M.Sc. Entrance 2014

Q1. Consider a solid cylinder of uniform mass density and a very thin cylindrical shell of uniform mass per unit area. They have the same radius, length and total mass. If they are both rolling on a plane without slipping with the same angular velocity, the ratio of the total kinetic energy of the solid cylinder to that of the cylindrical shell is

(a) $3:4$ (b) $1:2$ (c) $1:1$ (d) $2:1$ (e) $2:3$

Q2. The total mass of a spherical star of radius R and uniform density is M. If it explodes into a gas cloud of much lower density, the gravitational potential energy will

(a) decrease by $\frac{GM^2}{5R}$ (b) increase by $\frac{2GM^2}{5R}$ (c) increase by $\frac{3GM^2}{5R}$

(d) increase by $\frac{4GM^2}{5R}$ (e) decrease by $\frac{GM^2}{R}$

Q3. A large number of balls are thrown simultaneously from the same point at the top of a tower with the same initial speed, but in different directions. If air friction is neglected, the coordinates of the balls at any point of time after the throw (before any of the balls hits the ground) lie on a

(a) paraboloid of revolution

(b) surface obtained by rotating a segment of a parabola

(c) sphere, of which the radius increases and the centre moves downward with time

(d) sphere, of which the radius decreases and the centre moves downward with time

(e) sphere, of which the radius increases with time but the centre is stationary

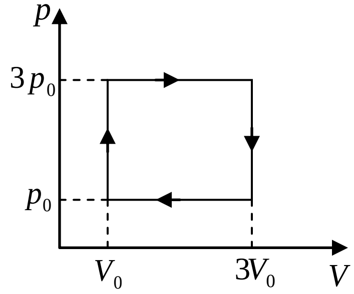
Q4. An ^{57}Fe nucleus decays from an excited state to the ground state by emitting a gamma ray. The energy of the emitted photon is $14.4\,keV$ when the nucleus is held fixed. If the nucleus is free to recoil, then the energy of the photon emitted will be

(a) lower by approximately $2\,keV$ (b) lower by approximately $2\,meV$

(c) higher by approximately $2\,eV$ (d) higher by approximately $20\,eV$

(e) lower by approximately $20\,meV$

Q5. The $p-V$ diagram of the working cycle of an engine is shown in the figure below:



If it is operated at 1500 revolutions per minute with the values of $p_0 = 1.5\times 10^5\, N/m^2$ and $V_0 = 1$ litre, then the power of the engine will be approximately

(a) $15\,kW$ (b) $8\,kW$ (c) $23\,kW$ (d) $38\,kW$ (e) $3.6\,kW$

Q6. The position vector of a particle moving in two dimensions, as a function of time t, is given as $\hat{r}(t) = \left(\hat{i}\,t + \hat{j}\sqrt{t}\right)e^{-t}$ for $t \geq 0$. Starting at $t=0$, the distance of the particle from the origin reaches its maximum value at $t = t_m$, and at this instant, the velocity vector $\vec{v}$ makes an angle α with $\vec{r}$. The values of t_m and α are

(a) $t_m = \frac{1}{2}$ and $\alpha = \frac{\pi}{2}$ (b) $t_m = \frac{1}{\sqrt{2}}$ and $\alpha = \pi$ (c) $t_m = \frac{1}{2}$ and $\alpha = 0$

(d) $t_m = \frac{1}{\sqrt{2}}$ and $\alpha = \frac{\pi}{2}$ (e) $t_m = 1$ and $\alpha = \pi$

Q7. The principal axes of the ellipse given by the equation $9x^2 + 6y^2 - 4xy = 4$ are along the directions of the vectors

(a) $2\hat{i}+3\hat{j}$ and $3\hat{i}-2\hat{j}$ (b) $2\hat{i}+\hat{j}$ and $\hat{i}-2\hat{j}$ (c) $3\hat{i}+2\hat{j}$ and $2\hat{i}-3\hat{j}$

(d) $\hat{i}+2\hat{j}$ and $2\hat{i}-\hat{j}$ (e) $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$

