An Institute of NET-JRF, IIT-JAM, GATE, JEST,TIFR & M.Sc Entrance in Physics & Physical Sciences

TIFR 2022: Question Paper Physics

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

GS- 2022 ADMISSION TEST

Instructions for all candidates appearing for the Physics test for Ph.D. or Integrated Ph.D.
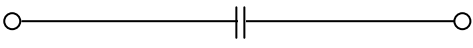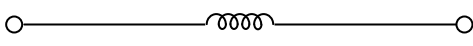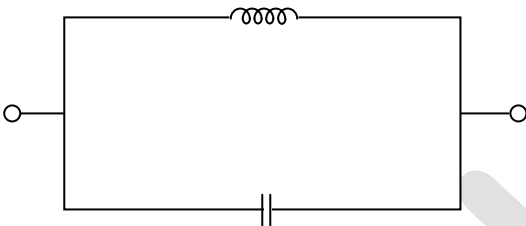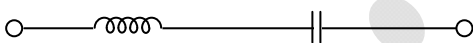
PLEASE READ THESE INSTRUCTIONS CAREFULLY BEFORE YOU ATTEMPT THE QUESTIONS

- You may not keep with you any books, papers, mobile phones, or any electronic devices which can be used to get/store information. Use of **scientific, non-programmable calculators is permitted.** Calculators which plot graphs are not allowed. Multiple use devices, such as smart phone, etc. CANNOT be used as calculator.

- This test consists of TWO sections.

  - ❖ **SECTION A comprises 25 questions, numbered Q1 - Q25. ----**These are questions on basic topics.

  - ❖ **SECTION B comprises 15 questions, numbered Q1 - Q15. ----**These may require somewhat more thought/knowledge.

- ALL questions are Multiple-Choice Type. In each case, ONLY ONE option is correct. Answer them by clicking the radio button next to the relevant option.

- If your calculated answer does not match any of the given options exactly, you may mark the closest one if it is reasonably close.

- The **grading scheme** will be as follows:

  - ❖ Section A : **+3** marks if correct; **–1** mark if incorrect; **0** marks if not attempted

  - ❖ Section B : **+5** marks if correct; **0** marks if incorrect or not attempted, i.e. NO negative marks.

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

1

Section A
(For both Integrated M. Sc. –Ph.D. and Ph.D. candidates)

Q1. It is required to design a circuit with an impedance $Z(\omega)$ such that $Z(\omega) = ik(\omega - \omega_0)$ for a range of frequencies ω such that $|\omega - \omega_0|/\omega_0 << 1$, where k and ω_0 are constant real numbers. A possible design for this circuit would correspond to

(a)

(b)

(c)

(d)

Q2. A commercial advertisement for a solar power converter claim that when the temperature of the plate (area $1.6\,m^2$) absorbing 20% of the solar energy (solar constant is about $1.36\,kW\,m^{-2}s^{-1}$) reaches 127^0C and the rest of the device is at room temperature (27^0C), the system will deliver a power of $100W$.

If a prospective customer comes to you for advice about buying this device, your advice should be that

(a)   The advertisement is false ad the device cannot deliver so much power.
(b)   The power delivered is very small for the given specifications.
(c)   Other similar devices are available which can deliver $1.5 - 2.0$ times the power with the same specifications.
(d)   It is an efficient device for the given specifications

Q3. The Principle of Linen Superposition of electron states in quantum mechanics is nicely illustrated by the
(a) Compton scattering experiment
(b) Millikan oil-drop experiment
(c) Franck-Hertz experiment
(d) Davisson-Germer experiment

Q4. Natural potassium contains a radioactive component of ^{40}K that has two decay modes

- In the first mode. ^{40}K undergoes a β decay to the ground state of ^{40}Ca.

- In the second mode, ^{40}K undergoes an election capture to the excited state of ^{40}Ar, followed by a single γ transition to the ground state of ^{40}Ar.

2

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

The amount of radioactive ^{40}K in a natural potassium (atomic weight of 39.089) sample is known to be 0.0118 percent. It is also known that in the decay of ^{40}K, for every 100β particles emitted, there number of γ-photons emitted is 12.

If the number of β-particles emitted per second by $1kg$ of natural potassium is 2.7×10^4

the means lifetime of ^{40}K in years is

(a) 1.9×10^9 (b) 1.7×10^9 (c) 1.3×10^9 (d) 1.1×10^8

Q5. A solid homogeneous sphere floats in water with a portion sticking out above the water, as shown in the figure below. The height of the highest point above the water surface is $R/2$ where R the radius of the sphere is.

If the density of water is $1\,g\,cm^{-3}$, the density of the material (in $g\,cm^{-3}$) must be

(a) $5/32$ (b) $5/18$ (c) $13/18$ (d) $27/32$
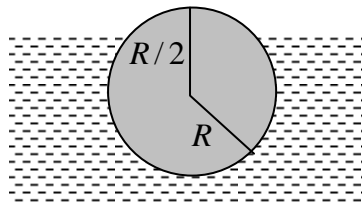
Q6. Consider the two-dimensional polar integral

$$P = \int dr d\theta\, r^{19} e^{-r^2} \sin^8\theta \cos^{11}\theta$$

If the integration is over only the first quadrant $\left(0 \le \theta \le \pi/2\right)$, the value of P is

(a) 180 (b) 88π (c) 16π (d) 20160

Q7. A particle moves in one dimension x under the influence of a potential $V(x)$ as sketched in the figure below. The shaded region corresponds to infinite V, i.e, the particle is not allowed to penetrate there.
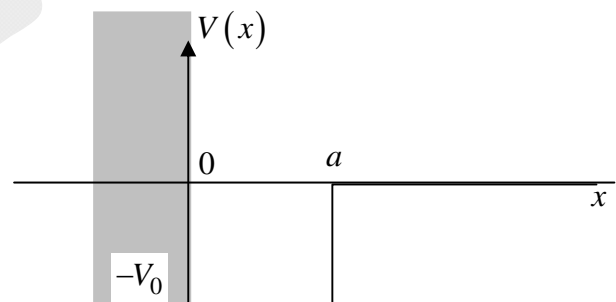
If there is an energy eigenvalue $E = 0$, then a and V_0 are related by

(a) $a^2 V_0 = \dfrac{n\pi^2}{2m}$ (b) $a^2 V_0 = \dfrac{\left(n+\frac{1}{2}\right)^2 \pi^2}{2m}$

(c) $a^2 V_0 = \dfrac{n^2\pi^2}{2m}$ (d) $a^2 V_0 = \dfrac{\left(n+\frac{1}{2}\right)\pi^2}{2m}$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

3

Q8. Two particles, as specified in the table below, both enter a region of uniform magnetic field in a direction perpendicular to the field direction.

