

Q21. The momentum operator

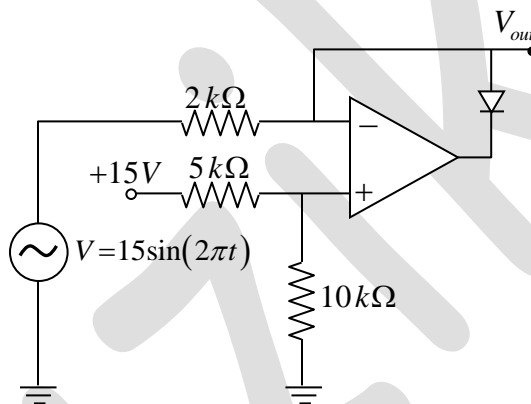
$$i\hbar \frac{d}{dx}$$

acts on a wavefunction $\psi(x)$. This operator is Hermitian

- (a) provided the wavefunction $\psi(x)$ is normalized
- (b) provided the wavefunction $\psi(x)$ and derivate $\psi'(x)$ are continuous everywhere
- (c) provided the wavefunction $\psi(x)$ vanishes as $x \rightarrow \pm\infty$
- (d) by its very definition

Ans. (c)

Q22.



In the above circuit, which of the following is the maximum value, in Volts, of voltage at V_{out} ?

- (a) 10
- (b) 15
- (c) 0
- (d) 5

Ans. : (a)

Q23. A badly-designed voltmeter is modelled as an ideal voltmeter with a large resistor (R) and a large capacitor (C) connected in parallel to it. Given this information, which of the following statements describes what happens when this voltmeter is connected to a DC voltage source with voltage V and internal resistance r ($r \ll R$) ?

- (a) The reading on the voltmeter rises slowly and becomes steady at a value slightly less than V
- (b) The reading on the voltmeter starts at a value slightly less than V and slowly falls to zero.

(c) The reading on the voltmeter rises slowly to maximum value close to V and then slowly goes to zero.

(d) The reading on the voltmeter reads zero even when connected to the voltage source.

Ans. : (a)

Q24. An OR gate, a NOR gate and an XOR gate are to be constructed using only NAND gates.

If the minimum number of NAND gates needed to construct OR, NOR and XOR gates is denoted $n(\text{OR})$, $n(\text{NOR})$ and $n(\text{XOR})$ respectively, then

(a) $n(\text{NOR}) = n(\text{XOR}) > n(\text{OR})$ (b) $n(\text{NOR}) = n(\text{XOR}) = n(\text{OR})$

(c) $n(\text{NOR}) > n(\text{XOR}) > n(\text{OR})$ (d) $n(\text{NOR}) < n(\text{XOR}) = n(\text{OR})$

Ans. (a)

Q25. On passing electric current, a tungsten filament is emitting electrons by thermionic emission. In order to maintain the energy of the electron beam obtained from this source at a value approximately 100 eV , which of the following methods will work in practice?

(a) Float the filament at -100 Volts with a grounded aperture in front of it.

(b) Heat the filament so that the emitted electrons will have 100 eV kinetic energy due to temperature.

(c) Apply a $+100$ Volts potential with respect to the filament potential to an aperture kept very close to the filament.

(d) Use an appropriate magnetic field to draw out the electron beam at the desired energy without applying any electric field.

