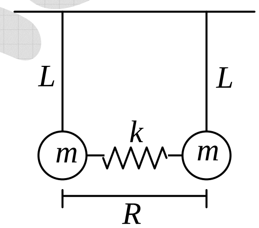
JNU (M.Sc) Entrance Examination, 2016

Q1. The maximum radius (R_0) that a black hole can have depends on its mass (M), the universal constant of gravitation (G) and the speed of light (c). On dimensional grounds, the expression for R_0 should be proportional to

(a) GM/c^2 (b) GM^2/c^3 (c) GM^2/c (d) GM/c

(e) G^2M/c

Q2. Two pendulums of identical length L and mass m are hanging from a horizontal rod. Their points of suspension are separated by R. A spring of unstretched length R and spring constant k, is now used to connect the masses. For this new system there is a mode of oscillation in which both the masses move sinusoidally with a common circular frequency ω and the displacements of the two masses are equal in magnitude but opposite in sign at all times. If ω has to be twice the common circular frequency of the two masses before the spring is connected, then the ratio $\frac{kL}{mg}$, where g is the acceleration due to gravity, must be



(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{2}{3}$

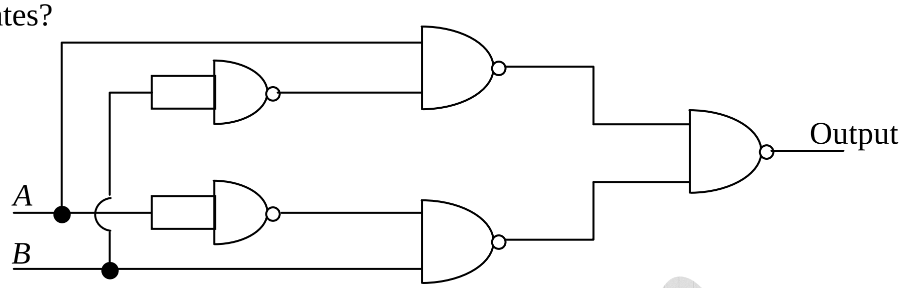
(e) $\sqrt{2}$

Q3. Consider an elastic collision between two objects A and B of mass m_1, and m_2, respectively. Before and after collision, motion takes place only along the x-axis. Before collision, A is moving with speed v and B is at rest. If, after collision, A keeps moving in the same direction, but with its speed reduced to $\frac{v}{2}$, the ratio $\frac{m_2}{m_1}$ must be

(a) 2 (b) 1 (c) $\frac{1}{3}$ (d) $\sqrt{5}-1$

(e) $\sqrt{2}-1$

Q4. Which of the following represents the output of the circuit implemented using only NAND gates?



(a) A AND B (b) A OR B (c) A NAND B (d) A NOR B

(e) A XOR B

Q5. Graphite, an allotrope of carbon, crystallizes in a simple hexagonal crystal structure with primitive translation vectors of the hexagonal space lattice given by

$$a_1 = \frac{\sqrt{3}}{2}a\hat{x} + \frac{a}{2}\hat{y}\ ; a_2 = -\frac{\sqrt{3}}{2}a\hat{x} + \frac{a}{2}\hat{y}\ ; a_3 = c\hat{z}$$

here $a = 2.46\overset{0}{A}$ and $c = 6.7\overset{0}{A}$. Given that there is one atom in the primitive cell, the density of graphite is given by (Take the mass number of carton to be 12)

(a) $0.56\,gm/cm^3$ (b) $5.10\,gm/cm^3$

(c) $2.40\,gm/cm^3$ (d) $1.40\ gm/cm^3$

(e) $4.32\,gm/cm^3$

Q6. A solid square slab of dielectric material with dielectric constant 5.0 fills up the space between the two square (5 cm x 5 cm) metallic plates of a capacitor. A potential difference of 10 volts is maintained between the two metal plates which are separated by 1 mm. Suppose the dielectric slab is now pulled out at a speed of 1 mm/s parallel to one of the edges of the plates, with neghgible friction. The power required to do this is nearest to