Particle	Rest Mass	Kinetic Energy
Alpha	3.7GeV	11.2GeV
Deuteron	1.9GeV	20.0Mev

If both the particles then follow circular trajectories in the magnetic field, the ratio of their time periods for one full revolution must be

(a) 1.0   (b) 2.0   (c) 3.0   (d) 4.0

Q9. Consider a square which can undergo rotations and reflections about its centre, where making no transformation at all is counted as a rotation by 0^0. The total number of such distinct rotations and reflections which will keep the square unchanged is

(a) 16   (b) 32   (c) 4   (d) 8

Q10. In a hydrogenic atom of atomic of number Z, the probability amplitude that the nucleus will capture an electron from its own K-shell is proportional to the overlap between the nuclear wave-fiction

$$\psi_n(\vec{r}) = \frac{1}{\sqrt{8\pi r_N^3}} e^{r/r_N}$$

and the electron wave-function

$$\psi_e(\vec{r}) = \frac{Z^{3/2}}{\sqrt{8\pi a_0^3}} e^{-Zr/a_0}$$

where a_0 is the Bohr radius and r_N is the nuclear radius, which is known to vary as $r_N \propto Z^{0.37}$. The probability of electron capture to a very good approximation, will be proportional to Z^{α} where α is

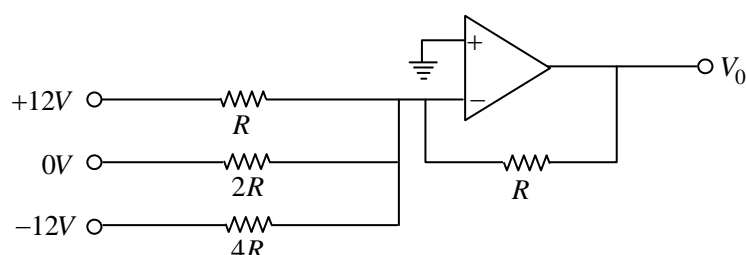(a) 2.22   (b) 1.11   (c) 2.05   (d) 4.11

Q11. A bicycle tyre is pumped with air to an internal pressure of 6 atm at $20°C$, at which point it suddenly bursts. Assuming the external pressure to be 1 atmosphere and the subsequent sudden expansion to be adiabatic, the temperature immediately after the burst is approximately

(a) $216.0°C$ (b) $-108.5°C$ (c) $-97.5°C$ (d) $45.5°C$

Q12. On a wet monsoon day at 12 noon, a thin film of oil of thickness $0.3\mu m$ is formed on a wet road. If the refractive index of oil and water are 1.475 and 1.333. respectively, which of the following wavelengths of light will be reflected with maximum intensity?

(a) $590\,nm$ (b) $407\,nm$ (c) $443\,nm$ (d) $640\,nm$

Q13. Consider a circuit with an operational amplifier (op amp) and four resistors as sketched below.

4

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

The output voltage V_0 is

(a) $-6V$ (b) $-9V$ (c) $0V$ (d) $-12V$

Q14. If an electron is set into oscillatory motion by the electric field of a laser of intensity $150 Wm^{-2}$ and wavelength $554\,nm$, the amplitudes of its displacement and velocity. respectively, are expected to be

(a) $5.1 \times 10^{-18} m$
$1.7 \times 10^{-2} ms^{-1}$

(b) $3.4 \times 10^{-18} m$
$1.7 \times 10^{-2} ms^{-1}$

(c) $3.4 \times 10^{-16} m$
$1.7 \times 10^{-1} ms^{-1}$

(d) $3.4 \times 10^{-17} m$
$1.0 \times 10^{-1} ms^{-1}$

Q15. A falling raindrop, spherical in shape, with a diameter of $1\,\mu m$, acquires a uniform negative charge due to friction with air. The electric field at a distance of $10\,\mu m$ from the surface of the droplet is measured to be $101 Vm^{-1}$.

The umber of excess electrons acquired by the droplet is

(a) 1414   (b) 1.4×10^{23} (c) 7   (d) 7.02×10^{6}

Q16. Since the refractive index of water is $4/3$, the angular velocity (in degrees per hour) of the Sun at noon is perceived by a fish in the ocean deep below the surface as around

(a) 15.0   (b) 11.3   (c) 13.2   (d) 20.0

Q17. Consider a set of three 3-dimensional vectors

$$A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

These vectors undergo a linear transformation.

$$A \to A' = MA \quad B \to B' = MB \quad C \to C' = MC$$

where M is given by

$$M = \begin{pmatrix} 1 & 1 & 4 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

The volume of a parallelepiped whose sides are given by the transformed vectors A', B' and C' is

(a) 8   (b) 2   (c) 4   (d) 16

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

5

Q18. An electromagnet is made by winding N turns of wire around a wooden cylinder of diameter of and passing a current I through it. When the current flows, a magnetic field of magnitude B is produced at a perpendicular distance z_0 from the axis of the cylinder.

where $z_0 >> d$.

If the number of turns N, she diameter of the wooden cylinder d and the current I are all doubled, then the magnitude of the magnetic field will be $B/2$ at a distance $z =$

(a) $0.5\,z_0$ (b) $3.2\,z_0$ (c) $4.8\,z_0$ (d) $2.4\,z_0$

Q19. The free election model of metals (Drude model) explains several physical properties but cannot be used to explain

(a) Thermal conductivity of the metal
(b) Positive value of Hall coefficient
(c) electrical conductivity of the metal
(d) magnetic susceptibility of the metal

Q20. A vertical cylinder of height H is filled with an ideal gas of classical point particles each of mass m and is allowed to come to equilibrium under gravity at a temperature T. The mean height of these particles is

(a) $\dfrac{H}{3}\dfrac{mgH/k_BT}{e^{mgH/k_BT}-1}$

(b) $\dfrac{H}{3}\dfrac{mgH/k_BT}{e^{mgH/k_BT}+1}$

(c) $\dfrac{k_BT}{mg}\left(1-\dfrac{2mgH/k_BT}{e^{mgH/k_BT}+1}\right)$

(d) $\dfrac{k_BT}{mg}\left(1-\dfrac{mgH/k_BT}{e^{mgH/k_BT}-1}\right)$

Q21. The circuit diagram on the right shows a block A representing a cubic structure comprising 12 identical resistances of $120\,\Omega$ each, whose body diagonal vertices are connected to the rest of the circuit with an inductor $L=10\,mH$, a resistor $R=100\,\Omega$, and a capacitor $C=1\,\mu F$.
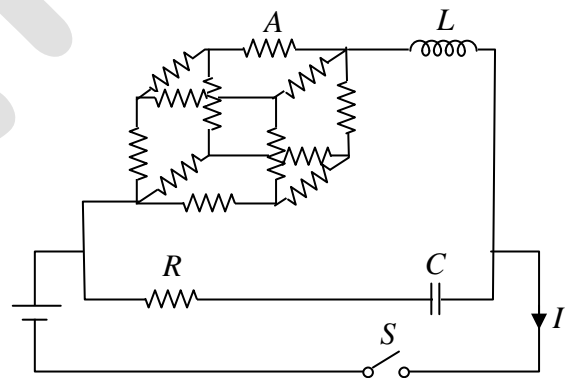
Now, the switch S is turned on at $t=0$.
The earliest time at which the current reaches a steady value l_0 is