SECTION B

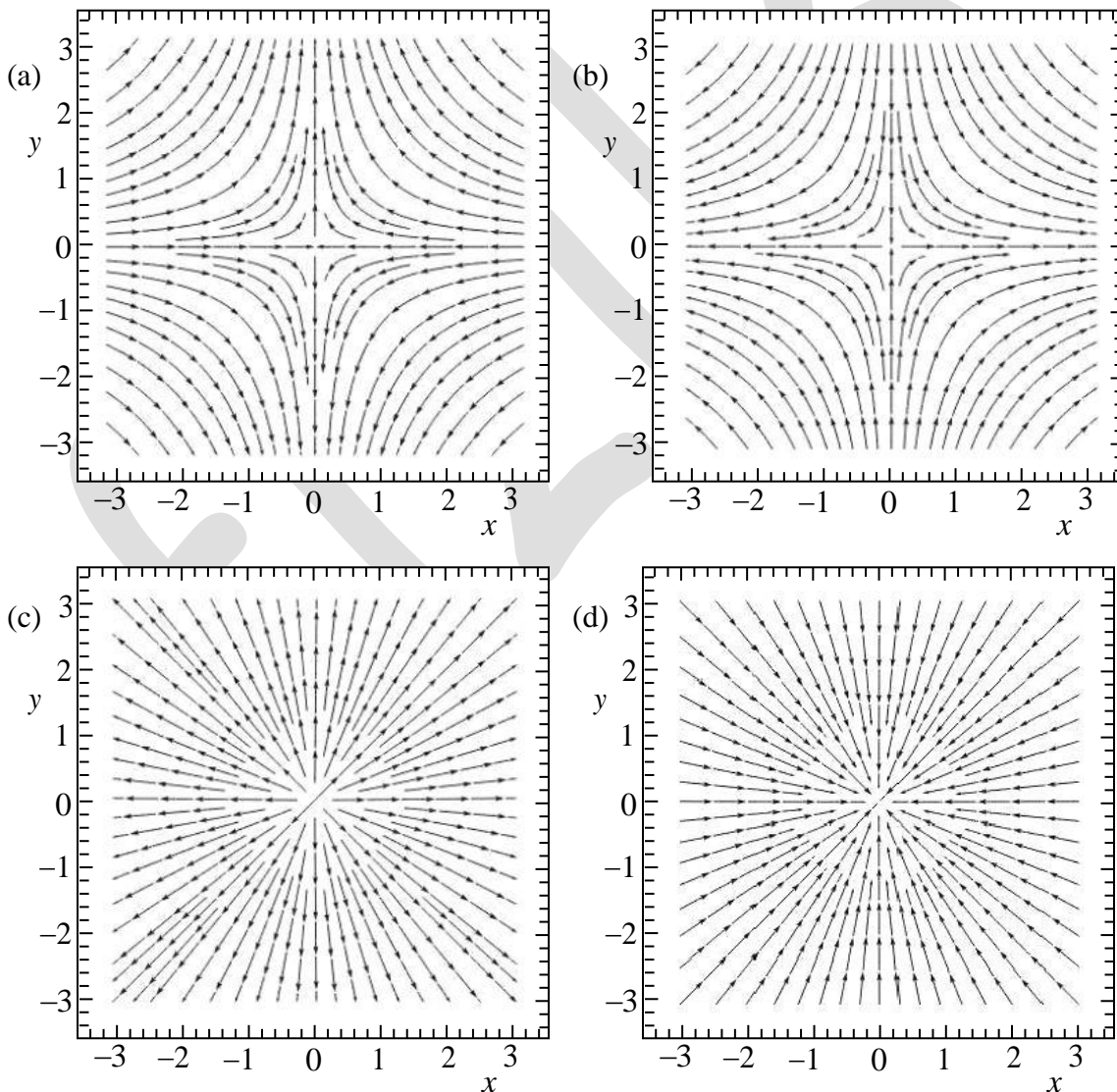
(only for Int.-Ph.D. candidates)

This Section consists of 15 Questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

Q26. A two-dimensional electrostatic field is defined as

$$\vec{E}(x, y) = -x\hat{i} + y\hat{j}$$

A correct diagram for the lines of force is



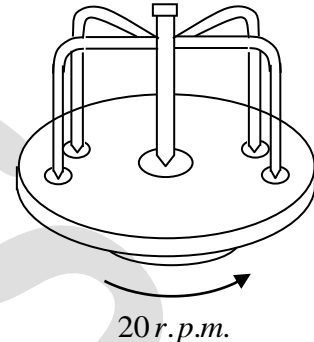
Ans. : (a)

Q27. The sum of the infinite series $S = 1 + \frac{3}{5} + \frac{6}{25} + \frac{10}{125} + \frac{15}{625} + \dots$ is given by

- (a) $S = \frac{125}{64}$ (b) $S = \frac{25}{16}$ (c) $S = \frac{25}{24}$ (d) $S = \frac{16}{25}$

Ans.: (a)

Q28. A roundabout rotating base is a heavy uniform disc of radius 2 m and mass 400 kg has a central pillar and handles which are of negligible mass (see figure). The roundabout is set rotating at a steady rate of 20 r.p.m.



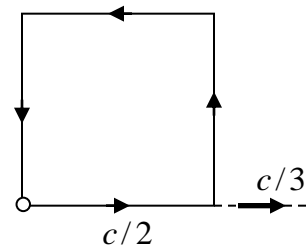
Four small children, of mass 10 kg , 20 kg , 30 kg and 40 kg respectively, step gently on to the edge of the roundabout, each with velocity 7.2 km/hr along a tangential direction and cling to the handles. After holding on for some time, the children step gently off the roundabout with the same velocity, but this time in a radial direction.

