JNU-ENTRANCE EXAMINATION- 2007

M.Sc. PHYSICS

Maximum Marks: 100

PART-A

NOTE: Answer **all** questions. Each question carries 10 marks.

Q.1 Consider a particle of mass m moving along the x-axis in a potential of the form

$$V(x) = x^3 - 3x + 3.$$

(a) Draw a schematic graph of this potential.

(b) Are there values of x at which the particle can be stationary in equilibrium? Indicate the nature of the equilibrium (stable or unstable) in each case. If there is a point of stable equilibrium, calculate the frequency of oscillation around it (for small enough amplitude).

Q.2 A particle of mass m moves in a one-dimensional box located between $x = 0$ and $x = L$. The wave function is given by $\psi(x) = Cx(L-x)$, where C is the constant of normalization. Calculate the expectation value of the kinetic energy of the particle.

Q.3 Let $f(= F/V)$ be the Helmholtz free energy per unit volume for a gas of N particles.

(a) Show that the equilibrium pressure P is obtained as

$$P = n\left(\frac{\partial f}{\partial n}\right)_T - f$$

where $n(= N/V)$ is the number of particles per unit volume, and T is the temperature of the gas.

(b) For an ideal gas, show that the free energy per particle $\tilde{f}(= F/N)$ is obtained as $\tilde{f} = k_B T \ln(n) + C_T$, where C_T is a temperature dependent constant.

Q.4 Consider a random walk of a person along a straight line. Each step is of length l and is equally likely to be in the forward or in the backward direction. After n such steps are taken

(a) Prove that the probability of the net displacement from the starting point being equal to rl in the forward direction is given by

$$P(r) = \frac{n!}{[(n-r)/2]![(n+r)/2]!}\left(\frac{1}{2^n}\right)$$

(b) Show that the average value of r is equal to 0;

(c) Show that the average value of r^2 is equal to n.

Q.5. Using Bohr's theory of an electron in a hydrogen atom and the classical theory of electromagnetism, calculate the strength of the magnetic field at the location of the nucleus due to electronic motion. Assume that the electron is in the ground state.

PART– B

NOTE: Answer **all** questions. Each question carries 5 marks.

Q.1 An L-C oscillator consists of a sinusoidal current with amplitude I_0 is flowing through the circuit. Calculate the frequency of the current and the average energy in the capacitor.

Q.2 Solve the ordinary differential equation

$$x\frac{du}{dx} - (x+1)u = 0$$

given that $u(x=1) = 1$.

Q.3 Estimate the maximum distance at which the eye is able to resolve the two headlights of an approaching car, given the following information: (a) Distance between the headlights is 1.4 m, (b) the pupil diameter of the observer's eye is 5 mm and (c) the wavelength of the light is 550 nm. Assume that the diffraction effects alone limit the resolution.

Q.4 A muon is an elementary particle of rest mass energy equal to 105 *MeV*. It decays with a mean lifetime of 2.2 x 10^{-6} second in its rest frame. Consider muons which are being produced with a kinetic energy of 70 *MeV*. What is the average distance that such a muon travels before it decays?

Q.5 An electric charge Q is placed at the point ($x = y = 0$, $z = d$). Calculate the electric field flux through the circle $x^2 + y^2 = a^2$ in the $z = 0$ plane.

Q.6 Find the average velocity of the free electrons producing a current of 5 amperes in a copper wire having a cross-sectional of area of 4.0 square mm. Assume that each copper atom contributes one mobile electron. Density of copper is 9 x 10^3 kg/m^3. Atomic weight of copper is 63.5 a.m.u.

Q.7 Suppose you are computing the derivative of a function f at the point x by evaluating the function numerically at two different points separated by h. For a given value of h which of the following two expressions constitutes a more accurate representation of the derivative?

(a) $[f(x + h/2) – f(x – h/2)] /h$

(b) $[f(x + h)+ f(x)] /h$

Explain your answer.

Q.8 Suppose you are putting identical hard spheres of diameter σ on a face centered cubic lattice. In the closest packed configuration of the spheres, calculate the lattice constant of the cubic unit cell in terms of σ.

Q.9 A reversible cyclic engine operates between two identical bodies of constant (temperature independent) heat capacity. Initially these two bodies are at temperatures T_1 and T_2. The engine comes to rest when these two bodies attain a common temperature T_F. Express T_F in terms of T_1 and T_2.

Q.10 For a particle of mass m moving in a spherically symmetric potential $V(r)$, it is found that the value of the kinetic energy is twice the value of the potential energy for all circular orbits. Derive the form of $V(r)$.