JNU-ENTRANCE EXAMINATION- 2010

M.Sc. PHYSICS

Maximum Marks: 90

INSTRUCTIONS FOR CANDIDATES

1. All questions are compulsory.
2. For each question, one and only one of the four choices given is the correct answer.
3. Each correct answer will be given +3 marks.
4. Each wrong answer will be given -1 mark.
5. Use of calculator is permitted.

Q.1 The eigenvalues of the matrix $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ are

(a) +1, +1, +1 and +1

(b) +1, +1, -1 and -1

(c) 0, 0, $\sqrt{2}$, and $-\sqrt{2}$

(d) $+\sqrt{2}, -\sqrt{2}, +1/\sqrt{2}$ and $-1/\sqrt{2}$

Q.2 Three dices with faces marked 1, 2, ….6 are thrown together. Assuming that they are unbiased, what is the probability that the sum of the numbers that turn up is 15?

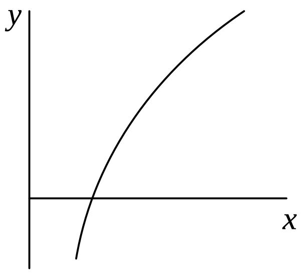
(a) 1/108 (b) 3/108 (c) 5/108 (d) 7/108

Q.3 What is the value of the definite integral $\int_0^1 (x \ln x)^2 dx$?

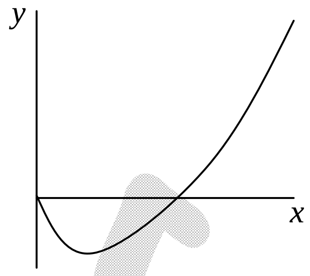
(a) 1/27 (b) 2/27 (c) 1/9 (d) 2/9

Q.4 Which of the following graphs gives the best representation of the real-valued function $y = x \ln x$ in the domain $x>0$?

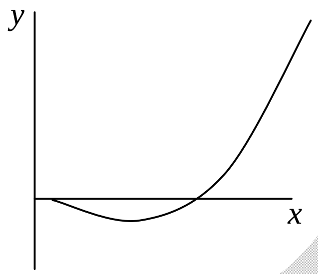
(a)



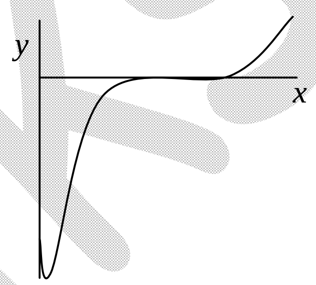
(b)



(c)



(d)



Q.5 A particle of mass m moves in the xy-plane in the presence of a potential $V(x, y)$ so that its Lagrangian is given by

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}\left(x^2 + y^2\right) + 2\ln|x-y|$$

Which of the following statements corresponds to the equilibrium of the system?

(a) There is no stable equilibrium at any finite values of (x, y)

(b) There is only one stable equilibrium at the point $(x, y)=(0, 0)$

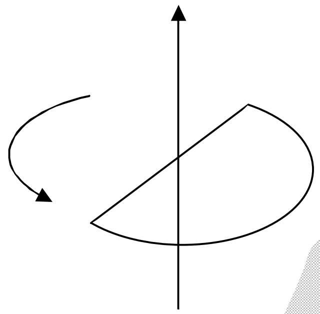
(c) There are two stable equilibria at the points $(x, y) = (1, 1)$ and $(-1, -1)$

(d) There are two stable equilibria at the points $(x, y) = (1, -1)$ and $(-1, 1)$

Q. 6 A uniform spring of spring constant k is cut into two pieces such that one piece is three times as long the other. The spring constant of the short piece is

(a) $k/4$ (b) k (c) $4k/3$ (d) $4k$

Q.7 A half disc of mass M and radius R rotates about an axis that passes through the centre of the straight side and is perpendicular to its plane (see figure). The moment of inertia about the axis of rotation is



(a) $MR^2/8$ (b) $MR^2/4$ (c) $MR^2/2$ (d) $2MR^2$

Q.8 Consider a pipe of length 65 cm with both ends open. How many modes of oscillations of the air column are possible up to a frequency of 1250 Hz? (Take the speed of sound to be 330 m/s)

(a) 2 (b) 3 (c) 5 (d) 6

Q.9. A plane wave of frequency ω propagates so that planes of constant phase move with speeds v_1, v_2 and v_3 along the x, y and z axes, respectively. The wave vector, $\vec{k}$ of the plane wave (in terms of the Cartesian unit vector $\hat{i},\ \hat{j}\ and\ \hat{k}$) is