(a) $100\,\mu s$ (b) $200\,\mu s$ (c) zero   (d) infinite

Q22. A gas of atoms, each of mass m, in thermal equilibrium at a temperature T, is radiating with a frequency v_0. The Doppler broadening (full width at half maximum of $FWHM$) of the observed spectral line would be given by

(a) $\dfrac{2v_0}{c}\sqrt{\dfrac{2\ln 2\,k_BT}{m}}$

(b) $\dfrac{v_0}{c}\sqrt{\dfrac{2k_BT}{m}}$

(c) $\dfrac{2v_0}{c}\sqrt{\dfrac{\ln 2\,k_BT}{m}}$

(d) $\dfrac{2v_0}{c}\sqrt{\dfrac{2k_BT}{m}}$

6

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q23. A particle of mass m moves under the action of a central potential

$$V(r) = -\frac{e^2}{r}$$

where e is a constant. Two vectors which rein conserved during the motion me

(i) the angular momentum $\vec{L} = \vec{r} \times \vec{p}$

(ii) the Runge-Lenz vector $\vec{K} = \vec{p} \times \vec{L} - me^2\hat{r}$ (where $\hat{r} = \vec{r}/r$)

The conserved energy of the particle can be written as

(a) $\dfrac{m^2e^4 - K^2}{2mL^2}$
(b) $\dfrac{K^2 - m^2e^4}{2mL^2}$
(c) $\dfrac{2mL^2}{m^2e^4 - K^2}$
(d) $\dfrac{2mL^2}{K^2 - m^2e^4}$

Q24. Treat the hydrogen molecule H_2 as a rigid rotator. The next-to-largest wavelength in its rotational spectrum is about $111\,\mu m$. From this it can be estimated that the separation between the pair of hydrogen atoms is about

(a) $61.4\,nm$
(b) $24.4\,nm$
(c) $3.07\,\mu m$
(d) $0.12\,nm$

Q25. Two students A and B measure the time period of a simple pendulum in the laboratory using the same stopwatch but following two different methods.

- Student A measures the time taken for one oscillation and repeats it for N_A number of times and finds the average.

- Student B on the other hand, measures the time taken for N_B number of oscillations and then computes the period.

Given that $N_A, N_B \gg 1$, to ensure that both students measure the time period with the same uncertainty, the relation between N_A and N_B must be

(a) $N_A = N_B$
(b) $N_A = N_B^2$
(c) $N_A = \sqrt{N_B}$
(d) $\ln 2N_A = N_B$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

7

Section B
(only for Ph.D. candidates)

Q1. There are two conceivable channels by which a vector ρ^0 meson can decay into a pair of pseudo scalar points. These are

$$\rho^0 \to \pi^0 + \pi^0 \text{ and } \rho^0 \to \pi^+ + \pi^-$$

The probability that the decay takes place through the process $\rho^0 \to \pi^+ + \pi^-$ is Approximately

(a) $m_{\pi^0}/2m_{m^+}$ 　　　(b) Zero 　　　(c) $m_{\pi^+}^2/m_\rho^2$ 　　　(d) 1

Q2. In the shell model of the nucleus, it is known that orbital get filled in the order $1s_{1/2}\ 1p_{3/2}\ 1p_{1/2}\ 1d_{5/2}\ 2s_{1/2}\ 1d_{3/2}$ and so on

For a nucleus of ${}_8^{18}O$ the two neutrons outside the doubly-magic core of ${}_8^{16}O$ will occupy the same orbital. The allowed value of J^p will be

(a) 5^+ 　　　(b) 4^+ 　　　(c) 3^+ 　　　(d) 2^-

Q3. From the knowledge that you already have about the length of one year and the fact that the Sun subtends $0.5°$ in the sky, the average density of the Sun can be computed in $kg\text{-}m^{-3}$ as

(a) 1.7×10^3 　　　(b) 7.5×10^2 　　　(c) 7.5×10^3 　　　(d) 1.7×10^2

Q4. Consider the inner product in the space of normal sable functions defined on the interval $[-1,1]$

$$\langle f \mid g \rangle = \int_{-1}^{1} dx \left(1+x^2\right) f(x) g(x)$$

The projection of the vector 1 along the vector x^2 is

(a) $\dfrac{16}{15}\sqrt{\dfrac{35}{24}}x^2$ 　　　(b) $\sqrt{\dfrac{35}{24}}x^2$ 　　　(c) $\dfrac{16}{15}x^2$ 　　　(d) $\dfrac{14}{9}x^2$

Q5. In a semi classical approach, the Hamiltonian of a He atom is modified by adding a magnetic interaction term between the two electrons, of the form

$$H_1 = A_2\vec{S}_1 \cdot \vec{S}_2$$

where $\vec{S}_1$ and $\vec{S}_2$ are the electron spins and A_2 is a coupling constant. This leads, for the configuration $1s^2$, to the energy shift

(a) $-A_2/4$ 　　　(b) $+A_2/4$ 　　　(c) $+3A_2/4$ 　　　(d) $-3A_2/4$

Q6. The power radiated by a point charge q moving rapidly with a uniform speed v in a circle of radius R will be

(a) $\dfrac{q^2c^3}{6\pi\varepsilon_0 R^3}\dfrac{v^2}{c^2-v^2}$

(b) $\dfrac{q^2c}{6\pi\varepsilon_0 R^2}\left(\dfrac{v^2}{c^2-v^2}\right)^2$

(c) $\dfrac{q^3c}{6\pi\varepsilon_0 R^4}\dfrac{v^2}{c^2-v^2}$

(d) $\dfrac{q^4c^2}{6\pi\varepsilon_0 R^2}\left(\dfrac{v^2}{c^2-v^2}\right)^2$

8

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q7. At very low temperatures, the electrical resistively of most metals is dominated by
(a) absorption of conduction electrons by ions in the lattice.
(b) collisions of conduction electrons with lattice phonons.
(c) collisions of conduction electrons with impurity atoms and lattice vacancies.
(d) transfer of conduction electrons to the valence band.