Neglecting all effects of friction and air drag the final rate of rotation of the roundabout will be about

- (a) 28 r.p.m. (b) 25 r.p.m. (c) 36 r.p.m. (d) 21 r.p.m.

Ans.: (b)

Q29. In the laboratory frame, a particle at rest starts moving with a speed $\frac{c}{2}$ from one corner of a square (see figure) and traverses the four sides of the square so that it returns to its original position. At each corner, it changes direction without any change in speed.

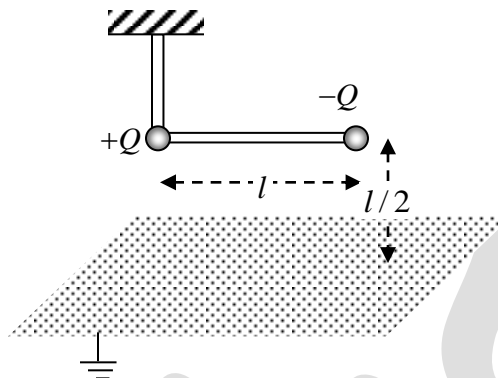


If the entire square now moves with a speed $\frac{c}{3}$ in the laboratory frame, as indicated in the figure, the speed of the particle (in the laboratory frame) when it returns to its original position will be

- (a) $\frac{2\sqrt{2}c}{15}$ (b) $\frac{c}{5}$ (c) $\frac{2\sqrt{2}c}{3}$ (d) $\frac{c}{5\sqrt{3}}$

Ans.: (a)

- Q30. A light rigid insulating rod of length ℓ is suspended horizontally from a rigid frictionless pivot at one of the ends (see figure). At a vertical distance h below the rod there is an infinite plane conducting plane, which is grounded.



If two small, light spherical conductors are attached at the ends of the rod and given charges $+Q$ and $-Q$ as indicated in the figure, the torque on the rod will be

- (a) $\frac{Q^2}{4\pi\epsilon_0\ell}\hat{k}$ (b) $-\frac{Q^2}{4\pi\epsilon_0\ell}\hat{k}$
 (c) $\frac{(4-\sqrt{2})Q^2}{16\pi\epsilon_0\ell}\hat{k}$ (d) $-\frac{(4-\sqrt{2})Q^2}{16\pi\epsilon_0\ell}\hat{k}$

Ans.: (d)

- Q31. The magnitude vector potential $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ is defined in a region R of space by

$$A_x = 5 \cos \pi y \quad A_y = 2 + \sin \pi x \quad A_z = 0$$

in an appropriate unit.

If L be a square loop of wire in the $x-y$ plane, with its end at

$$(0, 0) \quad (0, 0.25) \quad (0.25, 0.25) \quad (0.25, 0)$$

In appropriate unit and it lies entirely in the region R , the numerical value of the flux of the above magnetic field (in the same units) passing through L is

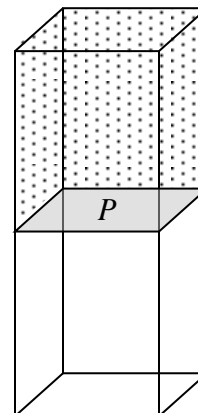
- (a) 0.543 (b) 3.31 (c) -0.75 (d) zero

Ans.: (a)

Q32. The volume V of a rectangular box is divided into two equal parts by a solid non permeable partition P . On one side of the partition P there is a vacuum, while the other side is filled with a real gas having equation of state

$$pVe^{a/RTV} = nRT$$

where a and b are constants, The gas was initially at a uniform temperature T_0 . Then the partition P was removed instantaneously, and the gas was allowed to expand to fill the full volume of the box and come to equilibrium. The final temperature of the gas, in term of its specific heat C_v will be



(a) $T - \left(\frac{na}{C_v}\right) \ln 2$

(b) $T + \left(\frac{na}{C_v}\right) \ln 2$

(c) $T - 2n \left(\frac{RTa}{C_v}\right)^{3/2}$

(d) $T + 2n \left(\frac{RTa}{C_v}\right)^{3/2}$

Ans. : (a)

Q33. A system is composed of a large number of non-interacting classical particles moving in two dimensions, which individually obey the Hamiltonian

$$\frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2)$$

and the system is connected to a heat bath at a temperature T .

