JNU-ENTRANCE EXAMINATION- 2011

M.Sc. PHYSICS

Maximum Marks: 90

INSTRUCTIONS FOR CANDIDATES

1. All questions are compulsory.
2. For each question, one and only one of the four choices given is the correct answer.
3. Each correct answer will be given +3 marks.
4. Each wrong answer will be given -1 mark.
5. Use of calculator is permitted.

Q1. You are given the following transformation of the coordinates (x, y) of a point in the two-dimensional plane:

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

which of the following statements describes the transformation?

(a) Rotation by an angle of 45° about the z-axis followed by translation by one unit along both x and y axis

(b) Rotation by an angle of 45° about the z-axis followed by an expansion of both x and y coordinates by a factor of $\sqrt{2}$

(c) Translation by one unit along both x and y axes followed by an expansion of both the x and y coordinates by a factor of $\sqrt{2}$

(d) Only a rotation by an angle of 45° about the z-axis

Q2: The elements of the infinite sequence 0, 1, 1, $\frac{3}{2}$, …. satisfy the recursion relation $F_{n+1} = F_n + \frac{1}{2} F_{n-1}$, where F_n denotes the nth element. What is the value of $\lim\limits_{n\to\infty} F_n / F_{n-1}$?

(a) 1.366 (b) 1.575

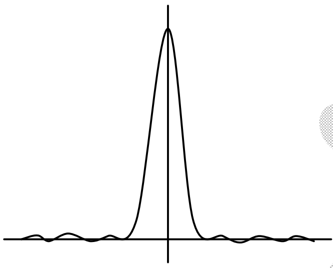
(c) 1.618 (d) The limit does not exist

Q3. For which of the following matrices both the eigenvalues are positive

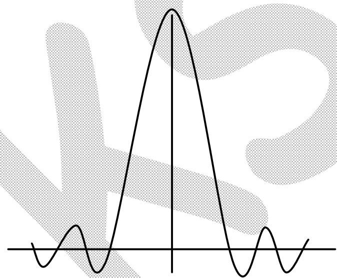
(a) $\begin{pmatrix} -7 & 2 \\ 2 & -7 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & 3 \\ -3 & 5 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 4 \\ 4 & -1 \end{pmatrix}$

Q4. Which of the following graphs gives the best representation of the real-valued function $y(x) = \frac{\sin x}{x}$ vs$\cdot x$?

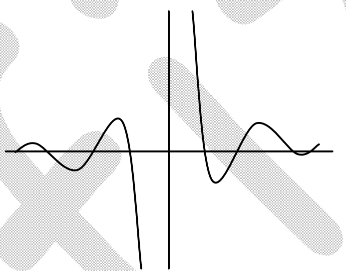
(a)



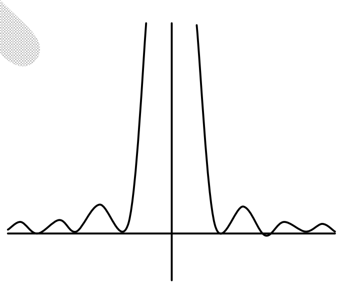
(b)



(c)



(d)



Q5. A particle of mass m moves along the x-axis under the influence of the potential $V(x) = V_0\left(e^{-2\alpha x} - 2e^{-\alpha x}\right)$. If the particle oscillates with small amplitude around the minimum of the potential, what is the period of oscillation? Take $m = 4$, $V_0 = 2$ and $\alpha = \frac{3}{4}$ (in appropriate units).

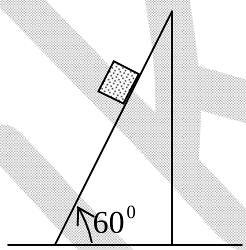
(a) 0.12 (b) 1.33 (c) 8.37 (d) 11.17

Q6. A simple pendulum, made of a point mass m attached to a massless string of length l, oscillates with a time period T_1. A second pendulum, in the shape of a thin uniform rod, has the same length l and the same total mass m and oscillates with the time period T_2. The thickness of the rod is negligible compared to l. The value of the ratio (T_2/T_1) is

(a) 0.44 (b) 0.67 (c) 0.82 (d) 1.00

Q7. A block of mass 1 kg is initially held at rest on the frictionless surface of a wedge of mass 5 kg (see figure). The wedge itself lies on a horizontal frictionless surface. The block is now released so that it is allowed to slide down the slope. How far down the slope does the block move when the wedge has moved a distance of 1 cm?

(a) 12.0 cm
(b) 10.0 cm
(c) 8.0 cm
(d) 6.9 cm



Q8. A planet of mass m moves in an orbit around the Sun (of mass M). The nearest and farthest distance from the Sun during the motion are a and b, respectively and G is the gravitational constant. The magnitude of the angular momentum of the planet around the Sun is [Hint: You may use conservation laws for energy and angular momentum]

(a) $m\sqrt{2GM(a+b)}$ (b) $m\sqrt{2GM}(ab)^{1/4}$

(c) $m\sqrt{2GM\left(\frac{ab}{a+b}\right)}$ (d) $m\sqrt{2GM\left(\frac{a^2+b^2}{a+b}\right)}$

Q9. A truck is moving along a straight highway with a speed of 108 km/hr towards a source (fixed on the highway) of microwave radiation of frequency 10 GHz. What will be the beat frequency if the radiation reflected by the truck is superposed with the emitted radiation?