Q8. Three students A, B and C are given identical counters and each is asked to measure the number of gamma rays emitted per second by a given radioactive source. They are expected to perform the counting many times and find the mean and the standard deviation. The students find the following:

student	A	B	C
Measurement (counts/second)	482 ± 22	495 ± 10	501 ± 22

If a counting experiment conducted previously by the instructor on this same sample with another identical counter had recorded exactly $30,000$ gamma rays in a minute. then which of the following interpretations is valid?
(a) The measurement by student B is too precise to be believable.
(b) The measurements of A and C have too large standard deviations.
(c) The measurement of student B is more correct than that of student A.
(d) The measurement of C is much more precise than that of A.

Q9. A pseudo-potential V_{12} between every pair of particles in an ideal gas is to be constructed which will reproduce the effects of quantum statistics if the gas particles are bosonic in nature. A correct formula for this, in terms of the inter-particle distance r_{12} and a mean distance λ, will be of the form

(a) $V_{12} = +k_B T \ln\left(1 + e^{-2\pi r_{12}^2/\lambda^2}\right)$
(b) $V_{12} = -k_B T \ln\left(1 + e^{-2\pi r_{12}^2/\lambda^2}\right)$

(c) $V_{12} = +k_B T \ln\left(1 - e^{-2\pi r_{12}^2/\lambda^2}\right)$
(d) $V_{12} = -k_B T \ln\left(1 - e^{-2\pi r_{12}^2/\lambda^2}\right)$

Q10. A system with two generalized coordinates (q_1, q_2) is described by the Lagrangian

$$L = m\left(\dot{q}_1^2 + 2\dot{q}_1\dot{q}_2 + \frac{3}{2}\dot{q}_2^2\right) - k\left(\frac{3}{2}q_1^2 + 2q_1 q_2 + q_2^2\right)$$

where m is the mass, and k is a constant.
This system can execute oscillations with two possible time periods

(a) $T = 2\pi\sqrt{\dfrac{2m}{k}}$ and $T = 2\pi\sqrt{\dfrac{m}{2k}}$

(b) $T = 2\pi\sqrt{\dfrac{2m}{3k}}$ and $T = 2\pi\sqrt{\dfrac{3m}{2k}}$

(c) $T = 2\pi\sqrt{\dfrac{m}{2k}\left(5 - 2\sqrt{6}\right)}$ and $T = 2\pi\sqrt{\dfrac{m}{2k}\left(5 + 2\sqrt{6}\right)}$

(d) $T = \pi\sqrt{\dfrac{m}{k}\left(1 - \sqrt{15}\right)}$ and $T = \pi\sqrt{\dfrac{m}{k}\left(1 + \sqrt{15}\right)}$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

9
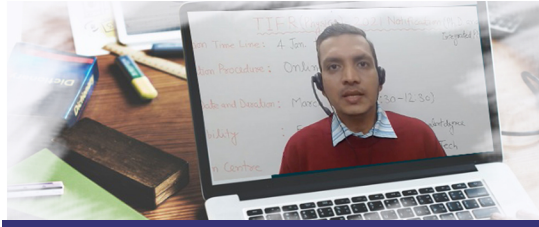
Q11. A particle is confined to a one-dimensional lattice with a lattice spacing δ. In the position space, the Hamiltonian operator for this particle is given by the matrix

$$H = E_0 \begin{pmatrix} \ddots & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \cdots & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & \cdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \ddots \end{pmatrix}$$

Noting that it commutes with the generator T of translations.

$$J = \begin{pmatrix} \ddots & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \cdots & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \ddots \end{pmatrix}$$

where $T = e^{ip\delta/\hbar}$ in terms of the momentum operator p, the energy of a state with momentum p will be

(a) $E_0 \cos(p\delta/\hbar)$ (b) $E_0 (p\delta/2\hbar)^2$ (c) $E_0 \sin(p\delta/2\hbar)$ (d) $4E_0 \sin^2(p\delta/2\hbar)$

Q12. A system was formed of three spin-$\frac{1}{2}$ particles A, B and C respectively and it was prepared in an initial state

$$|\psi\rangle = c_1|\uparrow\uparrow\uparrow\rangle + c_2|\uparrow\uparrow\downarrow\rangle + c_3|\uparrow\downarrow\uparrow\rangle + c_4|\uparrow\downarrow\downarrow\rangle + c_5|\downarrow\uparrow\uparrow\rangle + c_6|\downarrow\uparrow\downarrow\rangle + c_7|\downarrow\downarrow\uparrow\rangle + c_8|\downarrow\downarrow\downarrow\rangle$$

where the symbols $|\uparrow\rangle$ and $|\downarrow\rangle$ indicate states with $S_z = +\frac{1}{2}$ (spin-up) and $S_z = -\frac{1}{2}$ (spin-down) respectively.

A measurement was made on the system in the initial state and this identified the spin state of the particle A to be $|\downarrow\rangle$ (spin-down). Now the expectation value of $\langle S_z \rangle$ for the particle C could be calculated as

(a) $\dfrac{\left(C_5^* + C_7^* - C_6^* - C_8^*\right)\left(C_5 + C_7 - C_6 - C_8\right)}{|C_5|^2 + |C_7|^2 + |C_6|^2 + |C_8|^2}$

(b) $\dfrac{\left(C_5 + C_7\right)*\left(C_5 + C_7\right) - \left(C_6 + C_8\right)*\left(C_6 + C_8\right)}{|C_5|^2 + |C_7|^2 + |C_6|^2 + |C_8|^2}$

(c) $\dfrac{|C_5|^2 + |C_7|^2 - |C_6|^2 - |C_8|^2}{|C_5|^2 + |C_7|^2 + |C_6|^2 + |C_8|^2}$