(a) $9.00\times10^{-11}\,W$ (b) $1.11\times10^{-11}\,W$

(c) $0.22\times10^{-11}\,W$ (d) $4.90\times10^{-12}\,W$

(e) $2.16\times10^{-12}\,W$

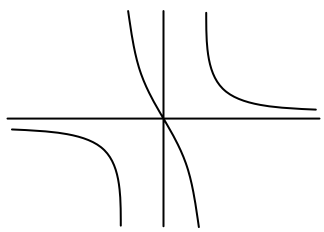
Q7. Consider a free quantum particle of mass m in a two-dimensional rectangular box covering the region $\{0 < x < 2L, 0 < y < L\}$. In the first excited state, the probability for the particle to be found in $\{L/2 < x < L,\ 0 < y < 3L/4\}$ is nearest to

(a) 0.13 (b) 0.19 (c) 0.16 (d) 0.23

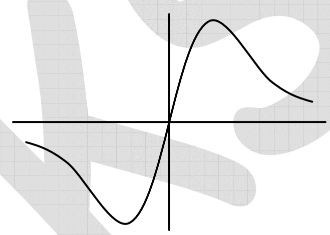
(e) 0.27

Q8. Which of the following figures provides the most accurate representation of the function $f(x) = \frac{x}{(x^2 - 1)}$?

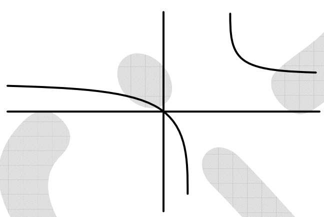
(a)



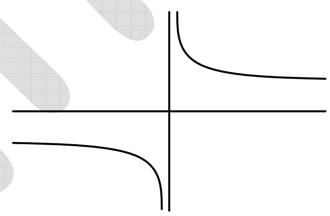
(b)



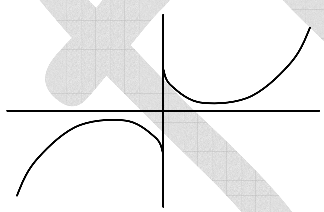
(c)



(d)



(e)



Q9. Cerenkov radiation is observed when the speed of a charged particle in a liquid is greater than the speed of light in that medium. In a liquid of refractive index 1.5, what is the minimum kinetic energy that an electron must have to give out Cerenk:ov radiation?

(a) 0.11 MeV (b) 0.17 MeV (c) 1.20 MeV (d) 0.51 MeV

(e) 0.61 MeV

Q10. Consider the following differential equation

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

it is given that $\frac{d^2y}{dx^2} = 1, \frac{dy}{dx} = 0$ and $y = 1$ at $x = 0$. Then the value of y at $x = 2$ is nearest to

(a) 30 (b) 42 (c) 21 (d) 63

(e) 12

Q11. Define I to be the value of the surface integral ($\int E.dS$, where dS points outwards from the domain of integration) of a vector field $E\left(E = \left(x + y^2\right)i + \left(z^3 + y^3\right)j + \left(x + z^4\right)k\right)$ over the entire surface of a cube which bounds the region $\left(0 < x < 2, -1 < y < 1, 0 < x < 2\right)$. The value of I is

(a) 0 (b) 16 (c) 72 (d) 80

(e) 32

Q12. Consider a gas of rubidium atoms (mass $= 85.5$ atomic mass unit) at a temperature of 1microkelvin. Let λ be the de Broglie wavelength of an atom moving with the average kinetic energy (per atom) of this gas. What is the value of n, the number of atoms per unit volume, that will satisfy the condition $n\lambda^3 = 1$?