Q8. A straight horizontal pipe carrying water has a cross-section of $0.4\,m^2$ in section A and $0.2\,m^2$ in section B. The speed of water in section A is $5.0\,m/s$ and the pressure in section B is $2.6\times 10^5\, N/m^2$ The value of water pressure in section A is nearest to (ignore the effect of viscosity)

(a) $1.0\times 10^5\, N/m^2$ (b) $3.0\times 10^5\, N/m^2$ (c) $1.25\times 10^5\, N/m^2$

(d) $5.0\times 10^5\, N/m^2$ (e) $2.5\times 10^5\, N/m^2$

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Q9. Consider a vector field $\vec{F} = xy^4\hat{i} + yz^2\hat{j}$. Its outward flux $\oiint \vec{F}.d\vec{S}$ over the surface of a cube bounded by the planes $|x| = 2, |y| = 2$ and $|z| = 2$ is nearest to

(a) 410 (b) -273 (c) 290

(d) -300 (e) 0

Q10. The exponential of a 2×2 matrix A is defined by the power series expansion $\exp(A) = \sum_{n=0}^{\infty} \frac{1}{n!} A^n$, with A^0 being the 2×2 identity matrix. If $S = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$ and $SAS^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, then the determinant of the matrix e^A is

(a) 1 (b) e (c) $\frac{1}{e}$

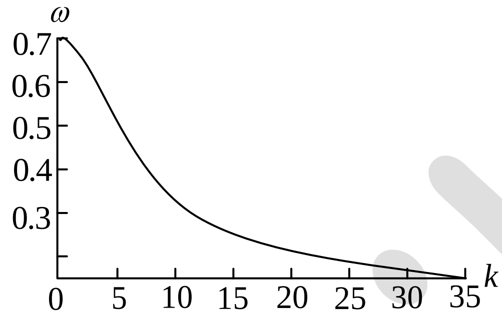
(d) $e + \frac{1}{e}$ (e) -1

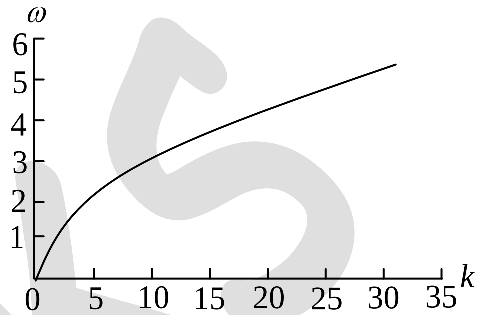
Q11. The distance between the successive intensity maxima seen on the screen in a Young's double-slit experiment is found to be s. Suppose now the experiment is repeated with the spacing between the two slits being doubled and the whole apparatus being immersed in water (earlier it was in air). The fringe spacing will now be (take the refractive index of water to be $\frac{4}{3}$)

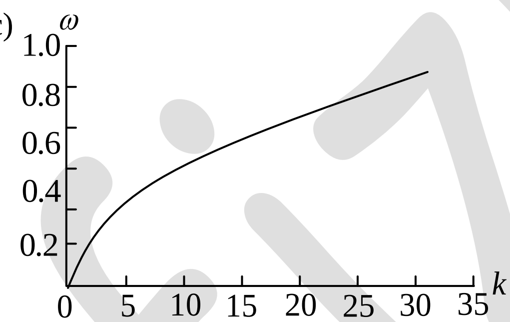
(a) $\frac{s}{2}$ (b) s (c) $\frac{3s}{2}$