(a) $\vec{k} = \omega\left(\frac{1}{v_1}\hat{i} + \frac{1}{v_2}\hat{j} + \frac{1}{v_3}\hat{k}\right)$ (b) $\vec{k} = \omega\left(\hat{i} + \hat{j} + \hat{k}\right)/\sqrt{v_1^2 + v_2^2 + v_3^2}$

(c) $\vec{k} = \omega\left(v_1\hat{i} + v_2\hat{j} + v_3\hat{k}\right)/\left(v_1^2 + v_2^2 + v_3^2\right)$ (d) $\vec{k} = \left(v_1\hat{i} + v_2\hat{j} + v_3\hat{k}\right)/\omega$

Q.10. The concentration $\rho(r, t)$ of ink diffusing in water is governed by the diffusion equation

$$\frac{\partial}{\partial t}\rho(r,t) = D\nabla^2\rho(r,t)$$

where D is a parameter known as the diffusion constant. What is the average time taken for a molecule of ink to spread by a root-mean-square distance R?

(a) $\sqrt{R/D}$ (b) $R/\sqrt{D}$ (c) R^2/D (d) RD

Q.11. The ratio F_C/F_G of the electrostatic Coulomb force F_C to the gravitational force F_G between the proton and the electron in the first Bohr orbit (of radius r_B) of a hydrogen atom is closest to the following value

(a) 2 x 10^{39} (b) 2 x 10^{49}

(c) 2 x 10^{42} r_B (d) – 2 x $10^{39}/r_B$

Q.12. Two point charges q and $4q$ are located at r_1 and r_2, respectively, on the xy-plane. The magnitude Q and location R of a third charge to be placed on the plane such that the total force on each of the three charges vanishes are

(a) $Q = -4q/9$ and $R = (2r_1 + r_2)/3$

(b) $Q = -4q/5$ and $R = (r_1 + 2r_2)/3$

(c) $Q = -4q/25$ and $R = (16r_1 + r_2)/\sqrt{17}$

(d) $Q = -9q/4$ and $R = (2r_1 + r_2)/3$

Q.13. Consider a spherical cavity in an infinite, homogenous and isotopic dielectric material of permittivity ε. When placed in an external electric field E, the electric field inside the cavity is

(a) $3\varepsilon E/(2\varepsilon + \varepsilon_0)$ (b) $(2\varepsilon + \varepsilon_0)E/3\varepsilon$

(c) $-\varepsilon E$ (d) E/ε

Q.14. A circular coil of radius 20 mm with 10 turns is placed with its plane parallel to the earth's magnetic field. When a current of 0.45 A flows through the coil, a compass needle placed at the centre of the coil is seen to be deflected by an angle of 45^o to the plane of the coil. The intensity of the earth's magnetic field is calculated to be

(a) 0.3 μT (b) 24 μT (c) 42 μT (d) 100 μT

Q.15. Electric current of uniform current density is flowing through an infinitely long wire with a circular cross-section of constant radius R. Which of the following relations gives the magnitude B of the magnetic field measured inside the wire at a distance r from the central axis of the wire (so that $r < R$)?

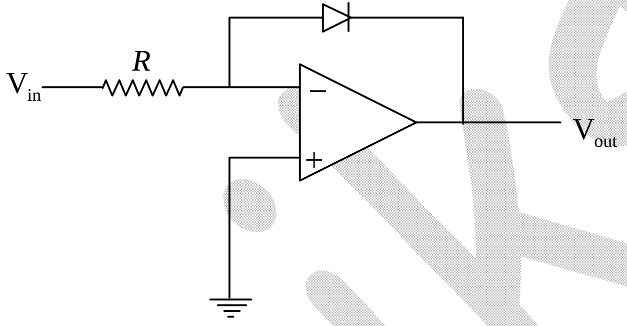
(a) $B \propto r$ (b) $B \propto \ln r$ (c) $B \propto 1/r$ (d) $B \propto 1r^2$

Q.16. An amplifier has a voltage gain $A_v = 1000$, input impendence 1 $k\Omega$ and output impedance 500Ω. A fraction $\beta_v = 0.1$ of the output voltage is fed back in series in opposition to the input voltage. The input and output impedance after the feedback are given respectively by the approximate values

(a) 100 $k\Omega$ and 5 Ω (b) 1 $k\Omega$ and 5 Ω

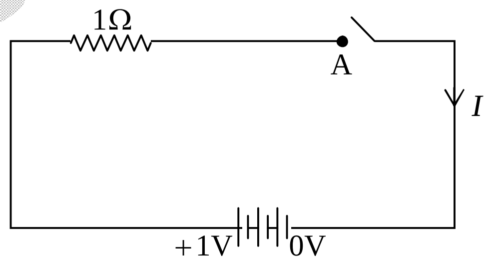
(c) 10 Ω and 50 $k\Omega$ (d) 100 $k\Omega$ and 100 Ω

Q.17. In the circuit shown below, the output voltage is proportional to



(a) $\ln V_{in}$ (b) $\exp(V_{in})$ (c) $|V_{in}|$ (d) $-|V_{in}|$

Q.18. Which of the following statements is true, regarding the voltage V measured at point A and the steady-state value of current I in the circuit shown below?