(d) $\dfrac{C_5 + C_7 - C_6 - C_8}{|C_5|^2 + |C_7|^2 + |C_6|^2 + |C_8|^2}$

10

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q13. A dumbbell consists of two small spherical masses M each, connected by a thin mass less rod of length 2ℓ. This dumbbell is centered at the origin, and is rotating about the z-axis with a uniform angular velocity ω, making an angle θ with the z-axis (see figure). Neglecting effects due to gravity, at the instant when the dumbbell is wholly in the yz-plane (as shown in the figure), the magnitude of torque about the origin will be
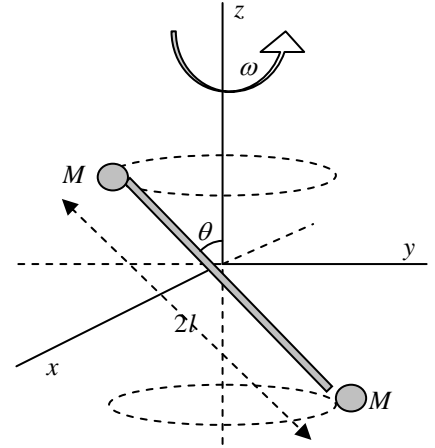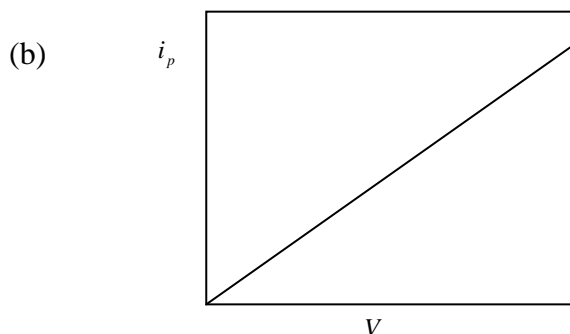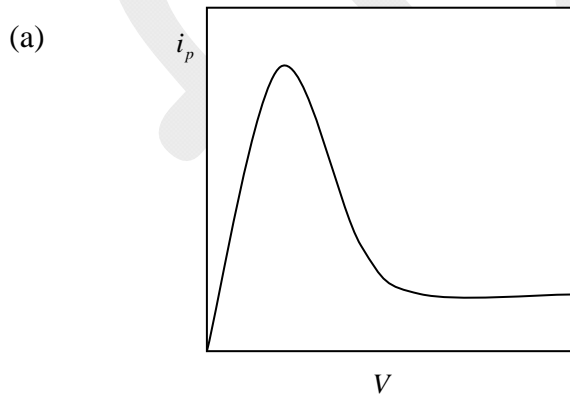
(a) $2M\ell^2\omega^2\sin^2\theta$

(b) $M\ell^2\omega^2\sin 2\theta$
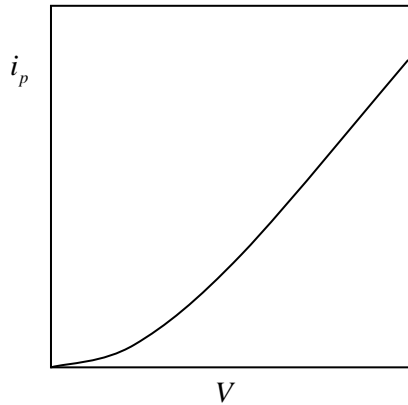
(c) $2M\ell^2\omega^2\cos^2\theta$

(d) zero

Q14. A thyratron consists of a tube filled with Xenon gas which can be used as a high power electrical switch. Electrons are emitted from a cathode K heated by a filament F, and made to accelerate to some energy E by a voltage V applied across the anode plate P.

Electrons that scatter from Xe atoms get deviated from their path and hit the shield S. which is a conducting envelope that transports the electrons back to ground potential (see figure on the right). The rest of the electrons strike the plate and contribute to the plate current i_p.
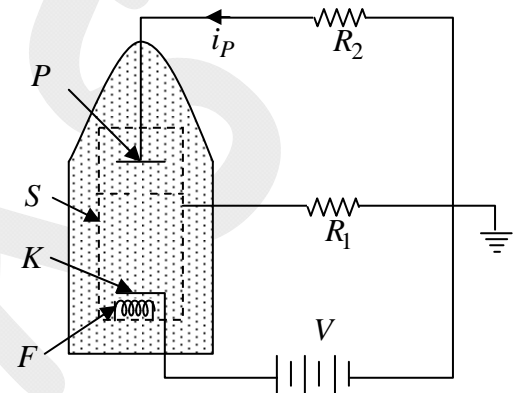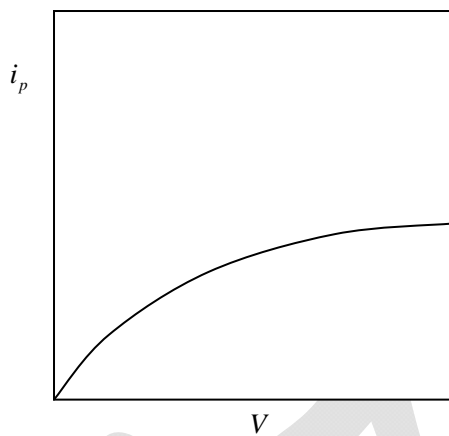
Which of the following graphs of the variation of the plate i_p with increase in the accelerating voltage V could indicate the wave nature of the electron?

(a)

(b)

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

11

(c)

(d)

Q15. According to a standard table, the refractive index of water at $4°C$ is 1.33 at a wavelength of $590\,nm$. However, a carefully performed experiment in the lab yielded a refractive index of 1.41.

Which one of the following statements could be the explanation of this discrepancy?

(a) The experiment was performed at a wavelength lower than 590 nm.

(b) The water sample was at a temperature lower than $4°C$.

(c) The experiment was performed at a wavelength higher than $590\,nm$.

(d) The water sample was at a temperature much higher than $4°C$.

12

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Section B
(only for Integrated M.Sc.-Ph.D. candidates)

Q1. A spectrographic method to search for exoplanets is by measuring its velocity along the line of sight, using the Doppler shift in the spectrum. If a star of mass M and a planet of mass m are moving around their common centre of mass, this component of velocity will vary periodically with an amplitude.

$$A = \left(\frac{2\pi G_N}{T} \right)^{1/3} \frac{m}{M^{2/3}}$$

For a particular planet-star system, if the time period is $T = (12 \pm 0.3)$ years, and A and M are measured with an accuracy of 3% each, then the error in the measurement of the mass m is

(a) 5.8%   (b) 8.5%   (c) 6.3%   (d) 3.7%

Q2. A cricket ball, bowled by a fast bowler, rises from the pitch at an angle of 30° with a speed of 72 km/hr, then moves straight ahead and, at a height of 0.5 m, strikes the flat surface of the bat held firmly at rest in a horizontal position (see figure). As a result, the ball bounces off elastically, providing a return catch straight back to the bowler.