The probability of finding a particle within a radius R from the origin is given by

(a) $1 - \exp\left(-\frac{m\omega^2 R^2}{2T}\right)$

(b) $\exp\left(-\frac{m\omega^2 R^2}{2T}\right)$

(c) $\operatorname{erf}\left(\sqrt{\frac{m}{2T}}\omega R\right)$

(d) $1 - \frac{m\omega^2 R^2}{2T}$

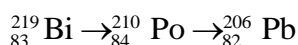
Ans. : (a)

Q37. A beam of X -rays is incident upon a powder sample of a material which forms simple cubic crystals of lattice constant 5.5 \AA . The maximum wavelength of the X -rays which can produce diffraction from the planes with Miller indices $(0,0,5)$ is

- (a) 2.2 \AA (b) 55.0 \AA (c) 1.1 \AA (d) 27.5 \AA

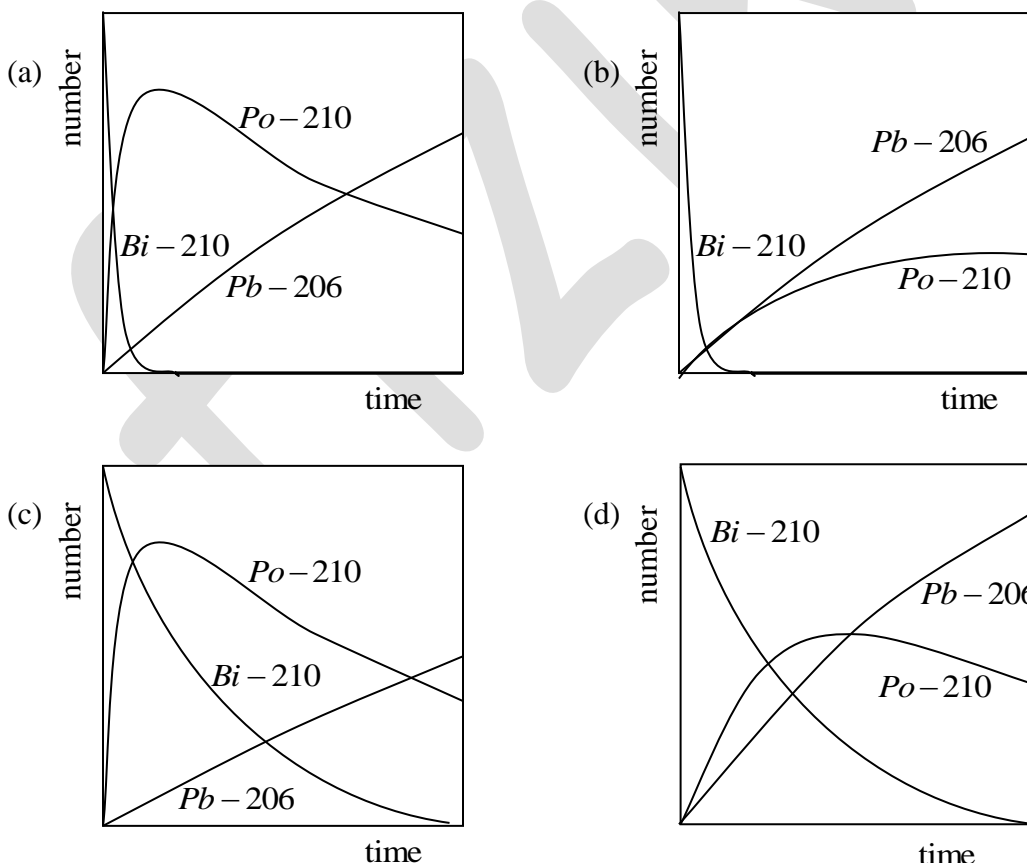
Ans.: (a)

Q38. Consider the nuclear decay chain of radio-Bismuth to Polonium to Lead, i.e.



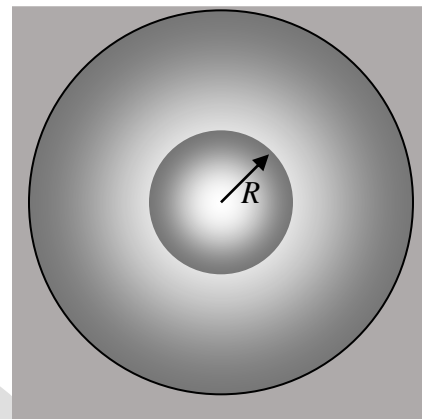
where $\text{Pb-206}({}_{82}^{206}\text{Pb})$ is a stable nucleus, and $\text{Bi-210}({}_{82}^{219}\text{Bi})$ and $\text{Po-206}({}_{84}^{210}\text{Po})$ are radioactive nuclei with half lives of about 5 days and 138 days respectively.