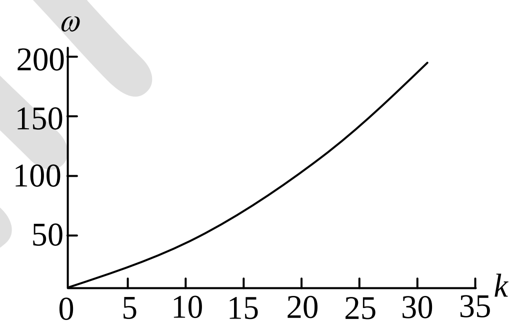
(d) $\frac{3s}{4}$ (e) $\frac{3s}{8}$

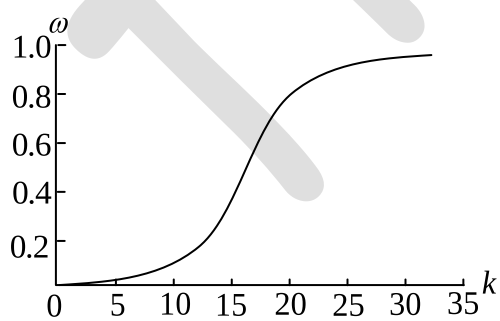
Q12. The phase velocity v of surface waves in a pool of water of depth h is given by the formula $v^2 = \frac{g}{k}\tanh(kh)$, where k is the wave vector and g is the acceleration due to gravity. Which of the following graphs gives the best qualitative approximation to the plot of the dispersion relation (angular frequency ω vs k) for these waves for a value of h around 1?

(a)


(b)


(c)


(d)


(e)


Q13. A spectral line of hydrogen is seen at $6500\mathring{A}$ in the light emitted from a far away galaxy. This line has a wavelength of $6250\overset{o}{A}$ when measured in a laboratory on Earth. This observation is consistent with the galaxy moving

(a) towards earth at a speed of $1200\ km/s$

(b) towards earth at a speed of $120 km/s$

(c) away from earth at a speed of $300 km/s$

(d) towards earth at a speed of $6000 km/s$

(e) away from earth at a speed of $12000 km/s$

Q14. A point charge $+q$ is placed outside a grounded conducting sphere of radius a. Which of the following statements is not true?

(a) There is an attractive force between the sphere and the point charge

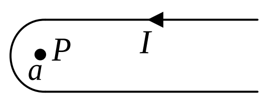
(b) The induced surface charge density on the sphere is not the same everywhere

(c) The electric field inside the sphere is zero

(d) The total induced charge on the sphere is $-q$

(e) The electrostatic potential at a large distance d (compared to the distance between the charge and the sphere) falls off as $\frac{1}{d}$

Q15. An infinite conducting wire, which is bent in the shape of a hairpin (the curved part being a semicircle of radius a) as shown in the figure, carries a current I.



The value of the magnetic field at the point P (the centre of the semicircular segment) when $I = 1\ A$ and $a = 1\ cm$, is nearest to $\left(1 \text{ weber/m}^2 = 1 \text{ tesla } = 10^4 \text{gauss}\right)$

(a) 1.3 gauss (b) 7.1 gauss (c) 31.4 gauss

(d) 3.14 gauss (e) 0.5 gauss

Q16. An AM radio receiver uses an $L-C-R$ circuit to tune to different stations by varying only the capacitance C. In order to ensure that broadcasts from various stations do not interfere, each radio station is allotted an exclusive range from $\omega_0 - 10kHz$ to $\omega_0 + 10\ kHz$, where ω_0 is the carrier frequency of that station. If the value of the inductance L in a receiver is $1.5\,mH$ (millihenry), which of the following best approximates the value of the resistance?

(a) 0.2 ohm (b) 2 ohm (c) 20 ohm

(d) 200 ohm (e) 2000 ohm

Q17. A thin and straight metallic rod of length L is rotating about its middle point with angular velocity ω in a uniform magnetic field of strength B. The axis of rotation is perpendicular to the length of the rod and parallel to the direction of the magnetic field. The strength of the induced electric field is

(a) constant throughout the rod

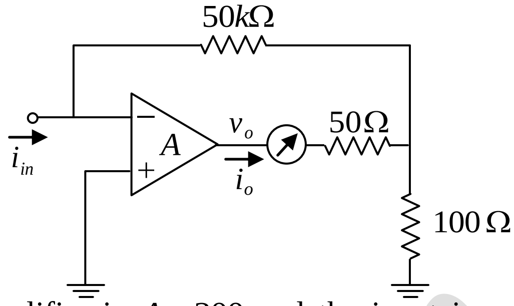
(b) maximum at the two points halfway between the centre and the two ends