(a) $I = 0$ A and $V = 0$ V

(b) $I = 0$ A and $V = 1$ V

(c) $I = 1$ A and $V = 1$ V

(d) $I = 0$ A and V fluctuates randomly until the switch is closed

Q.19. An ideal gas undergoes an isothermal expansion (at a constant temperature T) from an initial volume V_1 to a final volume V_2. The change in the entropy per mole is

(a) $R(V_1/V_2)$ (b) $R\ln|V_1 - V_2|$ (c) $R\ln(V_1/V_2)$ (d) $R\ln(V_2/V_1)$

Q.20. A gas at a pressure P_A and volume V_A is compressed adiabatically to a volume V_B at pressure P_B. If the pressure-volume relation for this gas during adiabatic compression is $PV^{5/3} = K$ (where K is a constant), the work done during the compression is

(a) $K\left(\frac{1}{V_B^{4/3}} - \frac{1}{V_A^{4/3}}\right)$ (b) $\frac{5}{2}K\left(\frac{1}{V_B^{2/3}} - \frac{1}{V_A^{2/3}}\right)$

(c) $\frac{3}{2}(P_BV_B - P_AV_A)$ (d) $\frac{5}{3}(P_BV_B - P_AV_A)$

Q.21. A gas, the molecules of which have mass m, is at equilibrium at absolute temperature T. The root-mean-square of the relative velocity between any two molecules of the gas is

(a) $4k_BT/\pi m$ (b) $3k_BT/m$

(c) $3k_BT/m$ (d) $6k_BT/m$

Q.22. The energy loss due to diffraction for a plane wave reflected back and forth between two plane mirrors

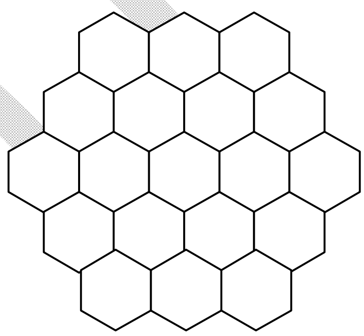
(a) decreases with an increase in the size of the mirrors

(b) increases with an increases in the size of the mirrors

(c) decreases with an increase in the separation between the mirrors

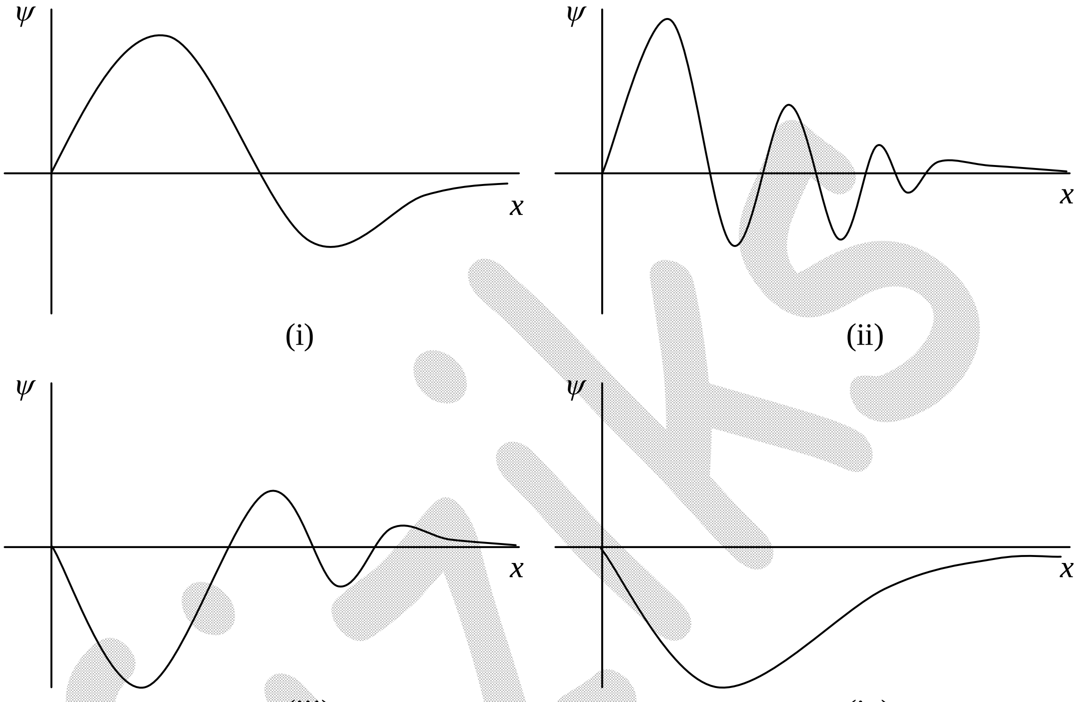
(d) increases with a decrease in the wavelength of the light

Q.23. In a single layer of graphite, called grapheme, the carbon atoms form a hexagonal lattice (see figure). How many carbon atoms are there in a unit cell of the lattice?



