JNU PhD PAPER 2020

Case Study- 1 to 3

A sphere of radius R carries a polarization $\vec{P}(r) = k\vec{r}$ where k is a constant and $\vec{r}$ is the vector from the center of the sphere. Answer the following three questions for this problem.

Q1. The surface bound charge σ_b is:

(a) $\frac{kr}{4\pi R^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{kr}{4\pi R^2}$ (c) $kR\hat{r}$ (d) $kR\hat{r}$

Ans. (d)

Q2. The volume bound charge (ρ_b) is:

(a) $\frac{1}{4\pi\varepsilon_0}\frac{3k}{4\pi R^3}$ (b) $-3kr$ (c) $-3k$ (d) $9k^3r^3\hat{r}$

Ans. (c)

Q3. The electric field outside the sphere is:

(a) $4\pi kR^2$ (b) $\frac{4}{3}\pi kR^3 + 4\pi kR^2$

(c) 0 (d) $\frac{1}{3\epsilon_0}\vec{r}$

Ans. (c)

Q4. Consider the differential equation $\frac{d^2y}{dx^2} + \omega^2 y = 0$. The solution of this equation can be expressed in the series form as: $y(x) = \sum_n^\infty c_n x^n$. Which of the following is the correct recursion relation for the coefficients of this series?

(a) $c_{n+2} = -\frac{\omega^2}{(n+2)(n+1)}c_n$ (b) $c_n = -\frac{\omega^2}{n(n+1)}c_{n+1}$

(c) $c_n = \frac{\omega^2}{n(n-1)}c_{n-1}$ (d) $c_{n+2} = \frac{\omega^2}{(n+2)(n+1)}c_n$

Ans. (a)

Q5. For an atom with an electronic configuration np^2 (where n is the principal quantum number of a shell), the possible values of total angular momentum L and total spin S in the ground state are:

(a) $L=2$ and $S=0$ (b) $L=2$ and $S=1$

(c) $L=1$ and $S=1$ (d) $L=1$ and $S=0$

Ans. (c)

Q6. Which one of the following two-particle state $\psi(\vec{r}_1,\vec{r}_2)$ correctly describes two identical bosons in the plane wave states given by the wave-vectors $\vec{k}_1$ and $\vec{k}_2$?

(a) $\psi(\vec{r}_1,\vec{r}_2)=e^{i(\vec{k}_1.\vec{r}_1+\vec{k}_2.\vec{r}_2)}$ (b) $\psi(\vec{r}_1,\vec{r}_2)=e^{i\vec{k}_1.\vec{r}_1}+e^{i\vec{k}_2.\vec{r}_2}$

(c) $\psi(\vec{r}_1,\vec{r}_2)=e^{i(\vec{k}_1.\vec{r}_1+\vec{k}_2.\vec{r}_2)}+e^{i(\vec{k}_1.\vec{r}_2+\vec{k}_2.\vec{r}_1)}$ (d) $\psi(\vec{r}_1,\vec{r}_2)=e^{i(\vec{k}_1.\vec{r}_1+\vec{k}_2.\vec{r}_2)}-e^{i(\vec{k}_1.\vec{r}_2+\vec{k}_2.\vec{r}_1)}$

Ans. (c)

Q7. Electrons are ejected from calcium surface when monochromatic light of wavelength $488\,nm$ falls on it. The work function of calcium is $2.28\,eV$. What is the maximum kinetic energy of the emitted electron?

(Planck's constant, $h=4.14\times10^{-15}\,eV\,\text{sec}$; speed of light, $c=3\times10^{8}\,m/\text{sec}$)

(a) $0.026\,eV$ (b) $26\,eV$ (c) $2.6\,eV$ (d) $0.26\,eV$

Ans. (d)

Q8. Which one of the following is not true about the superconductors?

(a) Type II superconductors realize a mixed state between the critical magnetic field H_{c1} and H_{c2}.