If we start with a sample of pure $\text{Bi-210}({}_{82}^{219}\text{Bi})$, then a possible graph for the time evolution of the number of nuclei of these three species will be



Ans.: (a)

Q39. A monochromatic laser beam is incident on a wet piece of filter paper atop a sheet of glass of thickness d . The pattern observed on the paper is shown in figure. If the radius of the inner ring observed is R , the refractive index of the glass must be



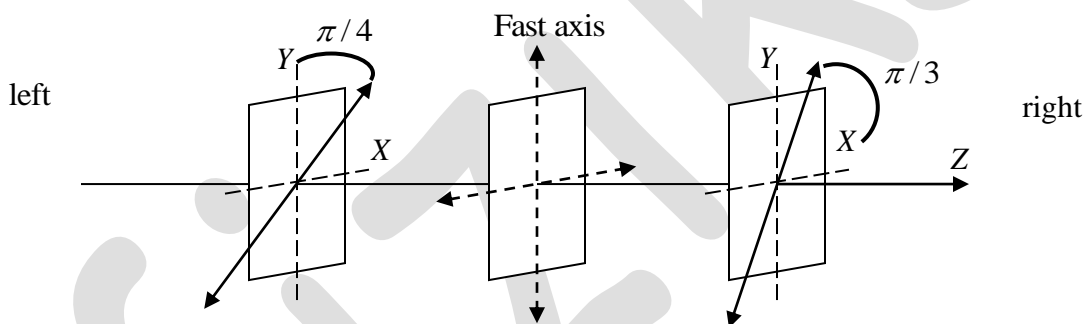
- (a) $\sin \left\{ \tan^{-1} \left(\frac{R}{2d} \right) \right\}$ (b) $\sin \left\{ \tan^{-1} \left(\frac{R}{d} \right) \right\}$
 (c) $\tan \left\{ \sin^{-1} \left(\frac{R}{2d} \right) \right\}$ (d) $\tan \left\{ \sin^{-1} \left(\frac{R}{d} \right) \right\}$

Ans.: (a)

Q40. A plane polarised light wave with electric field expressed as

$$\vec{E}(z, t) = E_0 \hat{j} \cos(kz - \omega t)$$

is incident from the left on the apparatus as sketched below.



The apparatus consists of (from left to right) a polariser with transmission axis at $\frac{\pi}{4}$ w.r.t. y -axis, followed by a quarter-wave plate with fast axis along the y -axis and finally, a polariser with transmission axis at $\frac{\pi}{3}$ about the x -axis.

If the incident intensity of the wave is I_0 , what will be intensity of the light emerging out of the apparatus (on the right)?

- (a) $\frac{I_0}{4}$ (b) $\frac{I_0}{8}$ (c) $\frac{3I_0}{8}$ (d) $\frac{I_0}{16}$

Ans.: (a)

SECTION B

(Only for Ph.D. candidates)

This Section consists of 15 Questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

Q26. The solution of the differential equation

$$\frac{dy}{dx} = 1 + \frac{y}{x} - \frac{y^2}{x^2}$$

for $x > 0$ with the boundary condition $y = 0$ at $x = 1$. is given by $y(x) =$

- (a) $\frac{x(x^2-1)}{x^2+1}$ (b) $\frac{x(x-1)}{x+1}$ (c) $\frac{x-1}{x+1}$ (d) $\frac{x^2-1}{x^2+1}$

Ans.: (a)

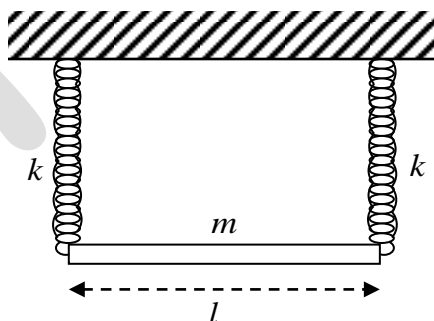
Q27. The value of the integral

$$\int_0^{\infty} \frac{dx}{x^4+4}$$
 is

- (a) $\frac{\pi}{8}$ (b) $\frac{3\pi}{8}$ (c) 2π (d) $\frac{\pi}{4}$

Ans.: (a)

Q28. A uniform rod of length ℓ and mass m is suspended horizontally from a rigid support by two identical massless springs, each with stiffness constant k , as sketched below.