(a) 0 kHz (b) 1 kHz (c) 2 kHz (d) 10 kHz

Q10. Consider the following four functions each representing a plane wave:

I. $f_1(x,y,z,t) = \sin\left(\sqrt{222}t - 2x - 7y + 13z\right)$

II. $f_2(x,y,z,t) = \sin\left(\sqrt{14}t - 2x - 3y - z\right)$

III. $f_3(x,y,z,t) = \sin\left(\sqrt{89}t - 4x - 3y + 8z\right)$

IV. $f_4(x,y,z,t) = \sin\left(\sqrt{125}t + 5x - 6y + 8z\right)$

which of the following pairs represents waves moving in mutually perpendicular directions?

(a) I and III (b) I and IV (c) II and III (d) II and IV

Q11. What is the maximum amount of work that can be extracted from an object of heat capacity C_p, initially at a temperature T_2, by a heat engine that operates between this object and a reservoir at temperature T_1?

(a) $C_p(T_2 - T_1)$

(b) $C_p(T_2 - T_1) - C_p T_1 \ln(T_2 / T_1)$

(c) $C_p(T_2 - T_1) \ln(T_2 / T_1)$

(d) $C_p(T_2 + T_1)/2$

Q12. Three concentric thin spherical shells of radii a, b and c ($a < b < c$) carry uniform surface electric charge of densities σ, $-\sigma$ and σ, respectively. The electric potential at the surface of the outermost shell is

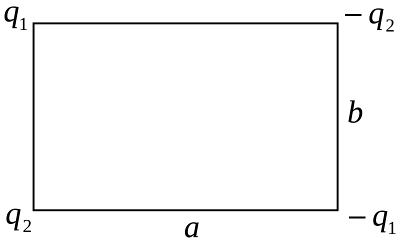
(a) $\frac{\sigma}{\varepsilon_0}(c - b + a)$

(b) $\frac{\sigma}{\varepsilon_0}\left(\frac{c^2}{a} - \frac{b^2}{a} + a\right)$

(c) $\frac{\sigma}{\varepsilon_0}\left(\frac{c^2}{b} - b + \frac{a^2}{b}\right)$

(d) $\frac{\sigma}{\varepsilon_0}\left(c - \frac{b^2}{c} + \frac{a^2}{c}\right)$

Q13. Four point charges $\pm q_1$ and $\pm q_2$ are placed at the corners of a rectangle of sides a and b as shown in the figure:



What is the magnitude of the dipole moment of the system?

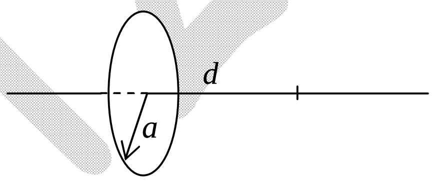
(a) $(q_1 + q_2)\sqrt{a^2 + b^2}$

(b) $(q_1 - q_2)(a - b)$

(c) $\sqrt{(q_1 + q_2)^2 a^2 + (q_1 - q_2)^2 b^2}$

(d) The dipole moment will depend on the choice of origin

Q14. A very thin circular wire of radius a carries electric charge of uniform linear density ρ. On its axis, the magnitude of the electric field attains its maximum value at a perpendicular distance d from the plane of the wire (see the figure below). The value of d is



(a) 0 (b) 0.32 a (c) 0.50 a (d) 0.71 a

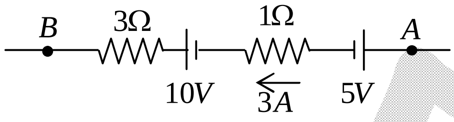
Q15. An electron with kinetic energy of 10 keV is moving in a circular orbit in a plane perpendicular to a uniform magnetic field of strength 2 tesla (= 2 × 10^4 gauss). The period of rotation is nearest to

(a) 2.8 × 10^{-12} s (b) 5.6 × 10^{-12} s (c) 1.8 × 10^{-11} s (d) 1.1 × 10^{-10} s

Q16. Consider a region of space in which there are uniform electric and magnetic fields which are mutually parallel. A charged particle released from rest in this region will move in a

(a) straight line (b) circle (c) helix (d) cycloid

Q17. A current of 3 amperes flows from A to B through the circuit shown below. The potential difference $V_A - V_B$ is

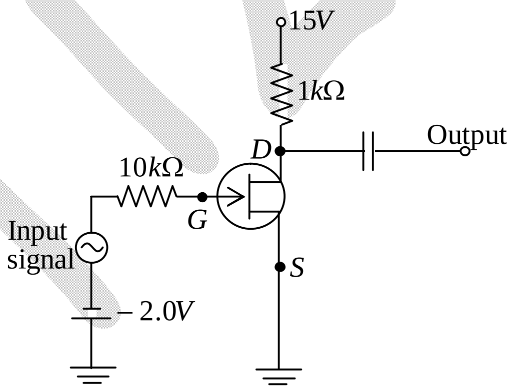


(a) +1 V (b) +7 V (c) – 11 V (d) +27 V

Q18. The peak value of the charge on the capacitor in an L-C oscillator is 10^{-5} coulomb. The values of L and C are 10^{-2} henry and 10^{-6} farad, respectively. The maximum value of energy stored in the inductor, during the oscillation, is nearest to