(b) Type I superconductors, the penetration depth (λ) is smaller than the coherence length (ζ)

(c) According to BCS theory, the copper pairs are formed due to electron-phonon interaction

(d) Superconductivity is characterized by strongly paramagnetic behavior

Ans. (d)

Q9. Consider a vector $\vec{v} = x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3$ in a real three dimensional vector space spanned by three basis vectors $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$. Consider a new basis of three vectors: $\vec{b}_1 = \vec{a}_1, \vec{b}_2 = \vec{a}_1 + \vec{a}_2$, and $\vec{b}_2 = \vec{a}_1 + \vec{a}_2 + \vec{a}_3$. Let the vector $\vec{v}$ given above be denoted in this new basis as: $\vec{v} = y_1\vec{b}_1 + y_2\vec{b}_2 + y_3\vec{b}_3$. If the transformation matrix V between the components of the vector $\vec{v}$ in the two bases is defined as: $x_i = \sum_{j=1}^{3} V_{ij} y_j$ for $i = 1, 2, 3,$, then

(a) $V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (b) $V = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $V = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ (d) $V = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Ans. (b)

Q10. Which of the following expressions is correct for the Helmholtz free energy $F(T,V,N)$ of a thermodynamic system in canonical ensemble? Here, P is pressure, V is volume, N is the number of particles, μ is chemical potential, and T is temperature.

(a) $F = -PV + \mu N$ (b) $F = PV + \mu N$

(c) $F = -PV - \mu N$ (d) $F = \mu N$

Ans. (a)

Q11. Let the angular momentum eigenstates with quantum number j be denoted as $|j,m\rangle$, where $m = -j, -j+1, \ldots j-1, j$. For a system of two angular momenta j_1 and j_2, any state can be described as linear superposition of their product states $j_1, m_1\rangle |j_2, m_2\rangle$. For $j_1 = 1$ and $j_2 = \frac{1}{2}$, which of the following is the correct expression for the total angular momentum eigenstate with quantum number $j_{total} = \frac{3}{2}$ and $m_{total} = \frac{1}{2}$?

(a) $\left|j_{total}=\frac{3}{2},m_{total}=\frac{1}{2}\right\rangle=\frac{1}{\sqrt{3}}\left(|1,1\rangle|1/2-1/2\rangle+\sqrt{2}|1,0\rangle|1/2,1/2\rangle\right)$

(b) $\left|j_{total}=\frac{3}{2},m_{total}=\frac{1}{2}\right\rangle=\frac{1}{\sqrt{2}}\left(|1,1\rangle|1/2,-1/2\rangle+|1,0\rangle|1/2,1/2\rangle\right)$

(c) $\left|j_{total}=\frac{3}{2},m_{total}=\frac{1}{2}\right\rangle=|1,0\rangle|1/2,1/2\rangle$

(d) $\left|j_{total}=\frac{3}{2},m_{total}=\frac{1}{2}\right\rangle=|1,1\rangle|1/2,-1/2\rangle$

Ans. (a)

Q12. Consider a gas of N free electrons confined in a volume V. (m is the electron mass, $\hbar$ is Planck's constant and k_B is Boltzmann's constant)

Answer the following three questions on the free electron gas problem. What is the density of states for the free electrons?

(a) $\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{1/2}E^{3/2}$ (b) $\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)E^{3/2}$

(c) $\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E^{1/2}$ (d) $\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)E^{1/2}$

Ans. (c)

Q13. What is the Fermi energy in terms of N and V?

(a) $\left(\frac{3\pi^2N}{V}\right)^{1/2}$ (b) $\frac{\hbar^2}{2m}\left(\frac{3\pi^2N}{V}\right)^{\frac{1}{3}}$

(c) $\frac{\hbar^2}{2m}\left(\frac{3\pi^2N}{V}\right)^{\frac{2}{3}}$ (d) $\left(\frac{3\pi^2N}{V}\right)^{\frac{3}{2}}$

Ans. (c)

Q14. How does the specific heat $\left(C_V\right)$ of free electron gas vary with temperature $\left(T\right)$ at low temperature?

(a) $C_V \propto T^3$

(b) $C_V \propto e^{\frac{-\Delta}{k_B T}}$, where Δ is the energy gap

(c) $C_V \propto T^2$

(d) $C_V \propto T$

Ans. (d)

Case Study- 15 to 17

Consider the function $f\left(z\right)=e^{1/z}$ of a complex variable $z=x+iy$ in a complex plane. Answer the following three questions on this function

Q15. The function $f\left(z\right)=e^{1/z}$ has:

(a) no singularity at $z=0$

(b)an essential singularity at $z=0$

(c) a simple pole at $z=0$

(d) a branch point at $z=0$

Ans. (b)

Q16. Evaluate the integral $\oint dz e^{1/z}$ over the closed contour given by the unit circle $|z|=1$ centered around the origin of the complex plane.