(a) $4.4 \times 10^{18} / m^3$ (b) $6.7 \times 10^{20} / m^3$

(c) $1.1 \times 10^{19} / m^3$ (d) $3.6 \times 10^{21} / m^3$

(e) $4.9 \times 10^{19} / m^3$

Q13. Protons of kinetic energy $10^{12}\,eV$ are injected into a uniform magnetic field of strength 10 Tesla. The magnetic field exists only inside a cylindrical region of diameter $50\,cm$ and is parallel to the axis of the cylinder. At the point of injection, the proton beam is directed towards the axis of the cylinder and is perpendicular to it. By the time the beam exits the magnetic field, it changes its direction by

[Hints : (i) The protons in the beam are ultrarelativistic, i.e., $v = c$]

(ii) Use the relativistic expressions for energy and momentum

(iii) $F = \dfrac{dp}{dt}$]

(a) 2.2×10^{-3} radian (b) 2.4×10^{-4} radian

(c) 1.0×10^{-2} radian (d) 1.5×10^{-3} radian

(e) 9.0×10^{-4} radian

Q14. A straight beam of ultrarelativistic electrons, confined within a very long cylindrical region of radius less than $1\,mm$, carries a current of $0.01\,\mu A$. The strength of the electric field at a point $2\,cm$ away from the beam axis will be nearest to

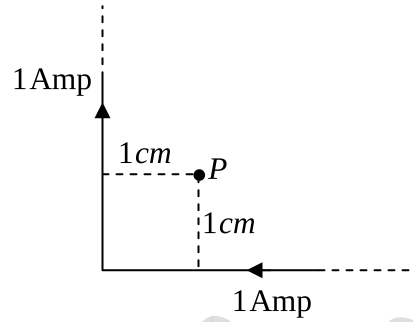
[Hint: Calculate the charge per unit length from the given information.]

(a) $6\times10^{-5}\,V/m$ (b) $1\times10^{-4}\,V/m$

(c) $3\times10^{-5}\,V/m$ (d) $2\times10^{-3}\,V/m$

(e) $8\times10^{-4}\,V/m$

Q15. Consider a L-shaped conductor carrying a current of 1 Ampere, as shown in the figure. Assume that both the arms extend to infinity. The strength of the magnetic field at the point P, which is at a distance of $1cm$ from both the arms, is nearest to

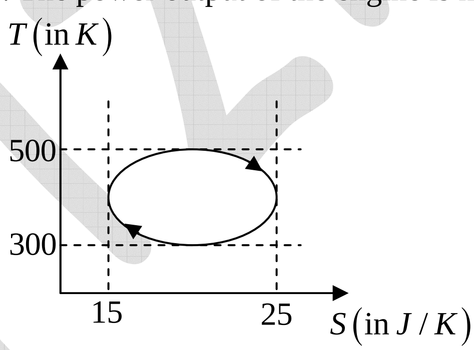


(a) $1.2\times10^{-7}\,T$ (b) $3.4\times10^{-5}\,T$

(c) $6.9\times10^{-6}\,T$ (d) $4.3\times10^{-4}\,T$

(e) $2.1\times10^{-5}\,T$

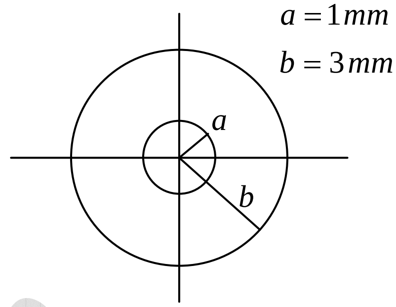
Q16. The thermodynamic cycle of a heat engine is shown in the figure below. The shape of the curve is elliptical with one axis parallel to the S-axis. This cycle is executed 20 times per second by the engine. The power output of the engine is nearest to



(a) $12.5\,kW$ (b) $32.0\,kW$ (c) $2.5\,kW$ (d) $10.0\,kW$

(e) $58.0\,kW$

Q17. A coaxial transmission line of length 1 m is made with a cylindrical inner metallic core of radius 1 mm and a thin cylindrical outer conductor of radius 3 mm. The cross-section of the transmission line (not to scale) is shown in the figure below. The capacitance of this transmission line is nearest to



