JNU-ENTRANCE EXAMINATION- 2008

M.Sc. PHYSICS

Maximum Marks: 100

PART-A

NOTE: Answer **all** questions. Each question carries 10 marks.

Q.1. Show that

$$\sum_{n=1}^{\infty}\frac{\sin(nx)}{n}=\frac{(\pi-x)}{2},\qquad 0<x\le\pi$$
$$=-\frac{(\pi+x)}{2},\ -\pi\le x<0$$

Q.2. A harmonic oscillator consists of a mass m attached to a spring with spring constant k. It is immersed in a fluid which damps the motion with friction coefficient b.

(a) Write down the differential equation which describes the time-dependence of the displacement $x(t)$ of the mass from its equilibrium position.

(b) Solve this equation, and identify parameter ranges for which the motion is damped oscillatory or only damped.

Q3. The free energy of a cluster with N molecules has contributions from both volume and surface terms as follows:

$$\Delta G=\Delta_0 V+\gamma S$$

Here Δ_0 is the free-energy cost (per unit volume) of bringing the particles together, γ is the surface tension, and V, S denote the volume and surface area of the cluster, respectively. Assume that the cluster is spherical in shape and each molecule has a volume v_m so that $V=v_m N$.

(a) Express ΔG in terms of N.

(b) Assuming $\Delta_0 < 0$, obtain the value of N for which ΔG is maximum. Sketch ΔG as a function of N.

(c) Compute the radius of the cluster in *(b)*.

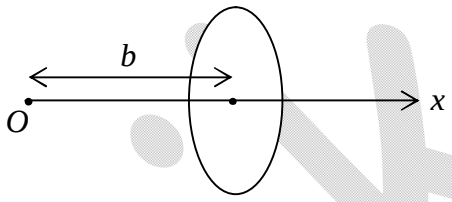
Q.4. A magnetic dipole of moment μ is placed along the x-axis at the origin.

(a) Write down the magnetostatic potential, and the value of the magnetic field at an arbitrary point $\vec{r}$. (No derivation is required.)

(b) The dipole executes oscillations along the x-axis with small amplitude A and angular frequency ω, so that the position in time is

$$x(t) = A\cos(\omega t)$$

A coil of small area S is placed perpendicular to the x-axis at a distance b as shown. Calculate the e.m.f. induced in the coil. (Neglect the variation of the field across the coil and take $A << b$)



Q.5. "A beam of thermal neutrons falling on a crystal gives rise to diffraction phenomena analogous to those observed with X-rays." Justify this statement by estimating de Broglie wavelength of a neutron with a speed corresponding to the average thermal energy at 300 K.

PART- B

NOTE: Answer **all** questions. Each question carries 5 marks.

Q.1. Compute the eigenvalues and normalized eigenvectors of the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Q.2. A factory has workers A, B, C and D who produce 1%, 3%, 5% and 7% defective items, respectively. Furthermore, A, B, C and D work for 40%, 30%, 20% and 10% of the time, respectively. If a defective item is produced, what is the probability that it was produced by A?

Q.3. A ball hits the ground at an angle of 45^0 with a speed of 1 m/s. After every bounce, the vertical component of velocity is reduced in magnitude by 10 percent. What horizontal distance does the ball travel between the first bounce and the tenth bounce?

Q.4. Write down the expression for Maxwell's velocity distribution law and speed distribution law. Find the most probable value of the particle speed. Obtain its ratio with the root-mean-square speed of the particles.

Q.5 Prove the thermodynamic relation

$$\left.\frac{\partial \mu}{\partial N}\right|_{V,T} = -\frac{V^2}{N^2}\left.\frac{\partial P}{\partial V}\right|_{N,T}$$

where the symbols have their usual meanings.

Q.6 A particle of charge q is placed near a large planar metallic sheet. The perpendicular distance of the charge from the sheet is d. Calculate the force of attraction exerted by the sheet on the charge.

Q.7 A proton is moving with a high speed so that it's mass exceeds the neutron rest mass. Under these conditions, can it undergo the following decay?

$$p \rightarrow n + e^+ + \nu_e$$

Q.8 A sample containing hydrogen atoms is subjected to an external oscillating magnetic field of frequency 34 *MHz*. When another constant magnetic field of magnitude 0.78 T is applied, proton spin-flipping is observed. Calculate the strength of the local magnetic field (due to the magnetic moments of the atoms and nearby nuclei) at the site of the protons that are undergoing spin flips, assuming the external and local fields are parallel there. The protons have a dipole moment $\mu = 1.41 \times 10^{-26}\ J/T$.

Q.9 A particle of mass m moves in a one-dimensional potential $V(x) = |x|$. Using the Bohr-Sommerfield quantization condition, or otherwise, find the n-dependence of the n^{th} energy level E_n.

Q.10 The compound *NaCl* has a cubic lattice with density 2.167 g/cm^3. What is the distance between adjacent atoms? *Na* has a molar mass of 23 *g/mol*, *Cl* has a molar mass of 35.4 *g/mol*.