(a) π (b) $i\pi$ (c) $i2\pi$ (d) 2π

Ans. (c)

Q17. The equation of the contour corresponding to a fixed value, A of the amplitude of the function $e^{1/z}$ is:

(a) $\left(x-\frac{1}{2\ln A}\right)^2+y^2=\frac{1}{4\left(\ln A\right)^2}$

(b) $\left(x+\frac{1}{2\ln A}\right)^2+y^2=\frac{1}{4\left(\ln A\right)^2}$

(c) $\left(x-\frac{1}{\ln A}\right)^2+y^2=\frac{1}{\left(\ln A\right)^2}$

(d) $\left(x+\frac{1}{\ln A}\right)^2+y^2=\frac{1}{\left(\ln A\right)^2}$

Ans. (a)

Q18. For a classical system described by a pair of canonical q and momentum p, consider the transformation $Q=-\sqrt{2p}\cos q$ and $P=\sqrt{2p}\sin q$. The Poissson bracket of the new variables Q and P is equal to:

(a) $-\cos 2q$ (b) $\cos 2q$ (c) 1 (d) 0

Ans. (c)

Case Study- 19 to 21

Answer the following three questions on the relativistic corrections to the hydrogen problem.

Q19. The leading relativistic correction to the kinetic energy term in the hydrogen atom Hamiltonian is:

(a) $\frac{p^4}{8m^3c^2}$ (b) $-\frac{p^3}{8m^3c^2}$ (c) $-\frac{p^4}{8m^3c^2}$ (d) $\frac{p^5}{8m^3c^2}$

Ans. (c)

Q20. The relativistic correction to the hydrogen atom problem leading to spin-orbit interaction is given by:

(a) $\xi(r)\vec{L}.\vec{S}$, where $\xi(r)\propto r$

(b) $\xi(r)\vec{L}.\vec{S}$, where $\xi(r)\propto r^{-3}$

(c) $\xi(r)\vec{L}.\vec{S}$, where $\xi(r)\propto r^{-2}$

(d) $\xi(r)\vec{L}.\vec{S}$, where $\xi(r)\propto r^{-1}$

Ans. (b)

Q21. The relavistic correction due to Darwin to the hydrogen atom problem is given by $\frac{1}{8\varepsilon_0}\left(\frac{\hbar e}{mc}\right)^2\delta(\vec{r})$ where $\delta(\vec{r})$ is Dirac delta function. Which of the following atomic states will be affected by the Darwin correction term?

(a) only $l=0$ states

(b) only $l=1$ states

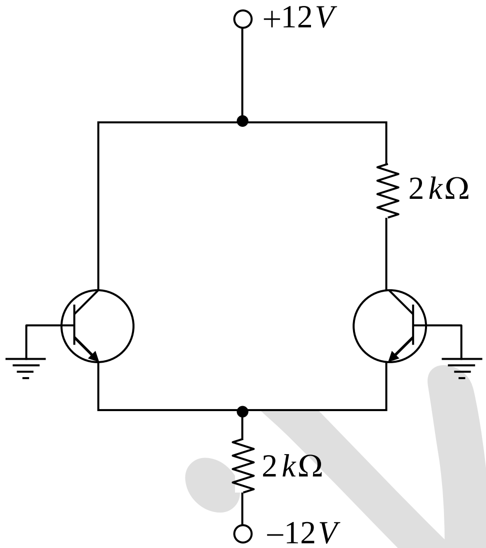
(c) only $l=2$ states

(d) All l states

Ans. (a)

Case Study- 22 to 24

For a single ended differential amplifier as given in the figure, answer the following three questions.



Q22. The tail current is:

(a) 5 mA (b) 10 mA (c) 6 mA (d) 8 mA

Ans. (c)

Q23. The value of emitter current is:

(a) 1 mA (b) 2 mA (c) 3 mA (d) 4 mA

Ans. (c)

Q24. The value of the collector voltage:

(a) 4V (b) 6 V (c) 8V (d) 10V

Ans. (b)

Q25. Which one of the following elements cannot be used as dopants in silicon to make it n-type semiconductor?

(a) Arsenic (b) Phosphorus

(c) Boron (d) Antimony

Ans. (c)

Q26. Consider a particle in a state given by the wavefunction $\psi(x,y,z)=(y+iz)^2$. This wavefunction is an eigenfunction of the angular momentum operator L_x with eigenvalues.

