Part-A

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Q1. Consider the quantum mechanical motion of a free particle of mass m in a one-dimensional box of length L. What is the expectation value of its kinetic energy in the state with wave function $\psi(x) = Cx(L-x)$ where C is a constant of normalization?

(a) $\frac{(h^2)}{(mL^2)}$ (b) $\frac{(5h^2)}{(4\pi^2 mL^2)}$

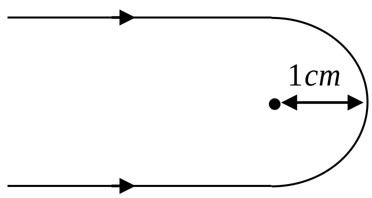
(c) $\frac{(2h^2)}{(3\pi^2 mL^2)}$ (d) $\frac{(4h^2)}{(3\pi^2 mL^2)}$

Q2. Consider a system of three interacting spins S_1, S_2 and S_3-each of which can have values $+1$ and -1. The Hamiltonian is given by $H = -J(S_1S_2 + S_2S_3 + S_1S_3)$. If T is the temperature and k_B is the Boltzmann constant, define $\beta = \frac{1}{k_BT}$. What is the thermally averaged value of $(S_1 + S_2 + S_3)$?

(a) $\frac{(\exp(4\beta J)-1)}{(\exp(4\beta J)+3)}$ (b) $\frac{\exp(3\beta J)}{(\exp(3\beta J)+2)}$

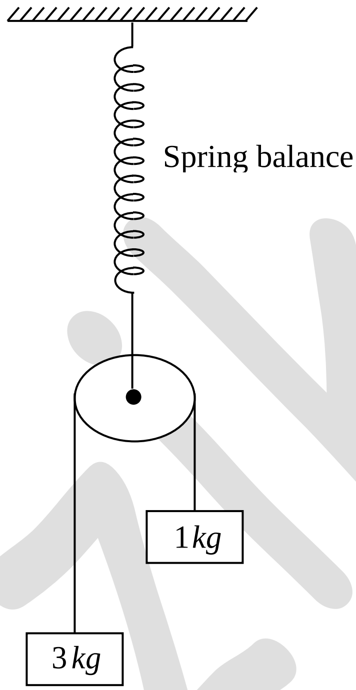
(c) $\frac{(\exp(3\beta J)+3)}{(\exp(3\beta J)+5)}$ (d) $\frac{\exp(4\beta J)}{(\exp(4\beta J)+5)}$

Q3. Consider an infinite wire bent into the shape of hairpin i.e. semicircle of radius $1cm$ connecting two semi-infinite wires (as shown in the figure) and carrying a current of $1.5A$. what is the strength of the magnetic field at the centre of the semicircle in the unit of Tesla?



(a) 1.3×10^{-7} (b) 2.9×10^{-5} (c) 7.7×10^{-5} (d) 4.6×10^{-6}

Q4. In the Atwood's machine shown here the pulley of negligible mass is suspended from a spring balance with a spring constant of $2000\,N/m$. Two bodies, of mass $1kg$ and $3kg$, are hanging from the two ends of the massless and frictionless rope passing over the pulley. When the two masses starts moving under the effect of gravity how much will the spring stretch from its length in the absence of any force?



(a) $1.0\,cm$ (b) $0.5\,cm$ (c) $2.0\,cm$ (d) $1.5\,cm$

Q5. What is the integral of the function $f(z)=\frac{1}{(z^2-z)}$ in the complex plane around the circle of radius 2 with the origin at its centre (The direction of integration is positive)?

(a) 0 (b) $2i\pi$ (c) $-2i\pi$ (d) 4π

Q6. Consider the differential equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=0$. If, at $x=0, y=0$ and $\frac{dy}{dx}=1$, what is the value of y at $x=1$?