If the springs can move only in the vertical direction, the frequency of small oscillations of the rod about equilibrium is given by

- (a) $\sqrt{\frac{2k}{m}}$ and $\sqrt{\frac{6k}{m}}$ (b) $\sqrt{\frac{2k}{m}}$ and $\sqrt{\frac{2\pi k}{m}}$
 (c) $\sqrt{\frac{\pi k}{2m}}$ and $\sqrt{\frac{6k}{m}}$ (d) $\sqrt{\frac{k}{m}}$ and $\sqrt{\frac{2\pi k}{m}}$

Ans.: (a)

Q29. The Lagrangian of a system described by generalised coordinates q_1 and q_2 is given by

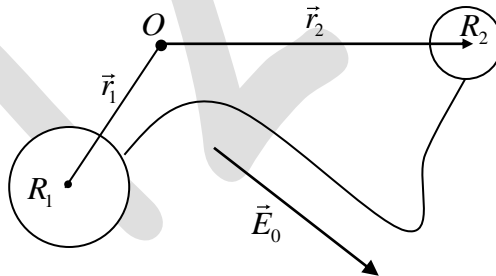
$$L = \frac{a}{2}(\dot{q}_1^2 + \dot{q}_2^2) - \frac{b^2}{\pi}(q_1^2 + q_2^2)$$

where a and b are constants. It follows that a conserved quantity in this system is

- (a) $q_1\dot{q}_2 - q_2\dot{q}_1$ (b) $q_1\dot{q}_2 + q_2\dot{q}_1$
 (c) $\frac{q_1\dot{q}_2 - q_2\dot{q}_1}{q_1^2 + q_2^2}$ (d) $2\pi(q_1^2\dot{q}_2 + q_2^2\dot{q}_1)$

Ans.: (a)

Q30. Two conducting uncharged spheres of radii R_1 and R_2 are connected by an infinitesimally thin wire. The centres of the spheres are located at \vec{r}_1 and \vec{r}_2 respectively with respect to the origin O . The system is subjected to a uniform external electric field \vec{E}_0 .



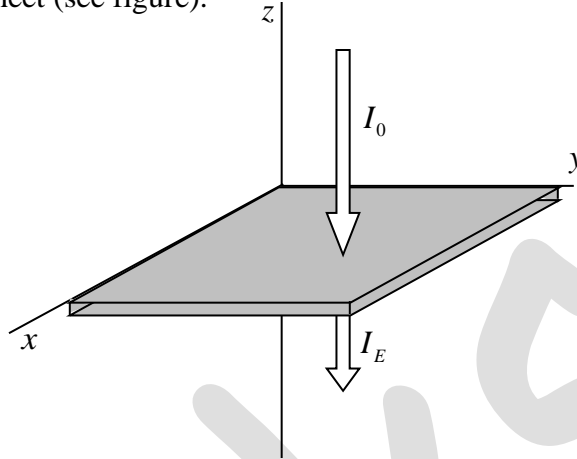
If the wire cannot support a net charge and two spheres are separated by distance much larger than the radii of each of them, the induced dipole moment in the system would be

- (a) $4\pi\epsilon_0 \frac{R_1 R_2}{R_1 + R_2} \{ \vec{E}_0 \cdot (\vec{r}_2 - \vec{r}_1) \} (\vec{r}_2 - \vec{r}_1)$ (b) $\frac{1}{4\pi\epsilon_0} \frac{R_1 R_2}{(R_1 + R_2)} \{ \vec{E}_0 \cdot (\vec{r}_2 - \vec{r}_1) \} (\vec{r}_2 - \vec{r}_1)$
 (c) $4\pi\epsilon_0 \frac{R_1 + R_2}{R_1 R_2} \{ \vec{E}_0 \cdot (\vec{r}_2 - \vec{r}_1) \} (\vec{r}_2 - \vec{r}_1)$ (d) zero

Ans.: (c)

Q31. Consider the following situation.

An infinite plane metallic plate of thickness 1.8 cm is placed along the $x-y$ plane, with z axis normal to the sheet (see figure).



A plane radio wave of intensity I_0 and frequency 29.5 MHz propagates in vacuum along the negative z -axis and strikes the metal foil at normal incidence.