(a) $-2\hbar$ (b) $-\hbar$ (c) $+\hbar$ (d) $+2\hbar$

Ans. (d)

Q27. By doing an elastic scattering experiment with a beam of electrons of momentum $p \geq 120 MeV/c$, we can determine:

(Planck's constant, $h = 6.63\times10^{-34}\,J.s$; speed of light, $c = 3\times10^{8}\,m/s$; electro charge, $e = 1.6\times10^{-19}\,C$)

(a) the size of a biomolecule

(b) the lattice constants of a crystal of gold

(c) the size of an atomic nucleus

(d) none of the above

Ans. (c)

Case Study- 28 to 30

A "two-level" atom is considered to have only two energy levels with energies 0 and $\in$. For a system of N non-interacting two-level atoms with total energy E, answer the following three questions.

Q28. What is the number of microstates $\Omega(N,E)$?

(a) $\dfrac{N!}{\left(N+\dfrac{E}{\in}\right)!\left(\dfrac{E}{\in}\right)!}$ (b) $\dfrac{N!}{\left(N-\dfrac{E}{\in}\right)!\left(\dfrac{E}{\in}\right)!}$

(c) $\dfrac{N!}{\left(N-\dfrac{E}{\in}\right)!\left(N+\dfrac{E}{\in}\right)!}$ (d) $\dfrac{N!}{\left(N-\dfrac{\in}{E}\right)!\left(\dfrac{\in}{E}\right)!}$

Ans. (b)

Q29. What is the entropy per particle in the limit of large N ?

(a) $-k_B\left[\left(1-\frac{E}{N_\in}\right)\ln\left(1-\frac{E}{N_\in}\right)-\left(\frac{E}{N_\in}\right)\ln\left(\frac{E}{N_\in}\right)\right]$

(b) $+k_B\left[\left(1-\frac{E}{N_\in}\right)\ln\left(1-\frac{E}{N_\in}\right)+\left(\frac{E}{N_\in}\right)\ln\left(\frac{E}{N_\in}\right)\right]$

(c) $-k_B\left[\left(1-\frac{E}{N_\in}\right)\ln\left(1-\frac{E}{N_\in}\right)+\left(\frac{E}{N_\in}\right)\ln\left(\frac{E}{N_\in}\right)\right]$

(d) $+k_B\left[\left(1+\frac{E}{N_\in}\right)\ln\left(1+\frac{E}{N_\in}\right)-\left(\frac{E}{N_\in}\right)\ln\left(\frac{E}{N_\in}\right)\right]$

Ans. (c)

Q30. What is the corresponding temperature T ?

(a) $\frac{1}{T}=\frac{k_B}{\in}\ln\left(\frac{N_\in}{E}-1\right)$

(b) $\frac{1}{T}=\frac{k_B}{\in}\ln\left(\frac{N_\in}{E}+1\right)$

(c) $\frac{1}{T}=\frac{k_B}{\in}\ln\left(\frac{E}{N_\in}+1\right)$

(d) $\frac{1}{T}=\frac{k_B}{\in}\ln\left(\frac{E}{N_\in}-1\right)$

Ans. (a)

Q31. The decay $n\rightarrow p+e^-$ of a neutron (n) into a proton (p) and an electron (e^-) is forbidden due to the violation of conservation of:

(a) Angular momentum and baryon number

(b) Energy and lepton number

(c) Angular momentum and lepton number

(d) Electric charge and baryon number

Ans. (c)

Case Study- 32 to 34

Consider a crystalline material which, under ambient conditions, is given to have the FCC (face-centered cubic) lattice structure with monoatomic basis. Answer the following three questions for this system.

Q32. A primitive unit cell of the monoatomic FCC crystal contains:

(a) 1 atom (b) 2 atom (c) 3 atom (d) 4 atom

Ans. (a)

Q33. The phonon dispersion of a monoatomic FCC crystal has:

(a) 3 branches of acoustic phonons only.

(b) 3 branches of acoustic phonons, and 9 branches of optical phonons.

(c) 1 branches of acoustic phonons, and 2 branches of optical phonons.