If the coefficient of restitution between the bat and the ball is 0.577, the acceleration due to gravity is 10 ms^{-2} and air resistance can be neglected, the catch will carry, before hitting the ground, to a distance of approximately

(a) 37.0 m   (b) 9.5 m   (c) 19.5 m   (d) 21.0 m

Q3. A quantum dot is constructed such that it has just three energy levels, with energies *E*, *2E* and *3E* respectively. The chemical potential in the system has the value $\mu = 2E$ and the temperature is given by

$$T = \frac{E}{2k_B}$$

The expected number of electrons populating the quantum dot will be

(a) 4.0   (b) 3.0   (c) 1.5   (d) 2.5

**Q4**. In a standardized entrance exam, the passing rates for the past 10 years are tabulated below.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Passing Rate	22%	16%	23%	21%	22%	14%	17%	20%	24%	21%

If 1000 candidates appear for the exam every year, the probability that more than 250 students will pass the exam this year is about

(a) 20%   (b) 6%   (c) 25%   (d) 0.1%

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

13

Q5. A hollow metal sphere filled with a thick, highly viscous oil is rotating about a vertical axis with an initial angular velocity ω_0. However, there is a small hole at the bottom of this sphere, through which drops of oil are leaking out vertically at a steady rate. The variation of the angular velocity $\omega(t)$ of the sphere with time t is best represented graphically by
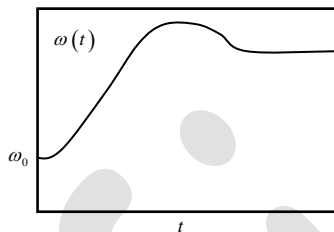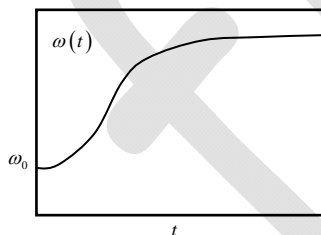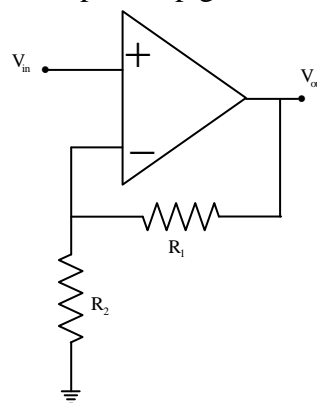
(a)
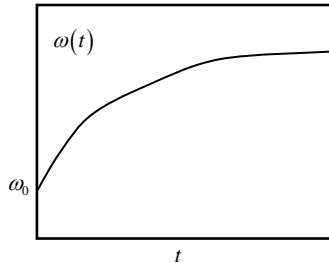
(b)
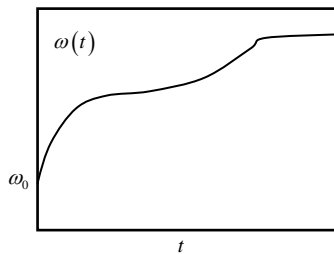
(c)

(d)

Q6. The non-inverting amplifier shown in the figure on the right is constructed using a non-ideal operational amplified (op amp) with a *finite* open loop gain A.

14

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

The value of feedback fraction is

$$B = \frac{R_2}{R_1 + R_2} = 0.1$$

If the gain A varies such that

$$10^4 < A < 10^5$$

then the approximate percentage variation in the closed loop gain will be.

(a) 0.9%   (b) 0.09%   (c) 9.0%   (d) 0.0%

Q7. In a matrix mechanics formulation, a spin-1 particle has angular momentum components

$$L_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 & -1 \\ 1 & \sqrt{2} & 0 \\ -1 & 0 & -\sqrt{2} \end{pmatrix} \qquad L_z = \frac{\hbar}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

It follows that $L_y =$

(a) $\sqrt{2}\hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(b) $\frac{\hbar}{2}\begin{pmatrix} 0 & i & -i \\ -i & 0 & i\sqrt{2} \\ i & -i\sqrt{2} & 0 \end{pmatrix}$

(c) $\frac{\hbar}{2}\begin{pmatrix} 0 & -i & i \\ i & 0 & -i\sqrt{2} \\ -i & i\sqrt{2} & 0 \end{pmatrix}$

(d) $\sqrt{2}\hbar\begin{pmatrix} 0 & -\sqrt{2} & 0 \\ -\sqrt{2} & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Q8. An observer *O*, moving with relativistic speed *v* away from a fixed plane mirror M in a line perpendicular to the mirror surface, sends a pulse of light of wavelength λ towards the mirror.

The wavelength of the light reflected back to the observer will be

(a) $\lambda\left(\frac{c-2v}{c+2v}\right)$ (b) $\lambda\left(\frac{c+2v}{c-2v}\right)$ (c) $\lambda\sqrt{\frac{c-v}{c+v}}$ (d) $\lambda\left(\frac{c+v}{c-v}\right)$

Q9. The value of the integral $\int\limits_{-\pi/2}^{+\pi/2} dx \cosh kx^2 \sin^2 x$ in the large-*k* limit, will be

(a) $\cosh\left(\frac{\pi^2}{4}\right)$

(b) $\frac{1}{2k\pi}e^{k\pi^2/4}$

(c) $\frac{1}{k^2\pi^2}\cosh\left(\frac{\pi^2}{4}\right)$

(d) $\frac{1}{k\pi}e^{k\pi^2/4}$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

15

Q10. The low-temperature specific heat of a certain material is primarily due to acoustic phonons. The frequency ω of a phonon is related to its wavevector k by $\omega = ck$, where c is the speed of sound in the material. The phonons have a Bose distribution

$$n(k) = \frac{1}{e^{\hbar ck/k_B T} - 1}$$

and the energy of a phonon has a maximum possible value ω_D.

In a two-dimensional sample, the specific heat at low-temperatures behaves as
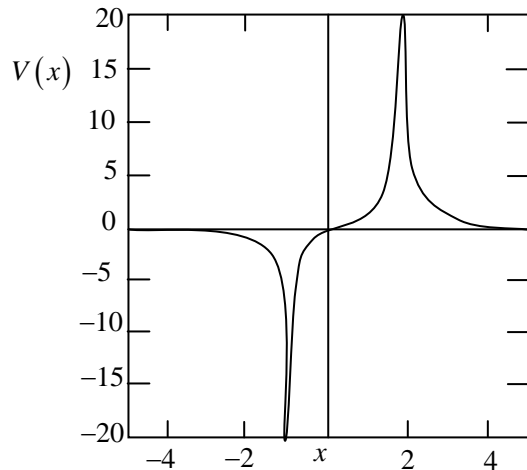
(a) $\dfrac{T}{\omega_D}$ (b) $\left(\dfrac{T}{\omega_D}\right)^3$ (c) $\left(\dfrac{T}{\omega_D}\right)^2$ (d) $\left(\dfrac{T}{\omega_D}\right)^{3/2}$

Q11. Two equal positive point charges $Q = +1$ are placed on either side of an x-axis normal to a grounded infinite conducting plane at distances of $x = +2$ units and $x = -1$ unit respectively (see figure) w.r.t. the point of intersection of the axis with the conducting plane as origin.
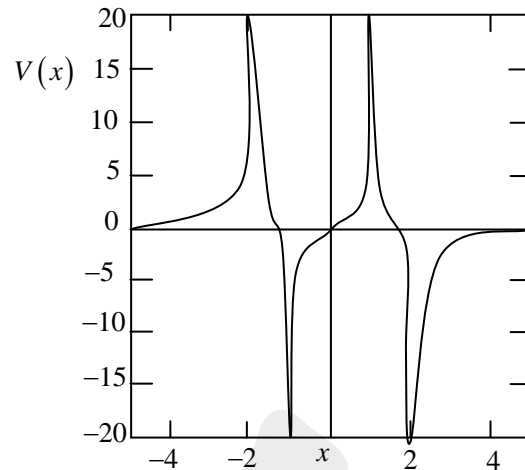
The electrostatic potential along the axis will correspond to the graph in