(c) minimum at the two points halfway between the centre and the two ends

(d) minimum at the ends and maximum at the centre

(e) maximum at the ends and minimum at the centre

Q18. A current feedback to an amplifier is applied in the following circuit



The voltage gain of the amplifier is $A = 200$ and the input impedance is $10^6\,\Omega$. The output current i_0 and the output voltage v_0 corresponding to an input current $i_{in} = 10\,\mu A$ are

(a) $i_0 = -5\,mA$ and $v_0 = -0.10V$

(b) $i_0 = -2\,mA$ and $v_0 = -0.55V$

(c) $i_0 = -5\,mA$ and $v_0 = -0.75V$

(d) $i_0 = -2\,mA$ and $v_0 = -0.75V$

(e) $i_0 = -5\,mA$ and $v_0 = -0.5V$

Q19. On a winter day, when the temperature outside is $17^o C$ the temperature inside a room is maintained at $27^o C$ by using a heat pump. The heat pump compensates for the loss of heat due to the difference of temperature. If, for the temperatures given, the rate of heat loss is $6000 cal/s$, what is the minimum rate of consumption of energy by the pump?

(a) 840 watt

(b) 9300 watt

(c) 15000 watt

(d) 200 watt

(e) 430 watt

Q20. Suppose you measured v_x (the x-component of the velocity) of every atom in a one-litre can of helium gas $\left({}^4He\right)$ kept in equilibrium at $300K$ and at atmospheric pressure, at a particular instant of time. The standard deviation $\sqrt{\left\langle v_x^2 - \langle v_x \rangle^2 \right\rangle}$ in the measurement is approximately equal to

(a) $450\, m/s$

(b) $1120\, m/s$

(c) $1400\, m/s$

(d) $800\, m/s$

(e) $14000\, m/s$

Q21. A proton moves in a circular orbit in a plane which is perpendicular to a uniform magnetic field of strength B. When the kinetic energy of the proton changes from $0.25m_pc^2$ to m_pc^2 (where m_p is the rest mass of the proton) the frequency of rotation changes from ω_1 to ω_2. The ratio $\frac{\omega_1}{\omega_2}$ is

(a) 1 (b) 8 (c) $\frac{8}{5}$

(d) $\frac{1}{\sqrt{8}}$ (e) $\sqrt{8}$

Q22. The typical de Broglie wavelength of an oxygen molecule at room temperature $(300\ K)$ is around (the mass of an oxygen molecule is approximately $5.3\times10^{-26}\,kg$)

(a) $2.6\times10^{-11}\,m$ (b) $6.6\times10^{-13}\,m$ (c) $2.4\times10^{-10}\,m$

(d) $8.9\times10^{-9}\,m$ (e) $1.2\times10^{-12}\,m$

Q23. The first ionization potential of helium atom is $24.6\,eV$. What is the minimum energy required to remove both the electrons of a neutral helium atom?

(a) $108.8\,eV$ (b) $79.0\,eV$ (c) $49.2\,eV$

(d) $51.8\,eV$ (e) $37.9\,eV$

Q24. Consider a particle of mass m which is occuping the lowest energy eigenstate of a one-dimensional box of length L. The force exerted by the particle on the boundary walls is

(a) proportional to $\frac{1}{L^2}$ (b) proportional to $\frac{1}{L^3}$ (c) proportional to $\frac{1}{L}$

(d) independent to L (e) proportional to L^2

Q25. If h is the Planck constant, c the speed of light in vacuum and G the universal gravitational constant, which of the following quantities has the dimension of length?

(a) $\frac{hG}{c^2}$ (b) $\frac{hG}{c^4}$ (c) $\sqrt{\frac{hc}{G}}$

(d) $\sqrt{\frac{hG}{c^5}}$ (e) $\sqrt{\frac{hG}{c^3}}$