(d) 3 branches of optical phonons only

Ans. (a)

Q34. Suppose by changing the temperature, if the crystal structure of the material changes from the monoatomic FCC to monoatomic BCC (body-centered cubic), then the number of optical phonon branches will change by:

(a) 0 (b) 2 (c) 3 (d) 6

Ans. (a)

Case Study- 35 to 37

Answer the following three questions on the semi-empirical formula for the binding energy of atomic nuclei in terms of the nuclear mass number A and the proton number Z

Q35. In the formula for binding energy per nucleon, the volume energy term is

(a) a constant (b) proportional to Z

(c) proportional to A (d) proportional to $A^{1/3}$

Ans. (a)

Q36. In the formula for binding energy per nucleon, the contribution from the Coulomb repulsion between protons is:

(a) proportional to Z only (b) proportional to $Z(Z-1)$ only

(c) proportional to $Z(Z-1)A^{-1/3}$ (d) proportional to $Z(Z-1)A^{-4/3}$

Ans. (d)

Q37. In the formula for binding energy per nucleon, the pairing energy term is:

(a) always zero

(b) zero only when A is an odd integer

(c)non-zero when A is an odd integer

(d) always non-zero

Ans. (b)

Q38. If the scalar and vector potentials are given by $\phi(\vec{r},t)=0$ and $\vec{A}(\vec{r},t)=-\frac{1}{4\pi\epsilon_0}\frac{qt}{r^2}\hat{r}$, the corresponding electric field $(\vec{E})$ is:

(a) 0 (b) $\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$ (c) $\frac{1}{4\pi\epsilon_0}\frac{q}{r}\hat{r}$ (d) $-\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

Ans. (b)

Case Study- 39 to 41

A body of mass m is thrown up vertically with an initial speed u. The air exerts a drag force $-kv$ upon it, where v is the instantaneous velocity of the body and k is a constant. The body also experiences gravitational acceleration g.

Answer the following questions on this problem.

Q39. What is the terminal speed attained by the body?

(a) $\frac{mg}{k}$ (b) $\frac{g}{k}$ (c) $\frac{k}{mg}$ (d) u

Ans. (a)

Q40. What is the time it will take to attain the maximum height?

(a) $\ln\left(1+\frac{mg}{ku}\right)$ (b) $\frac{k}{m}\ln\left(1+\frac{ku}{mg}\right)$

(c) $\frac{m}{k}\ln\left(1+\frac{ku}{mg}\right)$ (d) $\frac{m}{k}\ln\left(1+\frac{mg}{ku}\right)$

Ans. (c)

Q41. What is the maximum height attained by the body?

(a) $\frac{mu}{k}+g\left(\frac{m}{k}\right)^2\ln\left(1+\frac{ku}{mg}\right)$ (b) $\frac{mu}{k}-g\left(\frac{m}{k}\right)^2\ln\left(1+\frac{ku}{mg}\right)$

(c) $\frac{mu}{k}-g\left(\frac{m}{k}\right)^2\ln\left(1-\frac{ku}{mg}\right)$ (d) $\frac{mu}{k}+g\left(\frac{m}{k}\right)^2\ln\left(1-\frac{ku}{mg}\right)$

Ans. (b)

Q42. The Fourier transformation for a function $f(x)$ of a real variable x can be defined as:

$f(x)=\int_{-\infty}^{+\infty}dke^{ikx}g(k)$, where $g(k)$ is a function of another real variable k. If $g(k)=e^{iky}$ for a given y, then what is $f(x)$?

(a) $\delta(x+y)$ (b) $\delta(x-y)$

(c) $2\pi\delta(x+y)$ (d) $2\pi\delta(x-y)$

Ans. (c)

Q43. In spectroscopy, the selection rule for transition between the rotational energy levels of a diatomic molecule (given by the rotational quantum number J) states that the transition between two rotational levels is allowed if:

(a) $\Delta J=\pm 1$ (b) $\Delta J=\pm 2$

(c) $\Delta J=0$ (d) None of the above

Ans. (a)

Q44. For a classical system described by the Hamiltonian $H(q,p)$ in terms of the generalized coordinates q and p, the Hamilton's equation of motion (in the standard notation) are:

(a) $\dot{q}=\frac{\partial H}{\partial p}$, $\dot{p}=\frac{\partial H}{\partial q}$ (b) $\dot{q}=-\frac{\partial H}{\partial p}$, $\dot{p}=-\frac{\partial H}{\partial q}$

(c) $\dot{q}=\frac{\partial H}{\partial p}$, $\dot{p}=-\frac{\partial H}{\partial q}$ (d) $\dot{q}=-\frac{\partial H}{\partial p}$, $\dot{p}=\frac{\partial H}{\partial q}$

Ans. (c)

Q45. For a thermodynamic system of N particles at temperature T, which of the following relation is correct for the change in entropy S with respect to volume V?