If the metal of the foil has conductivity $5.9\ \Omega^{-1}\text{m}^{-1}$ and magnetic permeability $\mu = 1$, the intensity I_E of the emergent wave will be approximately

- (a) $0.26 I_0$ (b) $0.51 I_0$
 (c) $0.29 \times 10^{-7} I_0$ (d) $2.08 \times 10^{-4} I_0$

Ans.: (a)

Q32. In a certain atom, the ground state and first excited state of the valence electron are -7.8 eV and -3.9 eV , while all the higher excited states have energies very close to zero.

The ground state has a degeneracy of 2, while the first excited state has a degeneracy of 6.

It follows that if these atoms reside in the outer layers of a blue giant star at a temperature around $2.32 \times 10^4\text{ K}$, the average per atom will be approximately

- (a) -5.1 eV (b) -5.9 eV (c) -6.8 eV (d) -4.4 eV

Ans.: (c)

Q33. A square lattice consists $2N$ sites, of which alternate sites are labelled A and B . An example with $N=6$ is shown on the right. Now, N identical classical particles are distributed over these sites, such that each site can accommodate at most one particle.

A	B	A	B	A	B
B	A	B	A	B	A
A	B	A	B	A	B
B	A	B	A	B	A
A	B	A	B	A	B
B	A	B	A	B	A

The fraction of the total number N of particles occupying A sites is denoted α and the fraction occupying B sites is denoted β , so that $\alpha + \beta = 1$. If α, β are fixed and $N \gg 1$, the entropy S of the system can be written

- (a) $S = -2Nk_B T(\alpha \ln \alpha + \beta \ln \beta)$ (b) $S = 2Nk_B T(\alpha \ln \alpha + \beta \ln \beta)$
 (c) $S = -2Nk_B T(\alpha \ln \alpha - \beta \ln \beta)$ (d) $S = 2Nk_B T(\alpha \ln \alpha - \beta \ln \beta)$

Ans.: (a)

Q34. A particle of mass m is placed in one dimensional harmonic oscillator potential

$$V(x) = \frac{1}{2} m \omega^2 x^2$$

At $t = 0$, its wavefunction is $\psi(x)$. At $t = 2\pi / \omega$ its wavefunction will be

- (a) $\psi(x)$ (b) $-\psi(x)$ (c) $-\pi\psi(x)$ (d) $\frac{2\pi}{\omega}\psi(x)$

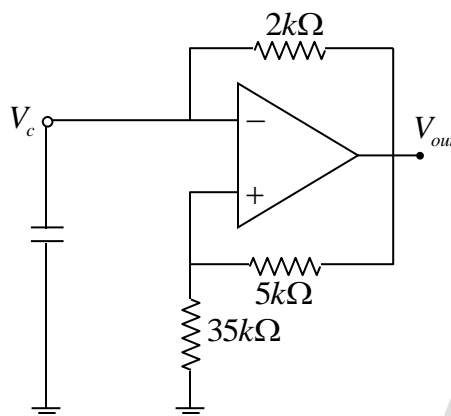
Ans.: (b)

Q35. A spin-2 nucleus absorbs a spin-1/2 electron and is then observed to decay to a stable nucleus in two stages, recoiling against an emitted invisible particle in the first stage and against an emitted spin-1 photon in the second stage. If the stable nucleus is spinless, then the spin of the invisible particle is

- (a) $\frac{3}{2}$ or $\frac{5}{2}$ (b) $\frac{3}{2}$
 (c) $\frac{1}{2}$ or $\frac{3}{2}$ (d) $\frac{1}{2}$

Ans.: (a)

Q36. The circuit sketched below is called a relaxation oscillator.

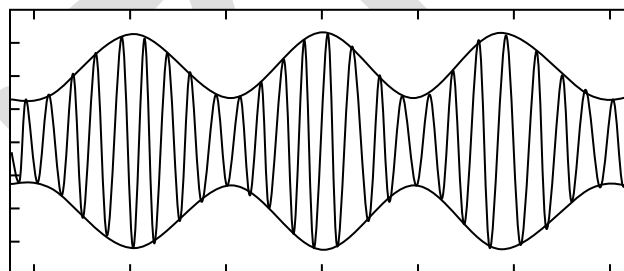


For the parameters indicated in the figure, the ratio of the maximum voltage at V_{out} to the maximum voltage at V_c is

- (a) $\frac{1}{8}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) $\frac{1}{4}$

Ans.: (a)

Q37. The figure below shows a carrier frequency 4kHz being amplitude-modulated by a sine wave-signal.

