JNU (Ph.D.) ENTERANCE 2016

PART - A

QA1. Use the series method to obtain the general solution of

$$x\frac{d^2u}{dx^2}+3\frac{du}{dx}+4x^3u=0$$

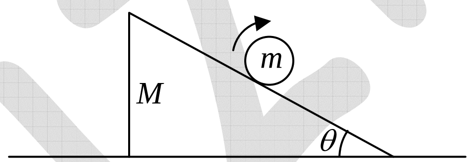
about $x=0$

QA2. A positronium atom is a bound state of an electron and its antiparticle (positron)

(a) Using the information that the energy levels of a hydrogen-like atom are proportional to the reduced mass, calculate the ground state energy for positronium. (The corresponding value for a hydrogen atom is $-13.6\ eV$)

(b) For this electron-positron system, what are the possible spin wave functions for the simultaneous eigenstates of S^2 and S_z?

QA3. A ball of mass m rolls down a rough wedge of mass M and angle θ. The wedge can also slide on a smooth horizontal table. Obtain Lagrange's equations for the appropriate generalized coordinates



QA4. A physical system is made up of a large number (N) of identical and non-interacting magnetic dipoles. In a magnetic field of strength H, the possible orientations of a dipole are such that its energy levels are given by $n\mu_B H$, where the values of n are $-J,-J+1,-J+2,..,J-2,J-1,J$. Here $J\geq 2$ and μ_B is positive constant.

(a) Evaluate the partition function of the system at a temperature T

(b) Calculate the entropy

QA5. Find the electric field (E), magnetic field (B), charge density (ρ) and current density (J) corresponding to following scalar (ϕ) and vector (A) potentials

$$\phi(r,t)=0, A(r,t)=-\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$$

where symbols have their usual their usual meanings,

use the gauge function, $\lambda=-\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{qt}{r}$ to transform the potentials. What do the potentials physically represent?

PART -B

QB1. Compute the eigenvalues to order α^2 of the matrix

$$\begin{pmatrix} 1+\alpha+\alpha^2 & \alpha+\alpha^2 & \alpha+\alpha^2 \\ \alpha+\alpha^2 & \alpha+\alpha^2 & \alpha+\alpha^2 \\ \alpha+\alpha^2 & \alpha+\alpha^2 & -1+\alpha+\alpha^2 \end{pmatrix}$$

Hint: You may use perturbation theory.

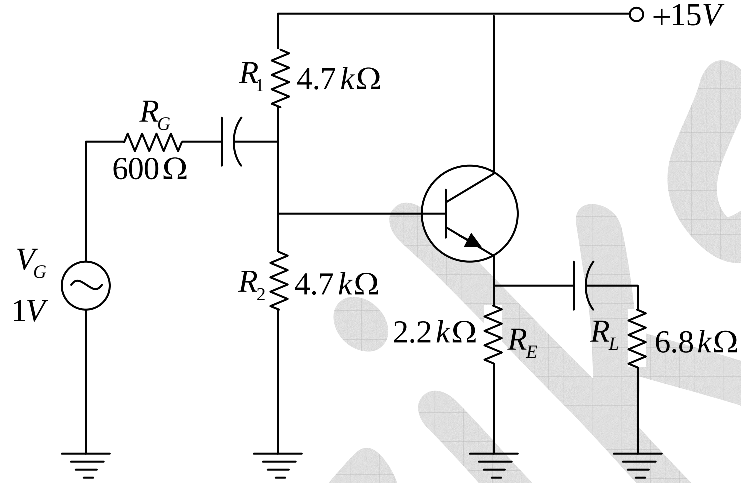
QB2. Consider a hydrogen atom in a weak electrostatic field $E\hat{z}$. It can be described by the Hamiltonian $H=H_0+V$, where H_0 denotes the hydrogen atom Hamiltonian, and $V=-eEr\cos\theta$ is the perturbation due to the electrostatic field. Here (r,θ,φ) denote spherical polar coordinates. Calculate the first-order corrections to the ground state energy of the hydrogen atom.

QB3. Consider two elementary particles, each of mass m and speed $0.6c$. They undergo a headlong collision and fuse together. Calculate the mass M of the composite particle.

QB4. Consider an extreme relativistic gas of mono-atomic molecules with the energy-momentum relationship $E=|p|c$, where c is the speed of light. Obtain the partition function in the (classical) canonical ensemble.

QB5. A grounded conducting sphere of radius a is placed in a uniform electric field $E = E\hat{z}$. The general form of the potential outside the sphere is $V(r,\theta,\varphi) = \left[Ar + \frac{B}{r^2}\right]\cos\theta$. Evaluate A and B

QB6. What is the voltage gain (A_V) of the emitter-follower circuit given below?



QB7. Consider a linear chain in which the force constants between nearest neighbour atoms are α and 10α alternately. Let the masses be equal and the nearest neighbour separation be $\frac{a}{2}$. For the normal modes, calculate $\omega(k)$ at $k = 0$ and $\frac{\pi}{a}$ and sketch the dispersion relation.

QB8. The bulk n type and P type semiconductors of a junction have conductivities of $10^4\ mho/m$ and $10^2\ mho/m$ respectively. Find the contact potential difference across the junction at room temperature $(300\,K)$. The intrinsic density (n_i) of both carries is 2.5×10^{19} per m^3 and the respective nobilities are $\mu_n = 0.38\,m^2/V-s$ and $\mu_p = 0.18\,m^2/V-s$.

QB9. The vibrational energy levels of a diatomic molecule are given by

$$E_n = \hbar\left(\frac{k}{\mu}\right)^{\frac{1}{2}}\left(n+\frac{1}{2}\right), n = 0,1,2,...$$

where μ is the effective mass and k is the force constant. Establish an expression for the observed infrared spectrum of a diatomic molecule

QB10. Write down the normalized Maxwell-Boltzmann velocity distribution for a gas at temperature T. Compute the average speed of a particle.