(a) -1.4 (b) 0.56 (c) 0.23 (d) 0.37

Q7. Two waves are superimposed to produce a resultant wave represented by $y = a\sin(\omega t - kx) + a\sin(\omega' t - k'x)$ with $k > k'$ and $\omega > \omega'$, where k and ω are are wave vector and frequency respectively. What will be their phase velocity (v) and how it is related with group velocity (u) and phase velocity (v)

(a) $\left(\frac{\omega+\omega'}{k-k'}\right)$ and $u = v - k\left(\frac{dv}{dk}\right)$

(b) $\left(\frac{\omega+\omega'}{k+k'}\right)$ and $u = v + k\left(\frac{dv}{dk}\right)$

(c) $\left(\frac{\omega-\omega'}{k+k'}\right)$ and $u = v + k\left(\frac{du}{dk}\right)$

(d) $\left(\frac{\omega-\omega'}{k-k'}\right)$ and $u = v - k\left(\frac{du}{dk}\right)$

Q8. A particle of mass m starts from rest at $x_0\,(x_0 > 0)$ in an attractive inverse cubic force field $F = -\frac{k}{x^3}\,(k > 0)$. How does the time taken depend on x_0?

(a) x_0^4 (b) x_0^2 (c) x_0 (d) x_0^3

Part-B

Q9. The ground state energy of a neutral lithium atom is $-203.2eV$. If the first ionization energy of lithium is $5.39\,eV$, what is its second ionization energy?

(a) $122.4\,eV$ (b) $75.4\,eV$ (c) $197.8\,eV$ (d) $192.4\,eV$

Q10. Consider a $100\,g$ block of very good insulator. If Q_1 and Q_2 denote the amount of energy required to raise the temperature of this block from $2K$ to $4K$, and from $4K$ to $6K$, respectively, what is the ratio $\frac{Q_2}{Q_1}$?

(a) 4.3 (b) 2.7 (c) 1.0 (d) 1.7

Q11. At the bottom of the conduction band of a semiconductor, corresponding to the wave-vector k_0, the band structure is given by $E(k) = A|k - k_0|^2$ where A is a constant. If a magnetic field B is applied on a sample of this semiconductor what will be the cyclotron frequency ω_c (m is mass of electron in free free space)?

(a) $\frac{\pi eBA}{2h^2}$ (b) $\frac{eB}{m}$

(c) $\frac{eB}{2m}$ (d) $\frac{8\pi^2 AeB}{h^2}$

Q12. A $10V$ unregulated DC power source is used to make a $6V$ regulated power supply using a $6V$ Zener diode. The maximum current that can be passed through the diode is $100\,mA$. Then the value of the series resistance R_z and its wattage are:

(a) $30\,\text{ohm}$ and $0.3W$ (b) $50\,\text{ohm}$ and $0.5W$

(c) $60\,\text{ohm}$ and $0.6W$ (d) $40\,\text{ohm}$ and $0.4W$

Q13. The potential energy of an isotropic harmonic oscillator in three dimensions is given by $V(x, y, z) = \left(\frac{1}{2}\right)k\left(x^2 + y^2 + z^2\right)$ where k is a positive constant. The degeneracy factor for the third excited energy level is:

(a) 10 (b) 7 (c) 6 (d) 3

Q14. The speed of a particle increases from $0.6c$ to $0.8c$, where c is the speed of light in vacuum. In this process the kinetic energy increases by a factor of:

(a) 1.8 (b) 2.7 (c) 1.3 (d) 3.6

Q15. In a photoelectric tube light of wavelength $500\ nm$ is incident upon a metal surface with a work function of $1.9\,eV$. What is the minimum absolute value of the negative potential (with respect to the electrode emitting the photoelectrons) that has to be applied to the anode so that there is no photocurrent ?

(a) $1.9V$ (b) $0.4V$ (c) $3.8V$ (d) $0.6V$

Q16. In a two body central force problem the initial position and velocity of one of the particles, of mass $1kg$, are given by $(0,0,2)m$ and $(1.6,1.6,0)m/s$. For the other particle of mass $2kg$, the initial position and velocity vectors are $(2,0,0)m$ and $(0,2,1)m/s$. During the motion the relative position vector always lies in a plane. Which of the following vectors are perpendicular to this plane?

(a) $(3,2,-2)$ (b) $(-2,2,3)$

(c) $(2,3,2)$ (d) $(-3,3,2)$

Q17. A positronium is a bound state of an electron and positron. What is the energy required to excite it from the ground state to the first excited state?