(a)

(b)
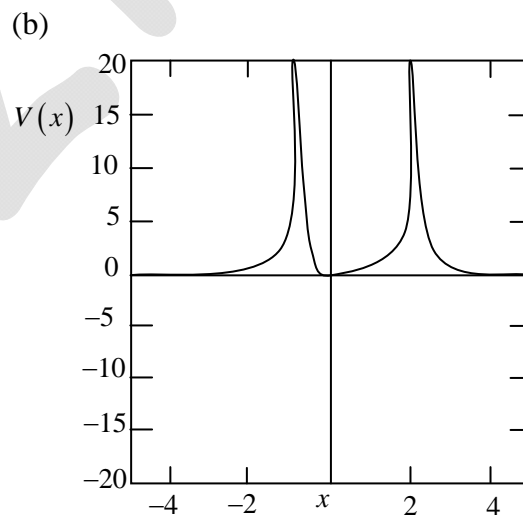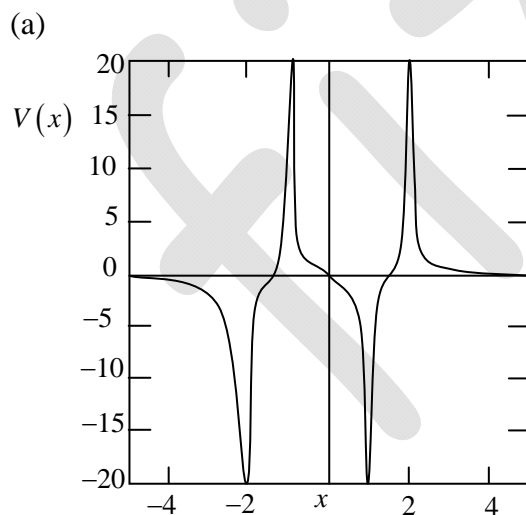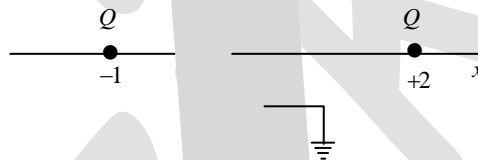
(c)

(d)

Q12. A particle of mass m in a three-dimensional potential well has a Hamiltonian of the form

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2}m\omega^2 x^2 + 2m\omega^2 z^2$$

where ω is a constant. If there are two identical spin-½ particles in this potential having a total energy

$$E = 6\hbar\omega$$

the entropy of the system will be

(a) $k_B \ln 12$　　　　(b) $k_B \ln 14$　　　　(c) $k_B \ln 10$　　　　(d) $k_B \ln 16$

Q13. Two co-axial solenoids A and B, one placed completely inside the other, have the following
parameters:

Solenoid	No. of turns	Length	Diameter
A	1000	50 cm	2 cm
B	2000	50 cm	4 cm

 The mutual inductance between the solenoids is

(a) 395.0 mH　　　　(b)1.58 mH　　　　(c) 125.7 mH　　　　(d)12.57 mH

Q14. A pendulum which is suspended from the ceiling of a train has time period T_0 when the train is stationary. When the train moves with a small but steady speed *v* around a horizontal circular track of radius *R*, the time period of the pendulum will be

(a) $T_0\left(1 + \frac{v^4}{g^2 R^2}\right)^{1/4}$　　　　　　(b) $T_0\left(1 - \frac{v^2 T_0^2}{4\pi^2 R}\right)^{-1/2}$

(c) $T_0\left(1 - \frac{v^4}{g^2 R^2}\right)^{1/4}$　　　　　　(d) $T_0\left(1 + \frac{v^2 T_0^2}{4\pi^2 R}\right)^{-1/2}$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

17

Q15. A statellite used to make Google Earth images carries on board a telescope which must be designed, when operating at a wavelength λ, to be able to resolve objects on the ground of length as small as δ.

If the satellite goes around the Earth in a circular orbit with uniform speed v, the minimum diameter D_{min} of the telescope mirror can be determined in terms of R, the radius of the Earth, and g, the acceleration due to gravity at the surface, to be

(a) $\dfrac{1.22\lambda}{\delta}\left(\dfrac{gR^2}{v^2} - R\right)$

(b) $\dfrac{1.22\lambda}{\delta}\dfrac{gR^2}{v^2}\left(1 + \dfrac{R}{\lambda}\right)$

(c) $\dfrac{1.22\lambda}{\delta}\dfrac{gR^2}{\lambda v^2}$

(d) $\dfrac{1.22\lambda}{\delta}\sqrt{\dfrac{gR^3}{v^2}}$

18

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498,   Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Physics by fiziks

An Institute of NET-JRF, IIT-JAM, GATE, JEST, TIFR & M.Sc Entrance in Physics & Physical Sciences

Achievement & Hall of Fame

Our Toppers in 2020-2022

Akash Naskar
IIT-JAM AIR - 5
Jadavpur Univ. Kolkata

Debosmita
NET AIR-10
IIT Delhi

Gourab Dutta
IIT-JAM AIR - 13, TIFR Quil.
Jadavpur Univ. Kolkata

Amarjeet
NET AIR-14
MDU Rohtak

Akshit Joon
NET AIR - 14
Kuk, S.D College
Panipat

Akash Bhardwaj
IIT-JAM AIR - 16
Ramjas College, DU

Anil
JEST AIR - 21
IGU Meerpur, Rewari

Siddhartha Paul
IIT-JAM AIR - 22, TIFR AIR - 32
Jadavpur Univ. Kolkata

Akshita Agarwal
JRF AIR-24
HNB Garhwal Univ.

Dikhya Joshi
NET AIR - 24
Techno India Univ. Kolkata

Vinay Kumar
IIT JAM AIR - 26
JMI, Delhi

Aditi
NET AIR-27, GATE AIR-688
BHU Varanshi

Satyaki Manna
GATE AIR-27
Jadavpur Univ. Kolkata

Shraddha Singhal
NET AIR - 27
Kumaun Univ. Nainital

Keshav Aggarwal
IIT-JAM AIR - 32,
Delhi Technical Univ.