(a) One (b) Two (c) Three (d) Six

Q.24. A quantum particle in one dimension x moves under the influence of a potential that supports bound states. Plotted below are the wave functions ψ vs. x corresponding to four eigenstates of energy. Identify the wave functions corresponding to the ground state and the highest energy state among those shown:



(i) (ii) (iii) (iv)

(a) (i) corresponding to the ground state and (iii) to the highest energy state

(b) (ii) corresponds to the ground state and (iv) to the highest energy state

(c) (iv) corresponds to the ground state and (ii) to the highest energy state

(d) None of these is the ground state and (ii) corresponds to the highest energy state

Q.25. An X-ray machine operates at a potential of 50000 V. What is the minimum possible value of wavelength present in the radiation?

(a) 0.0124 nm (b) 0.0248 nm (c) 0.124 nm (d) 0.248 nm

Q.26 The eigenstates of $\hat{S}_z$ of a quantum particle with spin ½ are $|\uparrow\rangle = \left|s = \frac{1}{2}, s_z = \frac{1}{2}\right\rangle$ and $|\downarrow\rangle = \left|s = \frac{1}{2}, s_z = \frac{1}{2}\right\rangle$. Which of the following statements about eigenstates of the operator $\hat{S}_x$ is true?

(a) $|\uparrow\rangle$ is an eigenstate of $\hat{S}_x$

(b) $|\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(c) $|\uparrow\rangle + |\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(d) No linear combination of $|\uparrow\rangle$ and $|\downarrow\rangle$ can be an eigenstate of $\hat{S}_x$

Q.27. Consider a quantum particle of mass m in a three-dimensional isotropic simple harmonic potential $V(x, y, z) = \frac{1}{2} m\omega^2 \left(x^2 - y^2 + z^2\right)$. It is known that the particle is in an energy eigenstate with eigenvalue $E = 7\hbar\omega/2$. which of the following *cannot* be the wave function of the particle? (In the following $\alpha = \sqrt{m\omega/\hbar}$ and $H_n(\xi)$ is the nth Hermite polynomial)

(a) $H_2(\alpha x)\exp\left(-\alpha\left(y^2 + z^2\right)\right)$

(b) $H_2(\alpha x)\exp\left(-\alpha\left(x^2 + y^2 + z^2\right)\right)$

(c) $H_1(\alpha y)H_1(\alpha z)\exp\left(-\alpha\left(x^2 + y^2 + z^2\right)\right)$

(d) $H_1(\alpha x)H_1(\alpha z)\exp\left(-\alpha\left(x^2 + y^2 + z^2\right)\right)$

Q.28. In order determine the age of ancient wooden tools, radiocarbon dating method is used. This is done by measuring the fraction of radioactive isotope ^{14}C of carbon compared to the normal (non-radioactive) isotope ^{12}C in a sample. An old sample is found to contain 1/10 times the fraction of ^{14}C as compared to a fresh piece of wood. Given that the half-life of ^{14}C is 5570 years, the approximate age of the old sample is

(a) 557 years (b) 12800 years (c) 18500 years (d) 55700 years

Q.29 In the Large Hadron Collider (LHC) in CERN, protons will be accelerated to an energy of 7 *TeV* (tetraectron volt), i.e., 7 x 10^{12} *eV*. (As a matter of fact, two proton beams, each with 7 *TeV* will collide so that the energy of collision in the centre of mass frame is 14 *TeV*.) The speed *v* of a proton of 7 *TeV* energy is in the range

(a) $0.999999000\, c < v < 0.9999999999\, c$

(b) $0.999900\, c < v < 0.999999\, c$

(c) $0.9900\, c < v < 0.9999\, c$

(d) $0.90\, c < v < 0.99\, c$

Q.30. In the special theory of relativity, consider a Lorentz boost by a velocity *v* along the *x*-direction. If $u = ct + x$, then the boosted value $u' = ct' + x'$ is

(a) $u' = \sqrt{\dfrac{1+v/c}{1-v/c}}\,u$ (b) $u' = \sqrt{\dfrac{1-v/c}{1+v/c}}\,u$

(c) $u' = \dfrac{1+v/c}{1-v/c}\,u$ (d) $u' = \dfrac{1-v/c}{1+v/c}\,u$