(a) 10 J (b) 5 J (c) 2×10^{-3} J (d) 5×10^{-5} J

Q19. A JFET amplifier has $I_{DSS} = 16\ mA$ and a pinch-off voltage $V_P = -4\ V$. If it is used in the circuit shown in the figure below, find the value of voltage amplification for small signals:



(a) -4.0 (b) -4.5 (c) -7.5 (d) -10.0

Q20. The isothermal compressibility $-\frac{1}{V}\frac{\partial V}{\partial p}$ of one mole van der Waals gas as a function of temperature and volume is (a and b are constants in the van der Waals equation of state)

(a) $V^2(V-b)/\left(k_B TV^2 - a(V-b)\right)$ (b) $V(V-b)^2/\left(k_B TV^2 - 2a(V-b)\right)$

(c) $V^2(V-b)^2/\left(k_B TV^3 - a(V-b)^2\right)$ (d) $(V-b)/k_B T$

Q21. The intensity of radiation emitted by the Sun has its maximum value at a wavelength of 500 nm. The corresponding value for the star Polaris is 380 nm. Assuming that these stars behave like blackbodies, the ratio (T_S / T_P) of the surface temperature of the Sun and Polaris is

(a) 1.70 (b) 1.32 (c) 0.76 (d) 0.58

Q22. A small block of copper with a specific of 100 calories/°C is initially at a temperature of 540° C. It is then dropped into a large tank of water maintained at 27°C. What is the change in entropy of the combined system after equilibrium is established?

(a) Decreases by 100 calories / K (b) Increases by 171 calories / K

(c) Decreases by 270 calories / K (d) Increases by 70 calories / K

Q23. Let v_x, v_y and v_z denote the components of the velocity along x, y and z directions, respectively, of an ideal particle. At the absolute temperature T, the average value of the product $v_x^2 v_y^2 v_z^2$ is proportional to

(a) T (b) $T^{3/2}$ (c) T^3 (d) T^6

Q24. An optical telescope of diameter 2 m is being used to observe stars. What is the order of magnitude of the minimum angular separation between two stars that can be resolved by this telescope?

(a) 10^{-5} (b) 10^{-7} (c) 10^{-9} (d) 10^{-11}

Q25. According to the uncertainty principle, the kinetic energy of an electron confined to a spherical region of volume 10^{-33} m^3 is of the order of

(a) 10^{-10} J (b) 10^{-12} J (c) 10^{-14} J (d) 10^{-16} J

Q26. In a Compton scattering process, a photon of wavelength λ is scattered off a charged particle of mass m (initially at rest) by an angle θ. If the final wavelength of the photon is λ', the difference $\lambda - \lambda'$

(a) depends on θ, but not on λ
(b) depends on λ, but not on m
(c) depends on both λ and θ
(d) depends on θ, but not on m

Q27. The time-dependent wave function of a particle of mass m moving in one dimension under the influence of a potential $V(x)$ is given to be

$$\psi(x,t) = \begin{cases} \alpha x e^{-\beta x} e^{i\gamma t/\hbar} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

where α, β and γ are real numbers. For $x > 0$, the potential $V(x)$ is of form (k_1 and k_2 are constants)

(a) $k_1 + \frac{k_2}{x}$ (b) $k_1 + \frac{k_2}{x^2}$ (c) $k_1 + k_2 x$ (d) $k_1 + k_2 x^2$

Q28. Consider an ensemble of quantum particles each of which can be in one of two states of energy E_1 and E_2. This system is in equilibrium at temperature T = 300 K. Let N_1 and N_2 denote the average number of particles in the two states. If the ratio N_2 / N_1 is $1/e$, the frequency of the radiation corresponding to transition between the two states is approximately

(a) 62×10^{9} Hz (b) 62×10^{11} Hz (c) 62×10^{13} Hz (d) 62×10^{15} Hz

Q29. A radioactive nucleus X decays to Y with a mean lifetime τ_1.The nucleus Y is also unstable and decays with mean lifetime τ_2 ($= \tau_1 / 2$). If N_0 nuclei of X (but no nuclei of Y) are present at t = 0, how many of Y will be there when the number of X nuclei becomes $N_0 / 2$?

(a) $N_0 / 2e$ (b) $N_0 / 4$ (c) $N_0 / 2$ (d) N_0 / e

Q30. A beam of light moves in a slab of glass of refractive index n in the positive x-direction. The slab itself is also moving in the positive x-direction with a speed v in the laboratory frame. What is the speed of the beam of light as measured in the laboratory frame?

(a) c (b) $(c^2 n + cv) / (c + nv)$

(c) $c\left(1 - \frac{1}{n}\right)$ (d) $(c^2 + vcn) / (cn + v)$