JNU PHD 2018

PART - A

NOTE: Answer all questions. Each question carries 10 marks

QA1. Consider the contour integral

$$I = \oint_C \frac{e^z dz}{i(1-2z)(z-2)}$$

where the contour C is given by $|z| = 1$

(a) using the residue theorem or otherwise, evaluate I

(b) Using (a), evaluate the integral

$$I_1 = \int_0^{2\pi} \frac{e^{\cos\theta} \sin(\sin\theta)}{5-4\cos\theta} d\theta$$

QA2. A particle of mass m moves in one dimension, in the region $x \geq 0$, under the influence of the potential

$$V(x) = e^{-\alpha x} - e^{-\beta x}; \alpha > \beta > 0$$

(a) Sketch $V(x) vs.x.$

(b) Show that there is only on point of equilibrium and it is a stable point

(c) What is the angular frequency (ω) of small oscillation around this point of stable equilibrium?

QA3. Consider a non-relativistic particle of mass m moving along, the x-axis in a potential $V(x)$. If the motion is bounded between the points x_1 and x_2, the Bohr-Sommerfeld quantization condition reads as $\int_{x_1}^{x_2} p(x)dx = \frac{1}{2}\left(n+\frac{1}{2}\right)h$, where $n = 0,1,2,...$ If $V(x) = \alpha x^4$, where α is a positive constant, show that, for large n, the nth energy level E_n is proportional to $n^{4/3}$. [Hint: In the integral change the variable from x to αx with an appropriate choice of α.]

QA4. Consider a system of two identical bosons that do not interact with each other. However, due to interaction with external fields, each of them has energy eigenstates with energies $\varepsilon, 2\varepsilon$ and 3ε. If the combined system is at temperature T, what is the mean square fluctuation $\left(\langle E^2\rangle - \langle E\rangle^2\right)$ of the total energy of the two particles? How would this result be modified if the two particles are identical fermions?

QA5. The electric potential of a charge distribution is given by

$$V(r) = B\frac{e^{-\lambda}}{r}$$

Where B and λ are the positive constants

(a) Calculate the electric field $E(r)$.

(b) Calculate the total charge $Q(r)$ within a sphere of radius r (centre of the sphere is at the origin).

(c) What are the limiting values of $Q(r)$ as (i) r goes to 0 and (ii) r goes to ∞?

(d) Interpret the results of part (c) physically.

QA6. Estimate the numerical value of the Fermi energy for conduction electrons in silver. The mass number of silver is 107.9 and its density is $10.49\, g/cm^3$. State your assumptions clearly.

PART – B

NOTE: Answer all questions. Each question carries 4 marks

QB1. Calculate the de-Broglie wavelength of an electron of kinetic energy $100\, eV$.

QB2. Calculate the determinant of the matrix $e^{i\omega A}$, where $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Here ω is a real number.

[Hint: Write down the definition of $e^{i\omega A}$ in terms of a power series in A and calculate the matrix A^2.]

QB3. The first ionization potential (derived from the formula $E = Z^2 e^2 / 2a_0$) for hydrogen is $13.6\, eV$. For helium, the experimentally measured value of the first ionization potential is $24.6\, eV$. What is the ground state energy of a neutral He atom?

QB4. A $10V$ unregulated DC power source and a $6V$ Zener diode are used to make a $6V$ regulated power supply. The maximum current that can be passed through the diode is $100\,mA$. Calculate the series resistance R_z and its wattage.

QB5. A rocket starts from rest and is propelled forward by ejecting the burnt fuel. Assuming that the burnt residues are ejected at a constant speed v_e (with respect to the rocket), calculate the final speed of the rocket when all the fuel has been used up. Initially, the mass of the fuel is half the mass of the rocket (without the fuel.)

QB6. The mean lifetime of a π^+ meson in its rest frame is $2.5\times10^{-8}\,s$. If a burst of π^+ mesons travels with a speed of $0.8c$ (c being the speed of light in vaccum), how far would it travel during its mean life in the laboratory frame?

QB7. X -rays with wavelength of $\lambda = 1\overset{o}{A}$ are scattered from a carbon block. The Compton shift observed is $\Delta\lambda = 0.025\,\overset{o}{A}$. What is the kinetic energy (in eV) imparted to the recoiling electron?

QB8. Calculate the electric polarization of helium gas with number density of $2.6\times10^{25}/m^3$, if it is subjected to an electric filed of $6\times10^5\,V/m$. (The atomic polarizability of helium is $0.23\times10^{-40}\,F-m^2$.)

QB9. A particle of mass m is moving in one dimension in a potential given by $V(x)$, where $V(x)=\infty$ for $x<0$ and is equal to $\frac{1}{2}m\omega^2x^2$ for $x\geq 0$. Using the results for a simple harmonic oscillator, calculate the energy levels of this system.

QB10. Consider a face-centered cubic lattice. Suppose you place identical spheres at each point of this lattice (with the centres of the spheres coinciding with the lattice points). Volume fraction is defined to be the volume of space occupied by the spheres divided by the total volume. What is its maximum possible value in the present case?