Amit Tyagi
JRF AIR 35, GATE AIR - 417
CCSU Meerut

Apoorva Asthana
IIT-JAM AIR - 39,
JEST AIR - 189
AKTU

Aditi Sindhu
IIT JAM AIR - 41
ARSD, Delhi University

Mani Shankar
IIT JAM AIR - 42
ARSD, Delhi University

Ananya Bansal
NET AIR - 43
Delhi University

Ajay Pratap Singh Rana
NET AIR - 45, GATE AIR - 640
IISER Thiruvananthpuram

Vaishali
JRF AIR - 46, GATE AIR - 762
GJUST Haryana

Tanu Sharma
IIT-JAM AIR - 50,
JEST AIR 85,
MLNC, Delhi Univ.

Rahul
IIT-JAM AIR - 50
S.V.C. Delhi University

Sapan Kumar Sahoo
JEST AIR - 50
NET AIR - 124, GATE 478
Central Univ. of South Bihar

Akash Rawat
JRF AIR - 54
SVNIT, NIT

Jaydeep Lohia
JEST AIR - 62
IIT-Bombay

Vijay Luxmi
NET AIR - 61
NIT Kurukshetra

Harsh Chaudhary
IIT-JAM AIR - 62
NIT, Kurukshetra

Devender Kumar
GATE AIR-63
Delhi University

Santanu Singh
IIT-JAM AIR - 67
RKMRC, West Bengal

Shubhrakanta Panda
JRF AIR-72
NIT Rourkela

Monika Redhu
NET AIR - 73
Kurukshetra, Haryana

Ayush Garg
JRF AIR - 79
Rajasthan Technical Univ.

Ekta
JEST AIR - 84

Ayush Kumar Shaw
JEST AIR - 91
IIT-JAM AIR - 432
Jadavpur Univ. Kolkata

Sagar Malik
IIT-JAM AIR - 96,
JEST AIR - 211,
NIT, SURAT

Abhishek T
JRF AIR - 97, GATE AIR - 121
NIT, Kolkata

Anu Sharma
GATE AIR-100
Punjabi Univ. Patiala

Seema Maurya
JRF AIR - 101
Guru Ghasidas Univ.

Sharmila Gunwal
NET AIR 109
Miranda House, DU

Jyoti
NET AIR 109, GATE AIR 515
Central Univ. of Punjab

Physics by fiziks

An Institute of NET-JRF, IIT-JAM, GATE, JEST, TIFR & M.Sc Entrance in Physics & Physical Sciences

Achievement & Hall of Fame

Our Toppers in 2015-2019

Pargam Vashishtha
JRF AIR-2
M.Sc. from CCS Univ.

Manish Singh
JEST AIR-3
B.E. from D.T.U. Delhi

Stav Haldar
IIT-JAM AIR-8
B.Sc. from BIT Mesra, Ranchi

Ritam Basu
JEST AIR-8, IIT-JAM AIR-18
RKMR College, Kolkata

Rashid Ali
GATE AIR-9, JRF AIR-17
SSVPG College, Meerut Univ.

Abhishek Singh
IIT-JAM AIR-9, JEST AIR-117
MLNC, DU

Kunal Vyas
IIT-JAM AIR-11, JEST AIR-141
SIMSR, Mumbai

Hemanshu Dua
GATE AIR-14
M.Sc. from IISER-Mohali

Sadhan Biswas
JRF AIR-14
M.Sc. from C.S.J.M. Univ.

Ruby Negi
JEST AIR-15, IIT-JAM AIR-251
MLNC, DU

Gaurav Mukherjee
IIT-JAM AIR-16
B.Sc. from BIT Mesra, Ranchi

Ankit Dulat
IIT-JAM AIR-16, JEST AIR-20
B.Sc. from DU

Surya Kant Verma
JRF AIR-17
M.Sc. from Rajasthan Univ

Mukaddar Shaikh
JRF AIR-19
M.Sc. from A.M.U. Aligarh
Ph.D. IIT-Kharagpur

Aman Kumar
IIT-JAM AIR-19
B.Sc. from ARSD, DU

Atul Dubey
JRF AIR-22
M.Sc. from D.D.U. Gorakhpur
Ph.D. IIT-Delhi

Banashree Baishya
JRF AIR-24, GATE AIR-177
Gauhati University

Ekta Kumawat
IIT-JAM AIR-25
B.Sc. from Rajasthan Univ (IIT-D)

Manjari Jain
GATE AIR-26, JRF AIR-93
M.Sc. from Dr. R.M.L.A Univ.

Akansha Gupta
JRF AIR-27
M.Sc. from Rajasthan Univ.

Mrityunjaya Goswami
IIT-JAM AIR-29
B.Sc. from IGNOU

Neeru Kundu
JRF AIR-33, GATE AIR-36
Kurukshetra University .

Radhika Prasad
IIT-JAM AIR-35
B.Sc. from DU

Vinay Vaibhav
IIT-JAM AIR-36
B.Sc. from Central Univ.
of Jharkhand, Int. Ph.D. CMI

Aman Chauhan
JEST AIR-45, IIT-JAM AIR-154
Agra College, DBRA Univ.

Mohit Mehta
JEST AIR-47
YMCA, Faridabad

Priyanka Garg
JRF AIR-47
M.Sc. from P.U.

Dhananjay Singh
IIT-JAM AIR-49
IIT Dhanbad

Gouri Patil
IIT-JAM AIR-50
B.Sc. from M.C. College, Bangalore
Int. Ph.D. IISc. Bangalore

Deepak Sharma
JRF AIR-57, GATE AIR-290
Kurukshetra University .

Desh Deepak Gaur
GATE AIR-61, JEST AIR-197
Aligarh Muslim University

Shree Hari Mittal
JEST AIR-65, IIT JAM AIR-340
ARSD, DU

Amanulla Karikar
JRF AIR-78
Aligarh Muslim University

Yash Chugh
JEST AIR-82, IIT-JAM AIR-595
IGNOU

Yogesh Arya
JEST AIR-82, GATE AIR-357
MNIT, Jaipur

Ramesh Kumar
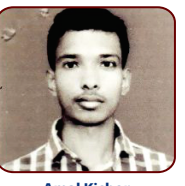JRF AIR-88
G.J.U.S.T, Hisar, Haryana

Amandeep Kaur
JRF AIR-94
IIT, Guwahati

Shashank Kumar
JRF AIR-99, GATE AIR-89
JEST AIR-107
IIT-Dhanbad

Shinjini Das
GATE AIR - 99
SCC, Calcutta University

Amal Kishor
CSIR-NET AIR 100
NIT, Durgapur

Rajesh Kumar Raul
CSIR-NET AIR 100
Annamalai University

Abhishek Navhal
TIFR
DTU, Delhi

☎ 011-26865455/+9871145498   Address: H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi - 110016