(a) $50.55\times10^{-12}\,F$ (b) $75.16\times10^{-12}\,F$

(c) $46.12\times10^{-9}\,F$ (d) $58.20\times10^{-10}\,F$

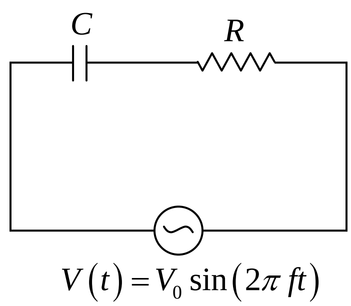
(e) $36.15\times10^{-11}\,F$

Q18. A particle of mass m is moving in a potential field given by $V(r)=(2x^2+y^3+2z^2)$. Out of the dynamical variables energy (E) and the three components of angular momentum (L_x, L_y, L_z), which of the following represents the complete set of conserved quantities?

(a) E and L_z (b) E, L_x and L_z (c) E, L_x, L_y and L_z

(d) E and L_x (e) E and L_y

Q19. In the following a.c. circuit $R = 10000\Omega, C = 0.32\,\mu F$, the frequency of the voltage $(f) = 50\,Hz$ and the root-mean-square value of $V(t) = 100\,V$. The average power absorbed by the resistor is nearest to



(a) $50W$ (b) $0.1\,W$ (c) $1W$ (d) $0.5W$

(e) $10W$

Q20. Three moles of an ideal monatomic gas occupy a volume of 20 m^3 at $300\,K$. If the gas expands adiabatically to $40\,m^3$ the final pressure is nearest to

(a) $331\,N/m^2$ (b) $1200\,N/m^2$ (c) $980\,N/m^2$ (d) $486\,N/m^2$

(e) $118\,N/m^2$

Q21. Ganymede is a moon of the planet Jupiter. It moves in a circular (orbit of radius 1.07 million kilometres with a time period of rotation of 7 days. The mass of Jupiter, deduced from this information, would be nearest to

(a) $5.3\times10^{29}\,kg$ (b) $1.2\times10^{31}\,kg$

(c) $4.8\times10^{28}\,kg$ (d) $1.9\times10^{27}\,kg$

(e) $3.2\times10^{30}\,kg$

Q22. An ice cube (of mass $5\,gm$), at a temperature of -23^0C, is dropped into a lake whose temperature is 27^0C. After equilibrium is established, what is the change: in the entropy of the universe nearest to? [Latent heat of fusion for the melting of ice $80\,cal/gm$, specific heat of water $=1\,cal/gm\,^0C$, specific heat of ice $=0.5\,cal/gm\,^0C$]

(a) $1.4\,J/K$ (b) $9.1\,J/K$ (c) $0.8\,J/K$ (d) $8.2\,J/K$

(e) $3.6\,J/K$

Q23. Which of the following force fields is/are conservative?

I. $\left(4x^3y+3x^2z^2\right)i+\left(x^4+3y^2z^2\right)j+2z\left(x^3+y^3\right)k$

II. $\left(yz^4+x^5\right)i+\left(x^4z+x^2y^3\right)j+\left(x^4y+z^5\right)k$

III. $x\left(3xy^2+4z^3\right)i+\left(z^4+2x^3y\right)j+2z^2\left(2yz+3x^2\right)k$

(a) II and III only (b) I, II and III

(c) I and III only (d) I and II only

(e) I only

Q24. In a long cylinder, a liquid of refractive index n is moving with a speed v. What will be the speed of light moving in this liquid, in the direction of motion of the liquid, when measured by an observer at rest in the laboratory (c is the speed of light in vacuum)?

(a) $(c/n)+v$
(b) $((c/n)+v)/(1+(v/cn))$
(c) c/n
(d) c
(e) $((c/n)+v)/(1-(v/cn))$

Q25. Two vertical radio frequency antennae (mounted on flat ground) are separated by $2m$, and are radiating in phase with $\lambda=1m$. Consider a circular path (at the ground level) of radius $50m$ around the two sources. If we move a detector once around this path, how many interference maxima will it detect?

(a) 2
(b) 24
(c) 16
(d) 4
(e) 8