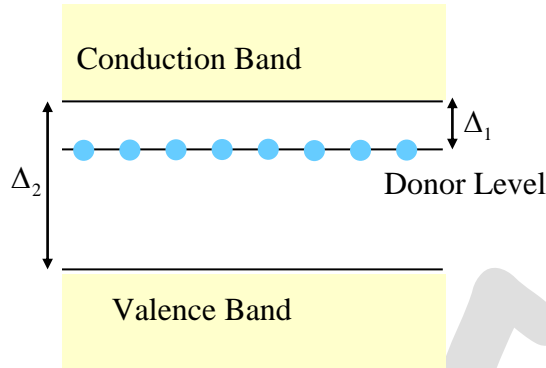


In order to transmit the signal (without distortion) the minimum bandwidth needed would be

- (a) 8 kHz (b) 2 kHz (c) 4 kHz (d) 6 kHz

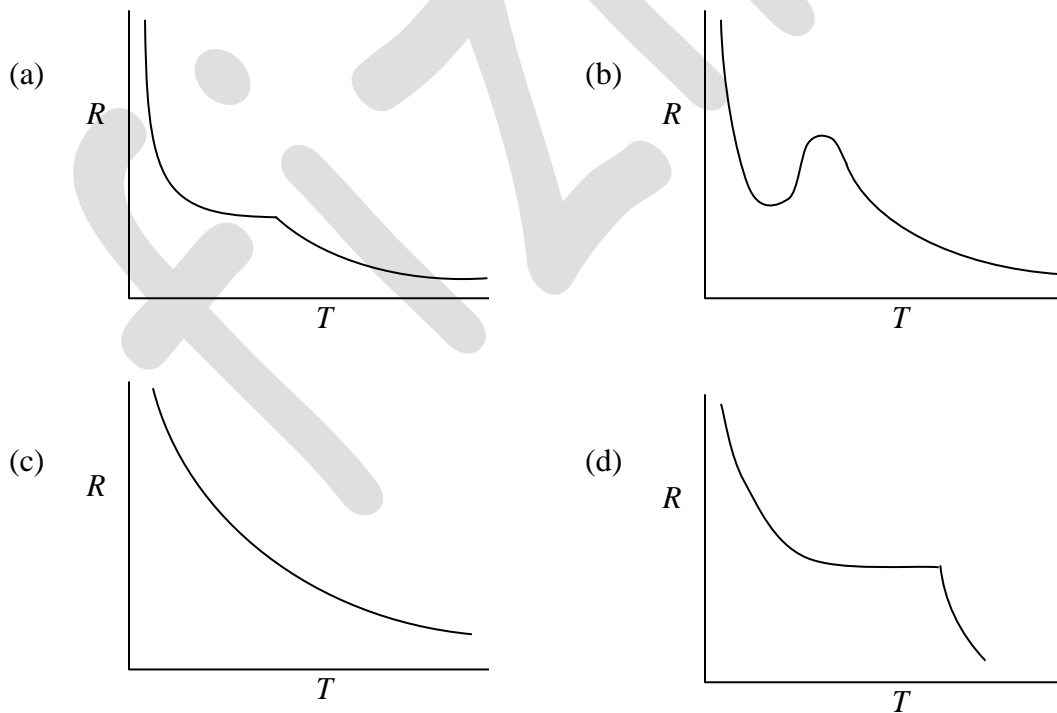
Ans.: (b)

Q38. A semiconductor with donor impurities can be thought in terms of a filled valence band, a filled donor level and an empty valence band at $T = 0$, as shown in the figure below



If the band gap between donor level and conduction band is Δ_1 and that between conduction and valence band is Δ_2 where $\Delta_2 \ll \Delta_1$, which of the following figures depict the qualitative features of the resistance (R) - vs-temperature (T) graph of the semiconductor?

(Assume temperature-independent scattering rates and a flat density of states for the bands.)



Ans.: (a)

Q39. Two atomic nuclei A and B have masses such that $m(B) = 2m(A)$, in the laboratory frame, the nucleus B is kept stationary, while the nucleus A is given a kinetic energy 300 MeV and is made to collide with B . It is found that the two nuclei fuse to form a compound nucleus C .

If the Q -value of the reaction is -30 MeV , the excitation energy of the compound nucleus can be estimated as

- (a) 81 MeV (b) 170 MeV (c) 330 MeV (d) 270 MeV

Ans.: (b)

Q40. Which of the following decays is forbidden?

- (a) $\pi^0 \rightarrow \gamma + \gamma$ (b) $K^0 \rightarrow \pi^+ + \pi^- + \pi^0$
(c) $\mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu$ (d) $n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$

Ans.: (c)