d) $\frac{8\hbar}{10}, \frac{3\hbar^2}{5}$

Q10. A system of two spin - $1/2$ particles 1 and 2 has the Hamiltonian

$$H = \epsilon_0 \hat{h}_1 \otimes \hat{h}_2$$

where

$$\hat{h}_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \quad \hat{h}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and ϵ_0 is a constant with the dimension of energy. The ground state of this system has energy

(a) $\sqrt{2}\epsilon_0$ (b) 0 (c) $-2\epsilon_0$ (d) $-4\epsilon_0$

Q11. At time $t=0$, the wavefunctionof a particle in a harmonic oscillator potential of natural frequency ω is given by

$$\psi(0) = \frac{1}{5}\left\{3\varphi_0 - 2\sqrt{2}\varphi_1 + 2\sqrt{2}\varphi_2\right\}$$

where $\varphi_n(x)$ denotes the eigenfunction belonging to the n-th eigenvalue of energy. At time $t=\tau$, the wavefunction is found to be

$$\psi(\tau) = \frac{i}{5}\left\{3\varphi_0 + 2\sqrt{2}\varphi_1 + 2\sqrt{2}\varphi_2\right\}$$

the minimum value of τ is

(a) $\frac{\pi}{2\omega}$ (b) $\frac{2\pi}{\omega}$ (c) $\frac{2\pi}{3\omega}$ (d) $\frac{\pi}{\omega}$

Q12. At low temperature, the measured specific heat C_V of a solid sample is found to depend on temperature as $C_V = aT^{3/2} + bT^3$

Where a and b are constants. This material has

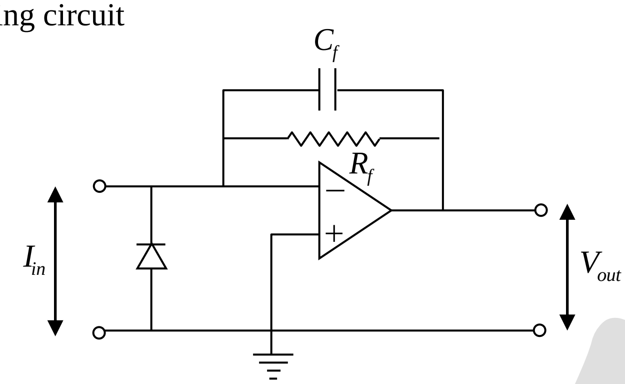
(a) one fermionic excitation with dispersion relation $\omega \propto k^4$ another bosonic excitation with dispersion relation $\omega \propto k$;

(b) one fermionic excitation with dispersion relation $\omega \propto k^2$ another bosonic excitation with dispersion relation $\omega \propto k^4$;

(c) one bosonic excitation with dispersion relation $\omega \propto k^2$ another bosonic excitation with dispersion relation $\omega \propto k$;

(d) one fermionic excitation with dispersion relation $\omega \propto k^2$ another bosonic excitation with dispersion relation $\omega \propto k$;

Q13. Consider the following circuit



It is given that $C_f = 100\,pF$, and for $I_{in} = 50\,nA$ $D.C$,, $V_{out} = 1V\ D.C$,. Therefore, the bandwidth of the above circuit is

(a) $15.8\,Hz$ (b) $79.6\,Hz$ (c) $145.3\,Hz$ (d) $200.4\,Hz$

Q14. The semi-empirical mass formula for a heavy nucleon (Z, A) can be written, to some approximation, as

$$M(Z,A)c^2 = ZM_pc^2 + (A-Z)M_nc^2 - \lambda_2 A^{2/3} - \lambda_3 \frac{Z(Z-1)}{A^{1/2}} - \lambda_4 \frac{(A-2Z)^2}{A} - \frac{\lambda_5}{A^{1/2}}$$

where $M_pc^2 = 938\,MeV, M_nc^2 = 939\,MeV$, and $\lambda_1 = 16$, $\lambda_2 = 18$, $\lambda_3 = 0.7$, $\lambda_4 = 23$ all in MeV, where

$$\lambda_5 = \begin{cases} +12\,MeV \text{ for even - even nuclei} \\ -12\,MeV \text{ for odd - odd nuclei} \\ 0 \text{ for other} \end{cases}$$

now, consider a spontaneous fission reaction

$$^{238}_{92}U \rightarrow ^{146}_{56}Ba + ^{91}_{36}Kr + ^{1}_{0}n$$

the energy released in this reaction will be close to

(a) $17.92\,keV$ (b) $19.2\,MeV$ (c) $170\,MeV$ (d) $190\,MeV$

Q15. The table below gives the properties of four unstable particles $\mu^+, \pi^+, n^0, \wedge^0$

Particle	Mass (MeV/c^2)	spin	Principal decay mode
Muon μ^+	105.66	$\frac{1}{2}$	$\mu^+ \to e^+ + v_\mu + \overline{v}_e$
Pion π^+	139.57	0	$\pi^+ \to \mu^+ + v_\mu$
Neutron n^0	939.56	$\frac{1}{2}$	$n^0 \to p^+ + e^- + \overline{v}_e$
Lambda hyperon $\wedge^0$	1,115.68	$\frac{1}{2}$	$\wedge^0 \to p^+ + \pi^-$

If arranged in order of DECREASING decay lifetime, the above list will read

(a) $n^0, \mu^+, \pi^+, \wedge^0$

(b) $\mu^+, \wedge^0, n^0, \pi^+$

(c) $n^0, \wedge^0, \mu^+, \pi^+$

(d) $\pi^+, n^0, \mu^+, \wedge^0$