(a) $5.1eV$ (b) $10.2eV$ (c) $2.55eV$ (d) $20.4eV$

Q18. In a Penning trap a static potential difference is applied between two electrodes in an evacuated region to produce the potential field $V(x,y,z)=c(az^2-x^2-y^2)$, where c and a are constants,. What must be the value of a?

(a) 1 (b) -1 (c) 0 (d) 2

Q19. In a system of three identical non-interacting bosons each particle can occupy one of three single-particle energy state of energy E, $2E$ and $3E$. What is the number of energy eigenstates for the combined system of three particles?

(a) 10 (b) 27 (c) 3 (d) 1

Q20. A satellite of mass $60\,kg$ is going around a planet in an elliptical orbit with an angular momentum of $5\times10^{12}\,kg-m^2/s$. If the time taken to go once around the orbit is 2 hrs, the area of the orbit is:

(a) $2\times10^9\,km^2$ (b) $5\times10^8\,km^2$

(c) $3\times10^8\,km^2$ (d) $9\times10^7\,km^2$

Q21. In a LRC resonance circuit $L=500\,mH, R=2\,\text{ohm}$ and $C=20\,\mu F$. It is driven by a voltage of the form $V(t)=V_0\cos(\omega t)$. What is the full width at half maximum in the resonance curve of this circuit (in radians/s)?

(a) 1.2 (b) 4.0 (c) 2.3 (d) 6.7

Q22. What is the total translational kinetic energy of all the air molecules in a room of dimensions $3m \times 5m \times 5m$ at standard temperature and pressure?

(a) $1.1 \times 10^7\ J$ (b) $2.9 \times 10^6\ J$

(c) $6.5 \times 10^3\ J$ (d) $3.4 \times 10^5\ J$

Q23. Cerenkov radiation is emitted when a charged particle moves faster than the speed of light in the medium in which the particle is moving. What is the minimum kinetic energy of an electron moving in water so that it may emit Cerenkov radiation?

(a) $0.68\,MeV$ (b) $0.38\,MeV$ (c) $0.51\,MeV$ (d) $0.26\,MeV$

Q24. For a $n \times n$ square matrix A for which A^2 equals the $n \times n$ identity matrix I, the value of $\exp(i\pi A)$ is:

(a) -1 (b) $-A$ (c) $I + iA$ (d) $-I + iA$

Q25. A proton is injected into a region of space with a uniform magnetic field of strength 10^{-4} Tesla with a speed of $10^7\ m/s$ and at an angle of 60^0 with the direction of the magnetic field. The resulting motion is helical. How much does it move along the axis of this helix during each full rotation around the axis?

(a) $540\,m$ (b) $3300\,m$ (c) $8200\,m$ (d) $15400\,m$

Q26. The normalized wave function of the electron in a hydrogen atom is given to be $\psi(r) = C\psi_{1,0,0}(r) + \left(\frac{3}{5}\right)\psi_{2,1,-1}(r)$ where C is a suitable constant of normalization and $\psi_{n,l,m}(r)$ denotes a normalized energy eigen function of the electron in the hydrogen atom in the standard notation. What is the expectation value of the energy of the electron for this state in the unit of eV?

(a) -12.8 (b) -14.2 (c) -7.6 (d) -9.9

Q27. A light of wavelength $500\,nm$ falls on a metal surface which has a work function of $1.9\,eV$. What is the photon energy of incident electron and the value of stopping potential?

(a) $1.9\,eV$ and $1.9\,eV$ (b) $2.48\,eV$ and $0.58\,eV$

(c) $4.96\,eV$ and $3.8\,eV$ (d) $1.24\,eV$ and $0.29\,eV$

Q28. Laser light with power output $18W/cm^2$ is normally incident on a non-reflecting surface of exposed surface area $20\,cm^2$. Calculate the total energy delivered to the exposed surface in 30 minutes and the total momentum delivered.

(a) $6.48\times10^5\,J$ and $2.16\times10^{-3}\,kg.m.s^{-1}$
(b) $540\,J$ and $0.54\times10^{-3}\,kg.m.s^{-1}$
(c) $360\,J$ and $1.08\times10^{-3}\,kg.m.s^{-1}$
(d) $18\,J$ and $0.27\times10^{-3}\,kg.m.s^{-1}$