(a) $\left(\frac{\partial S}{\partial V}\right)_{T,N}=-\left(\frac{\partial P}{\partial T}\right)_{V,N}$ (b) $\left(\frac{\partial S}{\partial V}\right)_{T,N}=\left(\frac{\partial P}{\partial T}\right)_{V,N}$

(c) $\left(\frac{\partial S}{\partial V}\right)_{T,N}=\left(\frac{\partial T}{\partial P}\right)_{S,N}$ (d) $\left(\frac{\partial S}{\partial V}\right)_{T,N}=-\left(\frac{\partial T}{\partial P}\right)_{S,N}$

Ans. (b)

Q46. A spin $\frac{1}{2}$ particle in a magnetic field B pointing along y-direction is described by Hamiltonion $H=\mu_b B\sigma_y$, where σ_y is the Pauli matrix corresponding to the y component of the spin $\frac{1}{2}$ operator (and μ_B is the Bohr magneton). For this system, the time evolution operator $e^{-iHt/\hbar}$ can be written as:

(a) $\begin{bmatrix}\cos\left(\frac{\mu_B Bt}{\hbar}\right) & -\sin\left(\frac{\mu_B Bt}{\hbar}\right)\\ -\sin\left(\frac{\mu_B Bt}{\hbar}\right) & \cos\left(\frac{\mu_B Bt}{\hbar}\right)\end{bmatrix}$ (b) $\begin{bmatrix}\cos\left(\frac{\mu_B Bt}{\hbar}\right) & i\sin\left(\frac{\mu_B Bt}{\hbar}\right)\\ -i\sin\left(\frac{\mu_B Bt}{\hbar}\right) & \cos\left(\frac{\mu_B Bt}{\hbar}\right)\end{bmatrix}$

(c) $\begin{bmatrix}\cos\left(\frac{\mu_B Bt}{\hbar}\right) & \sin\left(\frac{\mu_B Bt}{\hbar}\right)\\ \sin\left(\frac{\mu_B Bt}{\hbar}\right) & \cos\left(\frac{\mu_B Bt}{\hbar}\right)\end{bmatrix}$ (d) $\begin{bmatrix}\cos\left(\frac{\mu_B Bt}{\hbar}\right) & -\sin\left(\frac{\mu_B Bt}{\hbar}\right)\\ \sin\left(\frac{\mu_B Bt}{\hbar}\right) & \cos\left(\frac{\mu_B Bt}{\hbar}\right)\end{bmatrix}$

Ans. (d)

Case Study- 47 to 49

Consider the one-dimensional simple harmonic oscillator of mass m and frequency ω described by the Hamilton, $H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 x^2 = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$, with eigenvalues $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$ and eigenstates $|n\rangle$. The creation and annihilation operators a^+ and a are related to the coordinate x and momentum p as: $x = \sqrt{\frac{\hbar}{2m\omega}}\left(a^\dagger + a\right)$ and $p = i\sqrt{\frac{m\hbar\omega}{2}}\left(a^\dagger - a\right)$. Answer the following three questions on this problem.

Q47. The commutator $\left(a^\dagger a, a^\dagger a^\dagger\right)$ is equal to:

(a) $-2a^\dagger a^\dagger$ (b) $2a^\dagger a$ (c) $2a^\dagger a^\dagger$ (d) $-2a^\dagger a$

Ans. (c)

Q48. What is the uncertainty in position, $\sqrt{\langle x^2\rangle - \langle x\rangle^2}$, in the eigenstate $|n\rangle$?

(a) $\sqrt{\frac{\hbar}{m\omega}(2n+1)}$ (b) $\sqrt{\frac{\hbar}{m\omega}\left(n+\frac{1}{2}\right)}$

(c) 0 (d) $\sqrt{\frac{\hbar}{2}}$

Ans. (b)

Q49. Which of the following is the correct expression for the creation operator?

(a) $\sqrt{n+1}|n\rangle\langle n+1|$ (b) $\sum_{n=0}^{\infty}\sqrt{n+1}|n+1\rangle\langle n|$

(c) $\sum_{n=0}^{\infty}\sqrt{n}|n\rangle\langle n+1|$ (d) $\sqrt{n}|n\rangle\langle n-1|$

Ans. (b)

Q50. Consider a rectangular waveguide with a cross-section a dimension $2\,cm \times 1\,cm$. If the driving frequency is $1.7\times10^{10}\,Hz$, the transverse Electric (TE) mode that will propagate in this wave guide is:

(a) $0.53\times10^{10}\,Hz$ (b) $0.75\times10^{10}\,Hz$

(c) $1.9\times10^{10}\,Hz$ (d) $1.4\times10^{9}\,Hz$

Ans. (b)